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Ibn Khaldoun University **Tiaret**

Faculty of Applied Sciences

Department of Mechanical Engineering

Course Handbook :

Notions of electrical and electronic measurements

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This course is intended for **second-year** students.

Specialization : **Electromechanics**

Domain : Science and Technology

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Preface

Measurement is becoming increasingly crucial in the fields of electricity and electronics. It plays a key role in the experimental verification of circuits, modeling, tuning, troubleshooting setups, and certifying processes or products, particularly in industrial sectors, as well as in the maintenance and repair of electrical or electronic devices.

In this context, a variety of measuring instruments are used, such as analog and digital voltmeters for measuring voltages, ammeters for measuring currents, and wattmeters for measuring power, among others.

This course material, aligned with the LMD (Licence-Master-Doctorat) system, is primarily intended for second-year university students in science and technology. It is taught during the fourth semester, comprising 22.5 hours of instruction, with one session of 1.5 hours per week.

The objectives of this course are to :

⋇) Help learners acquire an understanding of error and **uncertainty in measurements.

⋇) Identify various techniques used in electrical measurement methods.

⋇) Explore different types of measuring instruments, both analog and digital.

This handout is structured into four parts :

1. An introduction to the fundamental concepts of measurement.
2. A presentation of different methods for measuring common electrical quantities.
3. A detailed study of measuring instruments, both analog and digital.
4. Exercises and solutions

Recognizing the vital importance of measurement in the scientific field, this course material is designed to equip students with the skills needed to use measurement systems effectively, select appropriate methods and equipment for their experiments, and obtain reliable results.

Although this contribution is modest, we encourage readers to provide feedback and suggestions to improve and enhance the content. The course adopts a competency-based approach, focusing on the development of students' knowledge, technical skills, and interpersonal abilities.

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Chapitre I

Generalities and Basic Concepts of Measurement

Objectives of this Chapter :

- ※ *Understand and apply fundamental concepts in measurement, meteorology, and units of measurement related to physical quantities.*
- ※ *Calculate and analyze uncertainties associated with basic measurement operations to ensure accuracy and reliability of experimental results.*
- ※ *Acquire the skills necessary to calculate and interpret uncertainties in measurements and calculations, using practical and reliable methods to improve the precision of experimental outcomes.*

1 Introduction

Measurement plays an increasingly essential role in the fields of electrical and electronic engineering. It serves various purposes, including :

- Experimental verification of a circuit ;
- Modeling, development, or troubleshooting of a setup ;
- Certification of a process or product in the industrial sector ;
- Maintenance or repair of an electrical or electronic device.

2 Concept of Measurement

2.1 Definitions

a. Measurement Process

The measurement process includes all experimental operations that contribute to determining the numerical value of the measurand. For example, when measuring the resistance R of a linear passive dipole, the measurand is the resistance R of this dipole, and the measurement is carried out, for instance, using an ohmmeter.

b. Metrology

Metrology is the branch of science concerned with measurement. As an applied science, it enables industries to control and manage various parameters or quantities. Metrology is also known as **the science of quantification**.

c. Measurand

The measurand is the physical quantity that is being measured.

Examples : displacement, temperature, pressure, voltage, etc.

2.2 Physical Quantities

A physical quantity is any natural property that can be quantified through measurement or calculation. The different possible values of a physical quantity are expressed as a number, typically accompanied by a unit of measurement. Examples include mass, length, refractive index, and density.

There are two types of physical quantities : fundamental or base quantities and derived quantities.

2.2.1 Units of Quantities

Units are abstract concepts selected to represent physical quantities. They are based on fundamental units established through primary standards.

The International System of Units (SI)

A system of measurement units (SI) is defined by a conventional selection of base quantities, each associated with specific units. Examples include :

- The **MTS** system (meter, ton, second) ;
- The **CGS** system (centimeter, gram, second) ;
- The **MKSA** system (meter, kilogram, second, ampere) ;
- The **SI** system (International System).

a. Base Units

The SI system is based on seven fundamental base quantities, each with a specific dimension and unit (see Table I.1). There cannot be more than seven dimensionally independent quantities.

The SI also includes two additional dimensionless units : plane angle (α, β, θ) and solid angle (Ω).

b. Derived Units

Derived units are combinations of base units corresponding to specific physical quantities. They can be expressed in terms of base units (see Table I.1). Certain frequently used derived units have specific names to simplify notation.

2.2.2 Electrical Quantities and Their Units

The main electrical quantities that an electrical technician is required to measure are as follows :

- **Voltage**, or the potential difference between two points ;
- **Current** intensity in a branch ;
- **Resistance** of a load ;

- **Capacitance** of a capacitor ;
- **Power** dissipated in a circuit ;
- **Frequency** and **period** of a signal.

The electrical quantities and their corresponding base units in the International System of Units (SI) are presented in Table I.1.

2.2.3 Dimensions

After covering the fundamental and derived units of quantities, as well as scientific notation, this section focuses on a concept known as the **dimension** of a quantity. A dimension provides the essential nature of a physical quantity and distinguishes it from others. Dimensions are represented by specific notations, for example, [M] for mass, [L] for length, and [T] for time.

a) Dimensional Equation

All derived quantities can be expressed in terms of fundamental quantities through a dimensional equation. This takes the form :

$$\left[X = M^{\alpha} L^{\beta} T^{\gamma} I^{\delta}\right] \tag{I.1}$$

where X represents the derived quantity, and α, β, γ, and δ are the powers associated with mass (M), length (L), time (T), and electric current (I), respectively.

Tableau I.1 – Representation of SI units, Symbols, and Dimensions

Quantity	**Symbol**	**SI Units**	**Unit Symbol**	**Dimensions**
Fundamental Units				
Mass	m	kilogram	kg	$[M]$
Length	l	metre	m	$[L]$
Time	t	second	s	$[T]$
Electric current	I	Ampere	A	$[I]$
Thermodynamic temperature	T	Kelvin	K	$[\Theta]$
Luminous intensity		Candela	cd	
Supplementary Units				
Solid angle	Ω	Steradian	sr	$[L^2]^{\circ}$

Plane angle	α, β, γ	Radian	rad	$[L]^\circ$
Derived Units				
Area	A	square metre	m^2	$[L^2]$
Volume	V	cubic metre	m^3	$[L^3]$
Density	ρ	kilogram per cubic metre	kg/m^3	$[L^{-3}M]$
Velocity	u	metre per second	m/s	$[LT^{-1}]$
Angular velocity	ω	radian per second	rad/s	$[L^\circ T]$
Acceleration	a	metre per second squared	m/s^2	$[LT^{-2}]$
Angular acceleration	α	radian per second squared	rad/s^2	$[L^\circ T^{-2}]$
Force	F	newton	$N(kgm/s^2)$	$[LMT^{-2}]$
Pressure, Stress	p	newton per square metre	N/m^2	$[L^{-1}MT^{-2}]$
Derived Units Electrical				
Work, Energy	W	joule	$J(Nm)$	$[L^2MT^{\breve{}2}]$
Frequency	f	hertz	Hz	$[T^{-1}]$
Power	P	watt	$W(J/s)$	$[L^2MT^{-3}]$
Quantity of electricity	Q	coulomb	$C(As)$	$[TI]$
Potential difference, Electromotive force	V	volt	V(W / A)	$[L^2MT^{\breve{}3}I^{-1}]$
Electric resistance	R	Ohm	$W(V/A)$	$[L^2MT^{-3}I^{-2}]$
Electric capacitance	C	farad	$F(As/V)$	$[L^{-2}M^{-1}T^4I^2]$
Electric field strength	E, e	volt per metre	V/m	$[LMT^{-3}I^{-1}]$
Magnetic fi eld strength	H	ampere per metre	A/m	$[L^{-1}I]$
Magnetic flux	F	weber	$Wb(vs)$	$[L^2MT^{-2}I^{-1}]$
Magnetic flux density	B	tesla	T(Wb / m2)	$[MT^{-2}I^{\breve{}1}]$
Inductance	L	henry	$H(Vs/A)$	$[L^2MT^{-2}I^2]$
Magnetomotive force	U	ampere	A	$[I]$
Luminous flux		lumen	$lm(cdsr)$	
Luminance		candela per square metre	cd/m^2	
Illumination		lux	$lx(lm/m^2)$	

Example 2 : Derive the dimensions of the following :

(a) Permeability of a medium

(b) Power using relation $P = I^2R$ and $P = V^2/R$.

Solution : The dimensions of the following quantities are obtained as :

(a) The permeability of medium μ is expressed as :

$$\mu = \frac{\text{Magnetic flux density}}{\text{Magnetic field strength}} = \frac{B}{H} = \frac{[MT^{-2}I^{-1}]}{[IL^{-1}]} = \left[MLT^{-2}I^{-2}\right] \tag{I.2}$$

(b) The power P is obtained from the relation as :

$$P = I^2R = \left[I^2\right]\left[ML^2T^{-3}I^{-2}\right] = \left[ML^2T^{-3}\right] \tag{I.3}$$

b). Anglo-Saxon units

Some units of Anglo-Saxon origin are also used :

✓ The inch (in) : 1 in = 2.54 cm = 25.4 mm ;

✓ The foot (ft) : 1 ft = 12 inches = 30.5 cm = 0.305 m ;

✓ The pound (lb) : 1 lb = 453.6 g ;

✓ The mile (mi) : 1 mi = 5280 ft = 1609 m ;

✓ The mil : 1 mil = 10^{-3} in = 25.4 μm.

2.2.4 Multiples and sub-multiples of units.

Different units can be divided into multiples and sub-multiples, as represented in the following table (I.2).

Tableau I.2 – Representation of Numbers, Scientific Notation, and Metric Prefixes

sub-multiples				multiples			
Number	**Scientific Notation**	**Prefix**	**Symbol**	**Number**	**Scientific Notation**	**Prefix**	**Symbol**
0.1	10^{-1}	deci	d	10	10	deka	da
0.01	10^{-2}	centi	c	100	10^2	hecto	h
0.001	10^{-3}	milli	m	1000	10^3	kilo	k

0.000 001	10^{-6}	micro	µ	1 000 000	10^{6}	mega	M
0.000 000 001	10^{-9}	nano	n	1 000 000 000	10^{9}	giga	G
0.000 000 000 001	10^{-12}	pico	p	1 000 000 000 000	10^{12}	tera	T

2.2.5 calibration

The definition of calibration is as follows : it involves a documented comparison between a measuring instrument to be calibrated and a traceable reference instrument.

The reference standard is sometimes also referred to as a "calibrator". Logically, the reference instrument is more accurate than the instrument being calibrated. The reference instrument itself must be calibrated traceably (we will revisit this topic later). Depending on the measured quantity, the reference standard is not necessarily an instrument, but can also be, for example, a mass, a mechanical part, a physical, liquid, or gaseous standard...

3 Error in measurement

The measurement obtained provides a quantitative assessment of the "expected value", as it's often challenging to precisely define the true value. Factors such as instrument connection and user influence can cause deviations from the expected value. These deviations, or errors, stem from various variables and are inherent in any measurement process. They are attributed to both instrument-related factors and user-related factors. The closeness of a measurement to the expected value is indicated by its measurement error.

Errors arise from various sources and can be categorized into three types :

1. Gross Errors
2. Systematic Errors
3. Random Errors

3.1 Gross Errors

Gross errors result from human mistakes in reading or using instruments. These errors encompass inaccuracies such as misreading, miscalculating, or improperly recording data, and occasionally stem from incorrect instrument adjustments.

While complete elimination of gross errors is impossible, they can be minimized through the following methods :

— Exercising caution during reading and recording of measurement data.

— Taking multiple readings of the same quantity, ideally by different individuals, with at least three or more readings.

3.2 Systematic Errors

A systematic error can be categorized into three distinct types : instrumental errors, environmental errors, and observational errors.

1. instrumental errors Instrumental errors arise from the instrument itself and are caused by inherent limitations, misuse, or loading effects. For instance, friction in bearings of moving components in a D'Arsonval movement can lead to inaccurate readings. Various types of instrumental errors exist depending on the instrument type.

To mitigate instrumental errors, the following steps can be taken :

(a) Choosing a suitable instrument for the specific measurement application.

(b) Applying correction factors once the amount of instrumental error is determined.

(c) Calibrating the instruments against a standard.

2. environmental errors

Environmental errors arise due to the impact of environmental factors on instruments. This includes conditions surrounding the instrument, such as temperature, humidity, barometric pressure, or magnetic and electrostatic fields.

For instance, when using a steel rule for measurements, the temperature during the measurement may differ from the temperature at which the rule was calibrated.

To mitigate environmental errors :

(a) Utilize appropriate correction factors and information provided by the instrument manufacturer.

(b) Employ setups that maintain consistent environmental conditions, such as air conditioning or temperature-controlled enclosures.

(c) Perform recalibration under the local environmental conditions.

3. observational errors

Observational errors arise from operator negligence during reading. They stem from various sources like parallax errors when reading a meter, incorrect scale selection, or individual observer habits.

To mitigate observational errors, instruments with features like mirrors and knife-edged pointers should be used. Nowadays, digital display instruments are also available, offering greater versatility.

3.3 Random Errors

Random errors, which have unknown origins, persist even after systematic errors are addressed. They can be significant in high-precision work, arising from factors like instrument friction and parallax errors. These errors limit the precision of measurement due to the discrete nature of measurement scales. Even with digital instruments, randomness occurs when values fall between display divisions. Random errors vary in magnitude and don't follow any known law, becoming evident when repeated measurements yield different results for the same quantity.

Note : Depending on the expression of the measurement, we have two types of errors :

1. Absolute error
2. Relative error

3.4 Absolute Error and Absolute Uncertainty

The absolute error, denoted as δX , is the difference between the measured value and its exact theoretical value expressed in the same unit.

$$\partial X = X_{mea} - X \tag{I.4}$$

where

➢ X : expected value

➢ X_{mea} : measured value

As the exact value of the quantity to be measured is unknown, it is necessary to evaluate an upper limit of the absolute error, which is none other than the **absolute uncertainty** denoted by :

$$\Delta X = \sup\left(|\partial x|\right) \tag{I.5}$$

3.5 Relative Error and Relative Uncertainty

The relative error is the quotient of the absolute error to the exact value. Mathematically, it can be expressed as :

$$\varepsilon_r = \frac{\Delta X}{X_{mea}} = \left|\frac{X_{mea} - X}{X_{mea}}\right| \tag{I.6}$$

Additionally, if the exact value of the quantity is inaccessible, we will take the upper limit of the relative error, which is none other than the **relative uncertainty**. It is generally expressed as a percentage (%) :

$$\varepsilon_r\,(\%) = \frac{\Delta X}{X_{mea}}100 = \left|\frac{X_{mea} - X}{X_{mea}}\right| 100 \tag{I.7}$$

Expression du results

The result can be expressed in two ways :

a. 1st method

The adopted value equals the measured value followed by the evaluation of the absolute uncertainty :

$$X = X_{mea} \pm \Delta X\,[\text{unit}] \tag{I.8}$$

b. 2nd method

The adopted value equals the measured value followed by the evaluation of the relative uncertainty :

$$X = X_{mea}\,[\text{unit}] \pm \left(\frac{\Delta X}{X_{mea}}\right)\% \tag{I.9}$$

Examples :

$$R = 20 \pm 0.5\,\Omega \text{ or } R = 20\,\Omega \pm 2.5\%$$

3.6 Calculation of uncertainty for basic operations

Generally, the value of the quantity to be measured (X) is obtained through a mathematical relationship : $X = f(a, b, c, d, ..)$ Therefore, we can use the mathematical tool 'differential calculus ΔX or logarithmic differentials $\frac{\Delta X}{X}$' to determine uncertainties.

The absolute uncertainty is expressed in the following form :

➤ ***1st step :*** we express the differential

$$df=\frac{\partial f}{\partial a}.da+\frac{\partial f}{\partial b}.db+\frac{\partial f}{\partial c}.dc+... \tag{I.10}$$

➤ ***2nd step :*** we calculate Δf, by increasing df

$$\Delta X=\left|\frac{\partial f}{\partial a}\right|_{b,c=cst}\Delta a+\left|\frac{\partial f}{\partial b}\right|_{a,c=cst}\Delta b+\left|\frac{\partial f}{\partial c}\right|_{a,b=cst}\Delta c+... \tag{I.11}$$

The relative uncertainty is expressed in the following form :

$$\frac{\Delta X}{X}=\left|\frac{\partial f}{\partial a}\right|_{b,c=cst}\frac{\Delta a}{X}+\left|\frac{\partial f}{\partial b}\right|_{a,c=cst}\frac{\Delta b}{X}+\left|\frac{\partial f}{\partial c}\right|_{a,b=cst}\frac{\Delta c}{X}+... \tag{I.12}$$

When the function f is in the form of a product or quotient, we can simplify calculations by using logarithmic differentials of f. The procedure involves the following steps :

Example :

$$f(a,b)=\frac{a^2}{b} \tag{I.13}$$

1. Evaluate the logarithmic differentials of each quantity in the set :

$$\ln(f)=2\ln(a)-\ln(b) \tag{I.14}$$

2. we express the differential :

$$\frac{df}{f}=2\left|.\frac{\partial f}{\partial a}.\right|da+\left|-\frac{\partial f}{\partial b}\right|.db \tag{I.15}$$

3. Take the relative value of each term :

$$\frac{\Delta f}{f}=2.\frac{\Delta a}{a}+\frac{\Delta b}{b} \tag{I.16}$$

4. Introduce absolute the errors Δf.

Table (I.3) illustrates the various basic operations in calculating absolute and relative uncertainty with examples.

Tableau I.3 – uncertainty for basic operations

Basic operations	Example	absolute uncertainty	Relative accuracy
1. Sum	$R = R_1 + R_2$	$\Delta R = \Delta R_1 + \Delta R_2$	$\frac{\Delta R}{R} = \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}$
2. Difference	$I = I_1 - I_2$	$\Delta I = \Delta I_1 + \Delta I_2$	$\frac{\Delta I}{I} = \frac{\Delta I_1 + \Delta I_2}{I_1 - I_2}$
3. Product	$W = U.I.t$	$\Delta w = It\Delta U + Ut\Delta I + UI\Delta t$	$\frac{\Delta W}{W} = \frac{\Delta U}{U} + \frac{\Delta I}{I} + \frac{\Delta t}{t}$
4. Quotient	$R = U/R$	$\frac{\Delta R}{R} = \frac{1}{I}\Delta U + \frac{U}{I^2}\Delta I$	$\frac{\Delta R}{R} = \frac{\Delta U}{U} + \frac{\Delta I}{I}$

3.7 Practical calculation of uncertainty

There are two types of measuring instruments : analog and digital

3.7.1 Case of analog (or deflection) instruments

This type of instrument operates on the principle of providing a needle deflection on a graduated scale proportional to the measured quantity. Thus, the measured value is given by the following relationship :

$$X_{mea} = \frac{Cal.L}{E} \tag{I.17}$$

with

- ✓ Cal : the caliber used [unit]
- ✓ L : the reading (number of graduations read on the scale)
- ✓ E : the scale (total number of graduations on the scale)

Example

Suppose you have a voltmeter with :

- **Caliber (*Cal*)** : 100 V
- **Reading (*L*)** : 25 graduations
- **Scale (*E*)** : 100 graduations

The measured value can be calculated using :

$$\text{Measured Value} = \frac{L}{E} \times \text{Cal}$$

Substituting the given values :

$$\text{Measured Value} = \frac{25}{100} \times 100 = 25 \text{ V}$$

In this case, the measured value is 25 volts.

A) **Instrumental absolute uncertainty for a deflection device :**

Instrumental uncertainty is the uncertainty attributed to the measuring instrument. It depends on the precision of the instrument.

This instrumental uncertainty is given by the following expression :

$$\Delta X_{inst} = \frac{\text{Range.Accuracy Class}}{100} \tag{I.18}$$

B) **Absolute reading uncertainty**

Generally, the reading error is estimated at 1/4 of a division.

$$\Delta X_L = \frac{\text{Range}}{\text{scale}} \tag{I.19}$$

C) **Method uncertainty**

The method is also a source of uncertainty to evaluate (method uncertainty notes by Δ_{meth})

D) **Total uncertainty of an analog device**

Hence, the total uncertainty incurred in a measurement using an analog instrument will be the sum of the class uncertainty, the reading uncertainty, and the method uncertainty if it exists.

3.7.2 Case of digital instruments

For digital display instruments, manufacturers provide an indication called precision, which allows calculating the total uncertainty in the measurement. Precision is typically given as a percentage of the reading for each range. It can be expressed in two ways :

1st method

$$\Delta X = \pm(x\%\text{Reading} + \text{y}\%\text{Range}) \tag{I.20}$$

We therefore obtain :

$$\Delta X = \frac{x.R}{100} \pm \frac{y.G}{100} \tag{I.21}$$

with

$X\%$ and $Y\%$: Provided by the manufacturer.

G : the range used [unit]

R : the reading (directly displayed on the device's screen)

2nd method

$$\Delta X = \pm(x\%\text{measured\_value} + \text{y}\%\text{resolution}) \tag{I.22}$$

We therefore obtain :

$$\Delta X = \frac{x.R}{100} \pm \frac{y.G}{N} \tag{I.23}$$

with

G : measurement range.

N : total number of points on the device.

4 Conclusion :

a solid understanding of electrical quantities and their units is essential for accurate measurements of electricity and electronics.

Absolute and relative errors provide valuable insights into the precision of measurements while calculating uncertainty for basic operations ensures meticulous data handling.

Practical uncertainty calculation is crucial for obtaining reliable measurements and is applicable across various chapters in electrical engineering, enhancing the overall accuracy and reliability of experimental results.

Chapitre II

Electrical measurement methods

The objectives of this chapter are as follows :

- ⋇ *Explore the different measurement methods used to assess resistance, impedance, and power.*
- ⋇ *Examine and compare the accuracy and efficiency of direct methods, indirect methods (Voltmeter–Ammeter), comparison techniques, and bridge circuits.*
- ⋇ *Apply these methods to various electrical circuits operating in direct current (DC) or alternating current (AC), including single-phase and three-phase systems.*
- ⋇ *Identify sources of measurement errors and propose strategies to enhance their accuracy.*

1 Introduction

Measurement methods can be categorized into four main types : deviation methods (both direct and indirect), comparison methods, bridge (or null) methods, and resonance methods. Each category provides a unique approach for accurately quantifying electrical and electronic quantities.

Deviation methods involve comparing the unknown quantity to a reference standard, such as resistance, capacitance, inductance, or electrical power. In comparison methods, the quantity to be measured is directly evaluated relative to a known value, allowing for precise determination.

Bridge methods utilize the balance of components to measure quantities like resistance, capacitance, and inductance with high accuracy. Lastly, the resonance method relies on resonance properties to determine values such as frequency or capacitance.

This chapter provides an in-depth exploration of the principles, applications, and relevance of each category of electrical and electronic measurement methods in diverse industrial and scientific fields.

2 Application of Different Measurement Methods

2.1 Resistance measurement

Resistance measurement can be performed using various techniques, such as the color code method, direct deviation, volt-ampere, and comparison methods.

a). color method

The color method is a widely used visual method for determining the value of a resistor. It relies on the use of colored bands printed on the resistor, which represent a specific numerical value (See Annex A Table (A.1)).

2.1.1 Deviation methods

I). Direct deviation method

The direct deviation method involves directly measuring resistance using an **Ohmmeter**r or **Multimeter** in Ohmmeter mode. The instrument is connected to the resistor terminals, and the resistance value is read from the digital display. While simple and quick, this method may be affected by the instrument's internal resistance, and its accuracy relies on the precision of the measuring device :

$$\text{uncertainty} = \frac{(\text{class.caliber})}{100} \tag{II.1}$$

II). Voltmeter–Ammeter method (indirect deviation)

This method involves determining the value of a resistor by applying Ohm's law. Specifically, one measures the voltage U across its terminals and the current I lowing through it. The resistance value is then deduced using the following relationship : $R_x = \frac{U}{I}$ where R_x is the resistance in ohms (Ω), U is the voltage in volts (V), and I is the current in amperes (A). This method is based on two possible setups in Figure (II.1) : the downstream configuration and the upstream configuration.

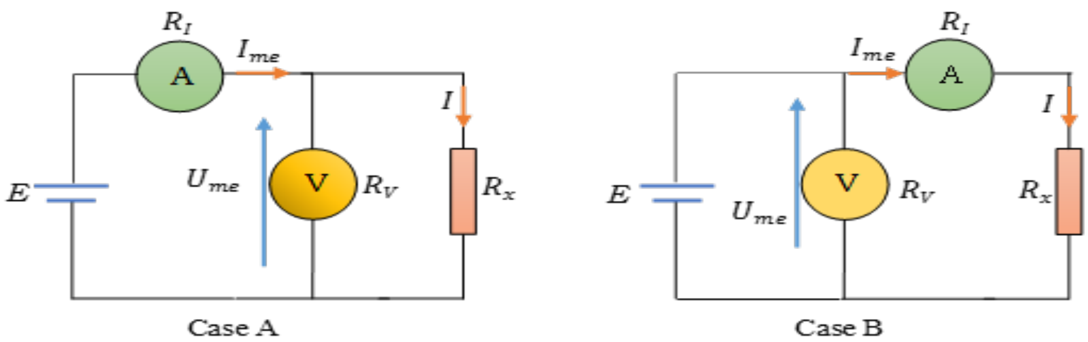


Fig. II.1 *Measurement of resistance by Voltmeter–Ammeter method*

R_x = true value of unknown resistance

R_{me} = measured value of unknown resistance

R_I = internal resistance of ammeter

R_V = internal resistance of voltmeter

Case A In this circuit, the voltmeter is connected directly across the unknown resistance, but the ammeter is connected in series with the parallel combination of the voltmeter and the resistance R_x.

Case B In this circuit, the ammeter is directly connected to the unknown resistance, whereas the voltmeter is connected across the series combination of the ammeter and the resistance R_x.

Table (II.1) illustrates the expressions for the measured resistance value, along with the associated absolute and relative uncertainties, in both configurations. The resistance value is determined using Ohm's law.

Tableau II.1 – Measured resistance value, as the uncertainties

The expression	Mounting	
	Case A	Case b
Measured value of unknown resistance R_m	$R_{me} = \frac{U_{me}}{I_{me}} = R_x + R_I$	$R_{me} = \frac{U_{me}}{I_{me}} = \frac{R_V R_x}{R_V + R_x}$
Absolute uncertainty	$\Delta R_x = \lvert R_{me} - R_x \rvert$	
Relative uncertainty associated with the method	$\varepsilon = \frac{R_{me}-R_x}{R_x} = \frac{R_{me}}{R_v}$	$\varepsilon = \frac{R_{me}-R_x}{R_x} = \frac{R_I}{R_x}$
Remark : Measurement	Low resistance	High resistance

2.1.2 Comparison Method

This method involves passing the same current through the resistance to be measured R_x and a known resistance R_e. Both resistances are placed in series within the same circuit, powered by a **DC** voltage source, and thus carrying the same current I (Figure II.2).

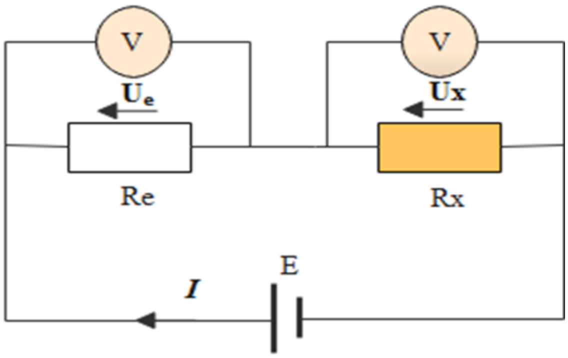


Fig. II.2 *Schematic diagram for the comparison method*

The expressions for the voltmeters are then :

$$\begin{cases} U_e = R_e I ....(1) \\ U_x = R_x I ....(2) \end{cases} \tag{II.2}$$

Where (1) and (2) give equation (II.3) :

$$R_x = \frac{U_x}{U_e} R_e \tag{II.3}$$

By using the same voltmeter for both measurements, instrumental errors are practically eliminated, leaving only reading errors.

The voltmeter deviations are then :

n_e and n_x for the measurement of the voltage drop U_e and U_x respectively.

Therefore, we can write equation (II.3).

$$R_x = \frac{n_x}{n_e} R_e \tag{II.4}$$

Measurement precision

The relative uncertainty is expressed as :

$$\frac{\Delta R_x}{R_x} = \frac{\Delta \mathrm{R}_e}{R_e} + \frac{\Delta n_x}{n_x} + \frac{\Delta n_e}{n_e} \tag{II.5}$$

Since the measurements are carried out using the same device and considering only the error introduced by the operator, it follows that :$\Delta n_x = \Delta n_x$,which we denote as Δn, thus :

$$\frac{\Delta R_x}{R_x} = \frac{\Delta \mathrm{R}_e}{R_e} + \Delta n \left(\frac{1}{n_x} + \frac{1}{n_e} \right) \tag{II.6}$$

2.1.3 Null Methods (Wheatstone Bridge)

Bridges, once widely employed for measuring resistances, inductance, capacities, and frequencies, lost popularity after the 1970s due to advancements in electronics. Despite their decline in metrology applications, the bridge structure continues to hold educational significance and finds use in various setups.

a). Principle Diagram

A Wheatstone bridge enables the measurement of low to medium resistances, as depicted in Figure (19).

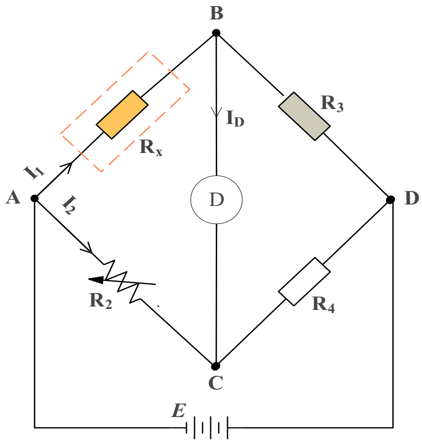


Fig. II.3 *Configuration of the Wheatstone Bridge.*

It consists of :

- Three calibrated and known resistancesR_2, R_3, and R_4, while the fourth resistance, R_x, is unknown and needs to be measured.
- A current detector, usually a highly sensitive center-zero galvanometer **D**.
- A DC power supply providing a constant current E.

Configuration of the Wheatstone Bridge.

b). Bridge Equilibrium Condition

The bridge is balanced when no current flows through the detector D due to adjusting the calibrated resistances R_2,R_3 and R_4 . if $I_D = 0$, then :

$$U_{BC} = U_{BA} + U_{AC} = 0 \tag{II.7}$$

- I_1 passes through R_3 and R_x, whileI_2 passes through R_2 and R_4.

And, by applying the voltage divider, we obtain :

$$U_{AB} = \frac{R_x}{R_x + R_3}E \tag{II.8}$$

and

$$U_{AC} = \frac{R_2}{R_2 + R_4}E \tag{II.9}$$

The bridge is said to be in equilibrium when $i = 0$, meaning

$$U_{BC} = U_{BA} + U_{AC} = 0 \tag{II.10}$$

Thus, from Eq (II.14), We have :

$$\frac{-R_x}{R_x + R_3}E + \frac{R_2}{R_2 + R_4}E = 0 \tag{II.11}$$

or

$$\frac{R_x}{R_x + R_3} = \frac{R_2}{R_2 + R_4} \tag{II.12}$$

or

$$R_x R_4 = R_3 R_2 \tag{II.13}$$

$$R_x = R_2 \frac{R_3}{R_4} \tag{II.14}$$

Therefore, the measurement of the unknown resistance is expressed about three known resistances. The arms BD and CD, housing the fixed resistances R_3 and R_4, are referred to as the ratio arms. The arm AC holds the known variable resistance R_2 and is the standard arm. Expanding the range of measurable resistance values can be achieved by adjusting the ratio R_3/R_4.

Note : The bridge is balanced when the cross products of the resistances are equal. In practice, we place the unknown resistance at R_x, R_2 is an adjustable known resistance, R_3 and R_4 are fixed resistances with a known ratio $K = \frac{R_3}{R_4}$. At bridge equilibrium, we can write

$$R_x = R_2 \frac{R_3}{R_4} = K.R_2 \tag{II.15}$$

2.2 Measurement of Impedance

2.2.1 Voltmeter–Ammeter method (indirect)

The voltmeter-ammeter method allows for the measurement of the impedance Z_x at the industrial frequency.

I). Inductor Measurement (Joubert Method)

The coil's impedance : $Z_{Lx} = r_x + jL_x$ is typically low ($Z_{Lx} << Z_V$). The downstream configuration is therefore the most suitable, , as shown in Figure (II.4(a))..

To measure the inductance of a real coil, two practical tests are performed :

DC test to determine the internal resistance of the coil r_x :

$$r_x = \frac{U_{DC}}{I_{DC}} \tag{II.16}$$

AC test to determine the impedance modulus $\bar{Z}_{Lx}$:

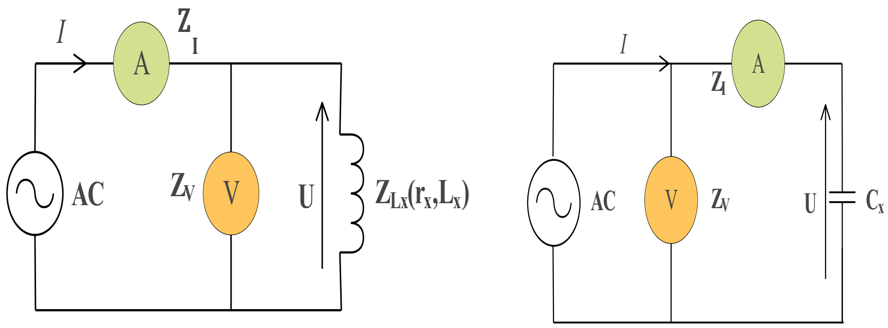
$$\bar{Z}_{Lx} = \frac{U_{AC}}{I_{AC}} \tag{II.17}$$

Impedance for an Inductor :

$$\bar{Z}_{Lx} = r_x + jL_x \Rightarrow Z_{Lx} = \sqrt{{r_x}^2 + (2\pi f L_x)^2} \tag{II.18}$$

or

$$L_x = \frac{1}{2\pi f} = \sqrt{{Z_{Lx}}^2 - {r_x}^2} \tag{II.19}$$



(a) *Case A : coil's impedance* **(b)** *Case b : Capacitor .*

Fig. II.4 *Measurement of Impedance : (a). for an Inductor and (b). for a Capacitor*

II). Capacitance Measurement

In most cases, the capacitor's impedance is quite high ($Z_{Cx} << Z_V$) as depicted in Figure (II.4(b)). The upstream configuration is therefore the most suitable

The impedance of a capacitor is :$\bar{Z}_{Cx} = \frac{1}{jC_x\omega}$.

$$Z_{Cx} = \frac{U_{AC}}{I_{AC}} \Rightarrow C_X = \frac{1}{Z_{Cx}} \tag{II.20}$$

Remark 2 : The capacitance of a capacitor can be directly measured using a capacitance meter.

2.2.2 AC Bridges for Measuring Impedance

An ac bridge in its general form is shown in Figure (II.5), with the four arms being represented by four unspecified impedances $\bar{Z}_x$, $\bar{Z}_2$, $\bar{Z}_3$ and $\bar{Z}_4$. Balance in the bridge is achieved by adjusting one or more of the bridge arms. Balance is indicated by zero response of the detector.

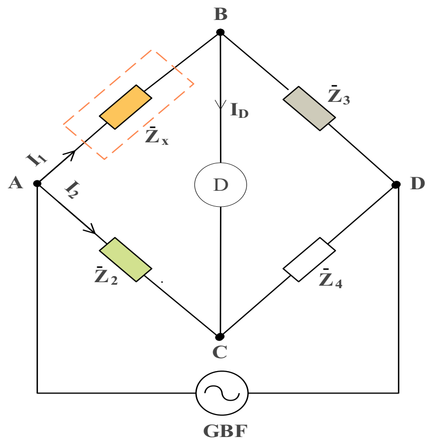


Fig. II.5 *Configuration of the Alternating Current Bridge.*

At equilibrium, the bridge satisfies the following conditions :

1. No current flows through the detector, implying no potential difference across it. In other words, the potentials at points B and C are equal.
2. This balance condition is achieved if the voltage $U_{AB} = U_{AC}$, both in magnitude and phase. The products of the impedance cross components are equal, ensuring equality between their real and imaginary parts.
3. The impedance of the unknown Z_x is given by Eq (II.21) :

$$\bar{Z}_x = \bar{Z}_2 \frac{\bar{Z}_3}{\bar{Z}_4} \tag{II.21}$$

. Typically, two arms consist of precision pure resistances, the third contains the unknown impedance, and the fourth comprises precision capacitors combined with precision resistances.

We avoid using inductance due to their frequency-dependent variation. There are several possible combinations, and we will explore the most commonly used ones.

A). Sauty's Bridge

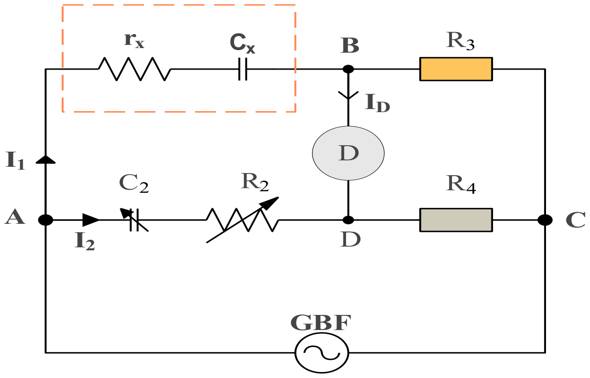


Fig. II.6 *Configuration of the Sauty Bridge.*

$\bar{Z}_x = r_x + \frac{1}{jC_x\omega}$: unknown impedance.

$\bar{Z}_2 = R_2 + \frac{1}{jC_2\omega}$: Variable and known impedance.

$\bar{Z}_3 = R_3$ and $\bar{Z}_4 = R_4$ fixed pure resistance.

At bridge equilibrium, we can express that :

$$r_x = R_2 \frac{R_3}{R_4} \tag{II.22}$$

and

$$C_x = C_2 \frac{R_4}{R_3} \tag{II.23}$$

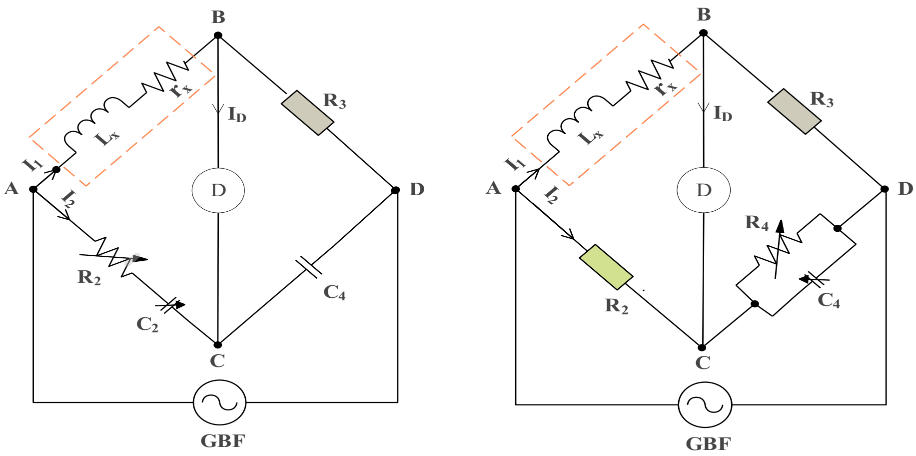
B). Maxwell's and Owen's Bridge

Figure (II.7) depicts Owen's and Maxwell's (Figure bridges in a state of balance. The following table(II.2) shows the characteristics of calculating inductance through bridges.

Tableau II.2 – Characteristics of calculating inductance through bridges

Bridge	
Owen's	**Maxwell's**
$\bar{Z}_x = r_x + jL_x$: Unknown impedance	$\bar{Z}_x = r_x + jL_x$: Unknown impedance
$\bar{Z}_2 = R_2 + j\frac{1}{C_2\omega}$: Variable and known impedance	$Z_2 = R_2$: fixed pure resistance
$\bar{Z}_3 = -j\frac{1}{C_3\omega}$ Known ideal capacitor	$Z_3 = R_3$: fixed pure resistance
R_4 : fixed pure resistance	$\frac{1}{\bar{Z}_4} = \frac{1}{R_4} + jC_4\omega$: Variable and known impedance

At bridge equilibrium, we can write : $r_x = R_3\frac{C_4}{C_2}$ and $L_x = R_2R_3C_4$	At bridge equilibrium, we can write : $r_x = R_3\frac{R_2}{R_4}$ and $L_x = R_2R_3C_4$.

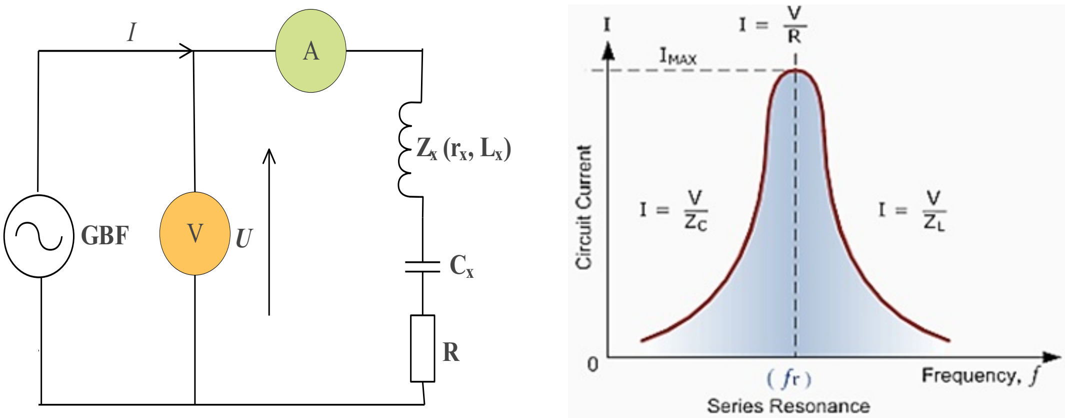


(a) *Owen Bridge.* **(b)** *Maxwell Bridge.*

Fig. II.7 *Bridge Configurations for Inductance Measurement*

2.2.3 Resonance Method

The resonance method is used to measure capacitance's and inductance's. It involves placing the quantity to be measured in a series or parallel resonant circuit and inferring the unknown quantity at resonance. For instance, in a series resonant circuit, at resonance, one can write $LC\omega^2 = 1$ and deduce the unknown quantity $(LorC)$. By plotting the curve $I = f(frequencies)$ (Fig. II.8(b)), one can derive the resonance frequency and the maximum current (Fig.II.8(a)) from the curve.



(a) *Series Resonance Circuit* **(b)** *Let's consider the plot of the signal $I = f(f)$.*

Fig. II.8 *Resonance Method*

$f_0 = \frac{1}{2\pi\sqrt{LC}}$ and $\mathrm{I}_{Max} = \frac{V}{R_T}$ with $R_T = R + r$

$$\begin{cases} \bar{Z} = R_T + j\left(L\omega - \frac{1}{C\omega}\right) \\ |Z| = \sqrt{R_T^2 + j\left(L\omega - \frac{1}{C\omega}\right)^2} \end{cases} \tag{II.24}$$

At resonance : $X_L - X_C = 0 \Leftrightarrow L\omega - \frac{1}{C\omega} = 0 \Rightarrow L = \frac{1}{C\omega^2}$

3 Power Measurement

Power measurement encompasses various techniques depending on the circuit type (DC or AC) and the desired measurement precision. Common methods include the indirect approach (Voltmetre-Amperemeter), direct methods like the Wattmeter and Varmeter, and the three-ammeter and three-voltmeter techniques. Method selection hinges on application requirements and accuracy needs.

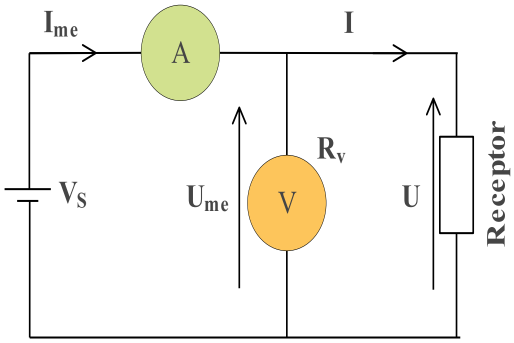
3.1 Power measurement in DC circuits

The Electric power consumed by any device is given by the following equation (II.25)

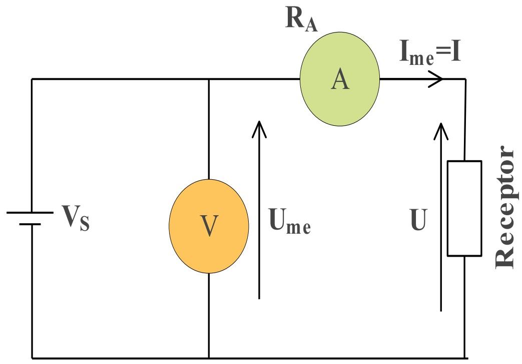
$$P = UI \tag{II.25}$$

3.1.1 Voltmeter–Ammeter method (indirect)

This method utilizes voltmeters and ammeters to measure voltage and electric current in the circuit. The electrical power P is then calculated by multiplying the voltage U by the current I. There are two categories : upriver and downriver (Figure.II.9).



(a) *Downriver mounting.*



(b) *Upriver mounting.*

Fig. II.9 *Measurement of electrical power by indirect method.*

The table (II.2) represents the specific relationships between electrical power and errors measured by two different methods.

Tableau II.3 – Expressions for the measured power value, as The method error

	Downriver mounting	**Upriver mounting**
Power measurement	$P_{me} = U_{me}I_{me} = P + \frac{U}{R_V}$	$P_{me} = U_{me}I_{me} = P + R_A I^2$
The method error	$\Delta P_{me} = \frac{U^2}{R_V}$	$\Delta P_{me} = R_V I^2$

3.1.2 Direct Measurement "Using a Wattmeter"

Direct electrical power measurement is carried out using a Wattmeter, an Electrodynamics type device (Fig(II.10)). It can be used for alternating (AC) and direct (DC) currents. The wattmeter is insensitive to external fields ; it consists mainly of current and voltage circuits. The wattmeter constant is given by $K = \frac{Caliber(U)*Caliber(I)}{Scale}$, which represents power by dividing the scale.

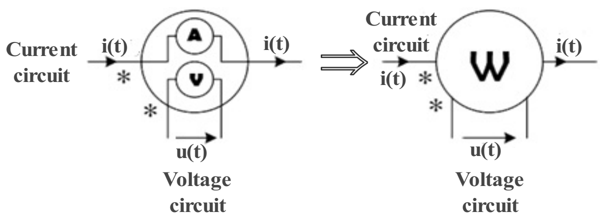
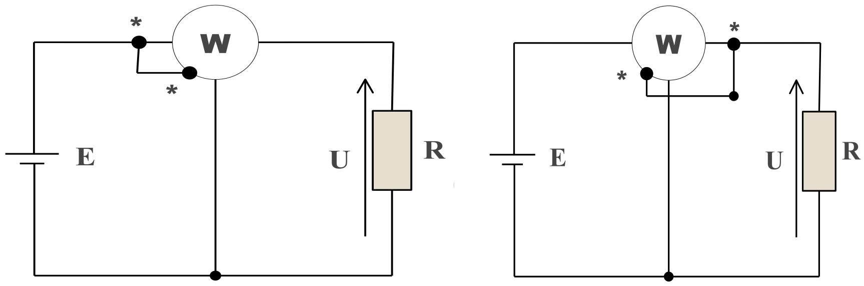


Fig. II.10 *Equivalent diagram and symbol of a wattmeter.*

a. Connection of a Wattmeter

There are two different ways to connect a wattmeter in an electrical circuit : downriver and upriver connection. In both cases, the current measurement circuit is connected in series with the load (represented by resistance R), while the voltage measurement circuit is connected in parallel with the load



(a) *Downriver mounting.* **(b)** *Upriver mounting.*

Fig. II.11 *Measurement of electrical power by indirect method.*

For upriver installation : $\Delta P_{tot} = \frac{Class*Caliber(U)*Caliber(I)}{100} + R_{AW}I^2$

For downriver installation : $\Delta P_{tot} = \frac{Class*Caliber(U)*Caliber(I)}{100} + \frac{I^2}{R_{VW}}$

R_{AW} : is the internal resistance of the current circuit of the wattmeter.

R_{VW} : is the internal resistance of the voltage circuit of the wattmeter.

3.2 Measurement of Power in Single-phase Alternating Current

The formulas for calculating powers in alternating current are as follows : $S = UI[VA]$: Apparent power, $P = UIcos(\varphi)[W]$: Active power. $Q = UIsin(\varphi)[VAR]$: Reactive power. Where : U and$I$ are the effective values of the single-phase voltage (between phase and neutral) and the current absorbed by the load. φ is the phase angle between the current and voltage.

I). Measurement of Apparent Power (Voltmeter-Ammeter Method)

To measure the apparent power S in alternating current, it is necessary to measure the effective values of current (I_{me}) and voltage U_{me} using an ammeter and a voltmeter.

$$S = U_{me}.I_{me} \tag{II.26}$$

II). Measurement of Active Power

a) Direct Method

To measure active power P, a wattmeter must be used. The connection mode of the wattmeter remains the same as that of direct current.

b) Three Ammeters Method

The principle of this method involves connecting three ammeters (see fig.17), where R represents a high-precision standard resistor.

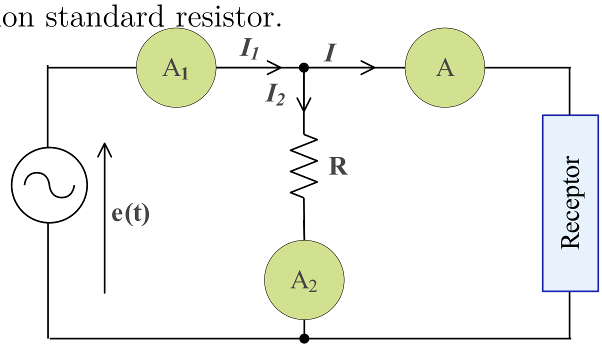


Fig. II.12 *Measurement of Power Using Three Ammeters.*

With i1, i2, and i denoting the instantaneous values of currents flowing through the three ammeters.

We have : $i_1 = i + i_2 \Rightarrow {i_1}^2 = i^2 + {i_2}^2 + 2ii_2 \Rightarrow 2ii_2 = {i_1}^2 - i^2 - {i_2}^2$

so : $P = UI$ and $i_2 = \frac{U}{R}$(With P representing the instantaneous power.)

$$2\frac{U}{R}i = {i_1}^2 - i^2 - {i_2}^2 \tag{II.27}$$

So

$$P = \frac{R}{2}\left({i_1}^2 - i^2 - {i_2}^2\right) \tag{II.28}$$

The active power consumed by the load is : $P = \frac{1}{T}\int_0^T Pdt$

Therefore, P is defined by the equation (II.28).

With I_1, I_2 and I representing the effective values of currents i_1, i_2 and i

c). Three Voltmeters Method

This method is analogous to the previous method. The voltmeters are connected according to figure (16), where R represents a standard resistor of value. Let u_1, u_2, and u be the instantaneous values of the voltages across the three voltmeters. We have :

$$u_1 = u + u_2 \Rightarrow {u_1}^2 = u^2 + {u_2}^2 + 2uu_2 \Rightarrow 2uu_2 = {u_1}^2 - u^2 - {u_2}^2 \tag{II.29}$$

The instantaneous power absorbed by the load is : $P = UI$and (where p represents the instantaneous power), thus i $= \frac{u_2}{R}$. Therefore, we obtain :

$$P = \frac{1}{2R}\left({u_1}^2 - u^2 - {u_2}^2\right) \tag{II.30}$$

The active power of a load is, by definition : $P = \frac{1}{2RT}\int_{0}^{T}({u_1}^2 - u^2 - {u_2}^2)$. Therefore, P is defined by the equation (II.30).

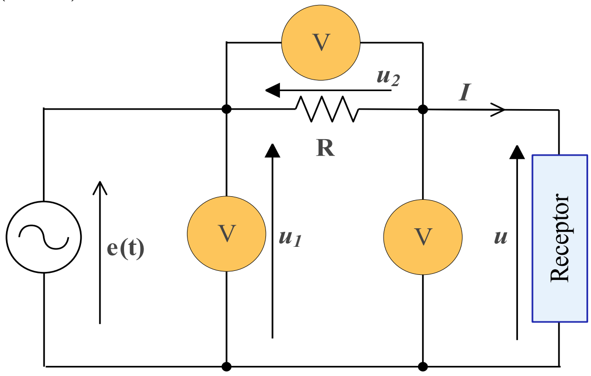


Fig. II.13 *Measurement of Power Using Voltmeters Method.*

3.2.1 Measurement of Reactive Power

To measure reactive power Q, it is sufficient to connect an ammeter, a voltmeter, and a wattmeter. Then calculate $Q = \sqrt{S^2 - P^2}$, taking into account the type of load :

- $Q = 0$ for a resistive load.
- $Q > 0$ for an inductive load.
- $Q < 0$ for a capacitive load

3.3 Measurement of Power in Three-phase Systems :

Regardless of the connection of the load, the powers in three-phase systems are expressed by the following formulas :

- Active power noted as :

$$\mathrm{P} = 3\mathrm{VJcos}\varphi = \sqrt{3}UI\cos\varphi\,[w] \tag{II.31}$$

- Reactive power noted as :

$$Q = 3\mathrm{VJsin}\varphi = \sqrt{3}UI\sin\varphi\,[VAR] \tag{II.32}$$

- Apparent power noted as :

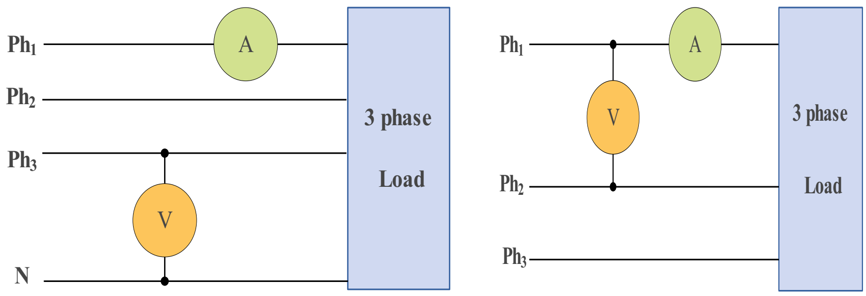
$$S = 3VI = \sqrt{3}UI\,[VA] \tag{II.33}$$

With :

- V and I : the effective values of the single-phase voltage (between phase and neutral) and the current absorbed by the load;
- φ : the phase angle between the current and the voltage;
- U : the effective composite voltage (between two phases).

3.3.1 Measurement of Apparent Power S

The power S is measured using the indirect (voltmeter-ammeter) method, as shown in Figure (II.14).



(a) *Three-phase Line with Neutral Wire ($S = 3VI$).* **(b)** *3 ∼ Line without Neutral Wire ($S = \sqrt{3}UI$) .*

Fig. II.14 *Measurement of electrical power S by indirect method.*

3.3.2 Measurement of Active Power P

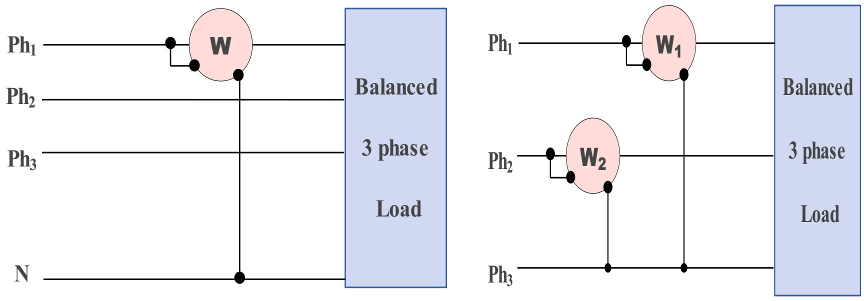
a). Single Wattmeter method with Neutral Wire

When the load is balanced, a single wattmeter is sufficient to measure the active power absorbed. The schematic diagram is provided in the following figure(II.15(a)).

The wattmeter, as connected, measures the power : $P_1 = VIcos(\varphi)$. The power absorbed by the balanced three-phase load is : $P = 3P_1$. We can write :

$$P = 3P1 = 3VIcos(\varphi) = 3UIcos(\varphi)[W] \tag{II.34}$$

This measurement requires the neutral wire to be accessible.



(a) *Single Wattmeter Method with Neutral Wire.* **(b)** *Two Wattmeter Method.*

Fig. II.15 *measurement of electrical power P by direct method.*

b). Two Wattmeter Method For an unbalanced system or a balanced system where the neutral is not accessible, the active power is measured using two wattmeter (see figure (II.11)). If we denote P_{me1} and P_{me2} as the powers measured by wattmeter w_1 and w_2 respectively, we determine the active power absorbed by the load using the relationship :

$$P = P_{me1} + P_{me2} \tag{II.35}$$

3.3.3 Measurement of Reactive Power Q

I). Measurement of Reactive Power with a Varmeter

For either an unbalanced system or a balanced system without accessible neutral, the reactive power is measured using Varmeter (refer to Figure (II.16)). Reactive power noted as :

$$Q= 3\text{VJ}\sin\varphi = \sqrt{3}UI\sin\varphi\,[VAR] \tag{II.36}$$

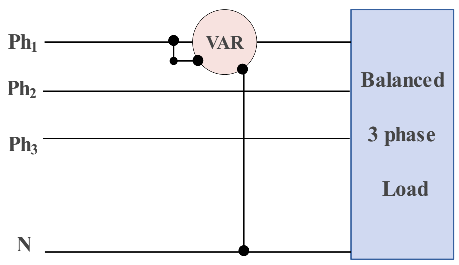


Fig. II.16 *Measurement of Three-phase Reactive Power with a Varmeter.*

II). Two Wattmeter Method :

This is the same method used for measuring active power (see figure (II.11)). However, we can determine reactive power using the following relationship :

$$P = \sqrt{3}(P_{me1} - P_{me2}) \tag{II.37}$$

4 Conclusion

The various methods for measuring electrical quantities offer diverse approaches to ensure optimal precision in various applications. From direct techniques like voltmeter-ammeter measurements to more advanced methods such as bridges and resonance, each method has its advantages and limitations. Additionally, precise measurement of electrical power, whether active, reactive, or apparent, is crucial in many areas of electrical engineering. Using instruments like wattmeter, ammeters, and voltmeters, it's possible to accurately assess the consumption and distribution of electrical energy in a variety of systems. These measurements play an essential role in the design, operation, and maintenance of electrical networks, ensuring their efficiency and reliability.

Chapitre

III Overview of Different Types of Electrical and Electronic Measuring Devices

The objectives of this chapter are as follows :

- I*dentify and understand the different types of electrical and electronic measuring devices by exploring their classification, operation, and applications, whether analog or digital.*
- *Evaluate the quality, accuracy, and reliability of analog measuring instruments, particularly deflection and indicating devices in direct current (DC) and alternating current (AC) systems.*
- *Analyze the role of analog-to-digital converters (ADC) and the key characteristics of digital measuring instruments.*
- *Compare analog and digital measuring devices to assess their advantages and limitations in various applications.*

1 Introduction

A measuring device is a system that translates a physical phenomenon, which is either not accessible to our senses or difficult to perceive, into another phenomenon that can be visualized and estimated.

There are two types of devices :

An analog measuring device typically consists of one or more fixed inductors (permanent magnet, electromagnet, etc.) acting on a movable element around an axis.

Digital devices provide a value representing the measured quantity at the quantization step. This value is presented in numerical form (digital display).

2 Analog Measuring Devices

The internal technology of these devices relies on three elements :

— The suspension of the movable element ;

— The reading device, which can be a needle or a light spot ;

— The damping device can be magnetic or air-based.

According to their terminology, there are several types of analog devices, including :

* Galvanometers (electrical balance) ;

* Integrating devices (counter, Flowmeter) ;

* Electronic devices ;

* Analog devices with digital display ;

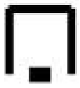
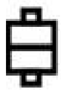
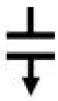
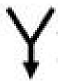
* Deflection devices.

We will focus on deflection analog devices in the following sections.

2.1 Classification of Analog Deflection Devices

We can classify analog measuring devices, of the deflection type, according to the nature of the physical phenomenon that determines their operation, and their symbols are given in the following table (III.1) :

Tableau III.1 – Classification of Analog Deflection Devices .

Designation	Symbol	Operation	Use
Magneto-electric		The fixed inductor is a ***U***-shaped permanent magnet, and the deflection of the needle is proportional to the average current "***i***" passing through a coil placed inside the magnetic field created by the fixed magnet.	Suitable for direct or stable current.
Ferromagnetic		The principle of operation of a ferromagnetic device is based on the action of a field created by a circuit carrying a current on one or more pieces of soft iron, some of which are mobile.	Usable in both direct and alternating current.
Electrodynamic		An electrodynamic device is mainly formed of a fixed circuit (usually two half-coils) creating a magnetic field within which a low-inertia mobile frame moves on two pivots and drives a needle.	Used in the manufacture of wattmeters.
Electrostatic		This type of device is characterized by a force exerted by the fixed armature of a capacitor on its movable armature.	Usable in both direct and alternating current.
Thermal		This type of device is based on the expansion of a conductive wire heating up when an electric current ***I*** passes through it. This effect is a direct consequence of the power dissipated by Joule heating in the expanding wire.	Usable in both direct and alternating current.

2.2 Quality of Analog Measuring Devices

The principle of operation and mode of construction are the main factors determining the quality of a measuring device.

2.2.1 Accuracy Class Index

The materials used, manufacturing techniques, and calibration methods ensure that a device never indicates the true value. The standard C42-100 defines the following class values :

Standard devices : class 0.5, 0.1, and 0.2 (used in laboratories).

Control devices : class 0.5 and 1 (used for inspection and verification).

Industrial devices : class 1.5 and 2.5.

Indicators : class 5.

2.2.2 Sensitivity

It is the ability of the device to detect small variations in the measured length.

2.2.3 Fidelity

It is the device's quality to consistently provide the same indication for the same value of the measured quantity. Fidelity can be disrupted by :

- Impacts on moving parts.
- Earth's magnetic fields or those produced by nearby equipment generating stray fields.
- Electrostatic phenomena.
- Moisture, which decreases the insulation resistance of the device's electrical circuits.
- Aging of the device, manifested by the weakening of the magnetic field of permanent magnets in magneto-electric devices.
- Temperature, which expands mechanical parts and affects the resistance of conductors.

2.2.4 Indication Speed

It is the quality of a device to provide the value of the measured quantity or its variations in minimal time.

2.2.5 Accuracy

It is the quality of a device to accurately translate the true value it measures.

2.3 Normalization

According to the UTE (Union Technique de l'Electricité), the approved publication concerning deflection measuring devices is the C42-100.

2.4 Symbols on the Dials of Analog Measuring Devices

In Table (III.2), the main symbols found on most devices are summarized :

Tableau III.2 – Symbology Found on Analog Measuring Device Dials.

Signification	Nature of Significance	Symbol
Nature of current	Direct current	– – –
	Alternating current	~
	Direct and alternating current	$\overline{\sim}$
Isolation voltage	Between the two terminals of the device is 2 KV	☆ 2
	Between the two terminals of the device is 500 V	☆
Reading position	Vertical	⊥
	Horizontal	⊓
	Inclined	∠
Frequency range	Within which the device can operate correctly	20 Hz ... 500 kHz
Accuracy class	Of the device is 0.5% of the scale	0.5
	Of the device is 1% of the scale	1
	Of the device is 2% of the scale	2

2.3 Types of Indicating Instruments

Indicating instruments can be divided into different types according to their working principle.

2.3.1 DC Indicating Instruments

a) D'Arsonval Movement

The primary sensing mechanism utilized in *DC* ammeters, voltmeters, and ohm meters is a current-sensing device known as a D'Arsonval meter movement (Figure(III.1)). The **D'Arsonval movement** is a *DC* moving coil-type mechanism in which an electromagnetic core is suspended between the poles of a permanent magnet. The measured current flows through the coils of the electromagnet, creating a magnetic field that opposes the field of the permanent magnet, resulting in the rotation of the core. Springs restrain the core, allowing the needle to deflect or move in proportion to the current intensity.

As the current through the core increases, the opposing magnetic field strengthens, leading to a larger deflection, up to the coil's current capacity limit. Upon interrupting the current, the opposing field diminishes, and the needle returns to zero due to the restraining springs. Typically, the maximum current capacity for this type of movement is less than one milliampere. A common variation of the D'Arsonval movement is the Weston movement, which employs a similar principle but with a more robust construction, utilizing jeweled supports for the core and a heavier winding in the electromagnet. It's essential to note that the D'Arsonval movement is designed for DC applications and can only measure DC current or AC current rectified to DC.

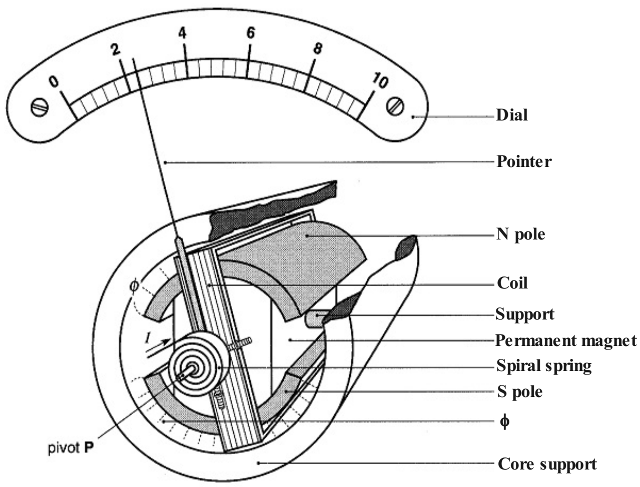


Fig. III.1 *Measurement of resistance by Voltmeter–Ammeter method*

The fundamental mechanism known as the Permanent Magnet Moving Coil (PMMC) is frequently referred to as the D'Arsonval movement. PMMC instruments offer precision and are specifically designed for DC measurements. As current (I) flows through the coil, it generates a deflecting torque on the coil. These devices demand minimal power consumption and current to achieve full-scale deflection. The sensitivity of a galvanometer can be expressed in terms of current sensitivity and voltage sensitivity.

√ **Current Sensitivity :** $S_I = d/I$

where :

d represents the deflection of the galvanometer in mm.

I denotes the galvanometer current in μA.

$\surd$ **Voltage Sensitivity** : $S_V = d/V$

V is voltage applied to the galvanometer in mV

b)Construction of an Ammeter

The Permanent Magnet Moving Coil (PMMC) galvanometer serves as the fundamental mechanism for a DC ammeter. Due to the small and lightweight coil winding in the basic movement, it can handle only very low currents. When measuring large currents, it becomes necessary to divert a significant portion of the current through a resistor known as a shunt, as depicted in Figure (III.2). The resistance of the shunt can be determined using conventional circuit analysis techniques.

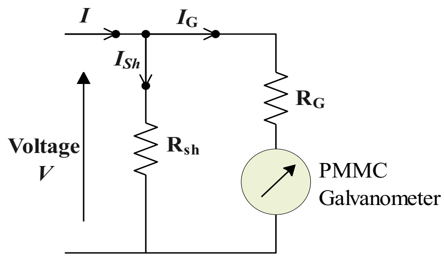


Fig. III.2 *Measurement of resistance by Voltmeter–Ammeter method*

Referring to Figure(III.2) R_G = internal resistance of the movement. I_{sh} = shunt current I_G = full scale deflection current of the movement I = full scale current of the ammeter + shunt (i.e. total current) Because the shunt resistance (R_{sh}) is connected in parallel with the meter movement, the voltage drop across both the shunt and the movement must be equal.

Calculation of the shunt :

We have :

$$\begin{cases} R_G.I_G = R_{sh}.I_{sh} \\ I = I_G + I_{sh} \end{cases} \Rightarrow R_G.I_G = R_{sh}(I - I_G) \tag{III.1}$$

Hence :

$$R_{sh} = \frac{R_G.I_G}{I - I_G} \tag{III.2}$$

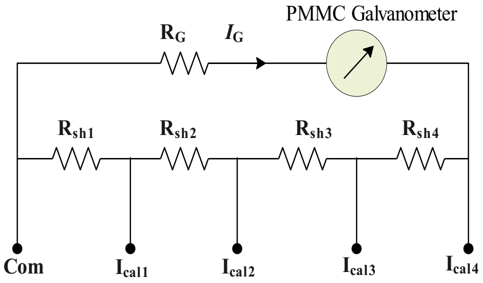
If we define $m = \frac{I}{I_G}$ as the multiplying factor,Mathematically, it can be represented as Equation (III.2) :

$$R_{sh} = \frac{R_G}{m - 1} \tag{III.3}$$

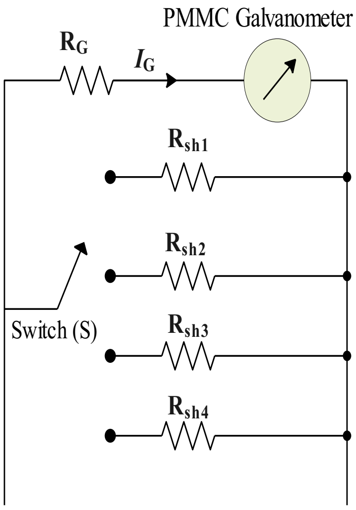
There are two main types of ammeters : universal shunt ammeters (figure (III.3(a))) and multi-range ammeters.

MULTIRANGE AMMETERS

To measure Direct Currents across multiple ranges, a *DC* ammeter utilizes multiple parallel resistors instead of a single one. This resistor combination is then connected in parallel with the PMMC galvanometer.



(a) *Universal Shunt Ammeter*



(b) *Multi-range DC ammeter.*

Fig. III.3 *Basic dc Ammeter*

This multi-range *DC* ammeter is incorporated in series with a branch of an electric circuit where the desired Direct Current range is to be measured. The specific current range is selected by connecting the switch (S) (figure (III.3(b))) to the corresponding shunt resistor. Let m_1, m_2, m_3 , and m_4 denote the multiplying factors of the DC ammeter for the respective total Direct Currents to be measured as I_1, I_2, I_3, and I_4, respectively. The following formulas correspond to each multiplying factor : $m_1 = \frac{I_1}{I_G}, m_2 = \frac{I_2}{I_G}, m_3 = \frac{I_3}{I_G}$ and $m_4 = \frac{I_4}{I_G}$.

In Figure (III.3), there are four shunt resistors are used, namely $R_{sh1}, R_{sh2}, R_{sh3}$ and R_{sh4}. The expression for finding the shunt resistance values are given below :

$R_{sh1} = \frac{R_G}{m_1-1}, R_{sh2} = \frac{R_G}{m_2-1}, R_{sh3} = \frac{R_G}{m_3-1}$ and $R_{sh4} = \frac{R_G}{m_4-1}$

b)Construction of an voltmeter

A basic D'Arsonval movement can be converted into a DC voltmeter by incorporating a series resistor, referred to as a multiplier, as illustrated in Figure (III.4). The purpose of the multiplier is to restrict the current flowing through the movement to ensure it does not surpass the full-scale deflection value. A DC voltmeter is employed to measure the potential difference between two points within a DC circuit or a circuit component. To measure this potential difference, the DC voltmeter is always connected across the points of interest with the correct polarity. The necessary value of the multiplier is determined as follows, referring to Figure (III.4) :

R_G = internal resistance of the movement.

R_{ad} = multiplier or additional resistance

I_G = full scale deflection current of the movement(I_{fsd})

V = full range voltage of the instrument

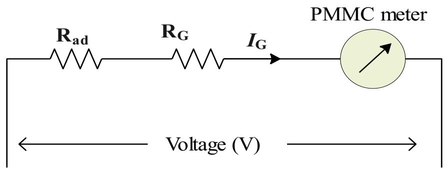


Fig. III.4 *Measurement of resistance by Voltmeter–Ammeter method*

From the circuit of Figure(III.4)

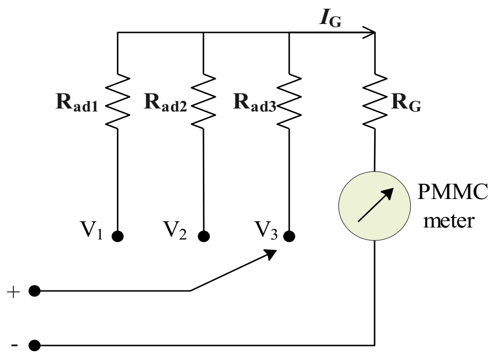
$$V = I_G(R_{ad} + R_G) \Rightarrow R_{ad} = \frac{V}{I_G} - R_G \tag{III.4}$$

The multiplier limits the current through the movement, so as to not exceed the value of the full-scale deflection I_{fsd}.

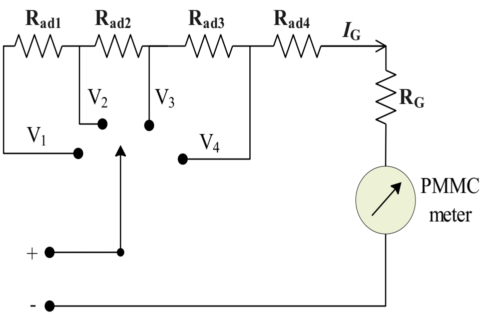
Note : Equation (III.4) is also utilized to expand the measurement range of the DC voltmeter further.

Multi-Range Voltmeter

A multi-range voltmeter is constructed by integrating multiple multipliers with a range switch.



(a) *Parallel connection*



(b) *Series connection.*

Fig. III.5 *Multi-range voltmeter*

This configuration is illustrated in Figure (III.5), parts (a) and (b).

Voltmeter sensitivity : $\left(\frac{\Omega}{V_{rating}}\right)$, $S = \frac{1}{I_{fsd}}\left[\frac{\Omega}{V}\right]$

The total resistance of the voltmeter is : $R_{Tot} = S.V$

$$R_{ad} = SV - R_G \tag{III.5}$$

C)Construction of an Ohm meter

An ohmmeter measures the resistance of a circuit or component. This device is divided into two types :

series-type ohmmeter

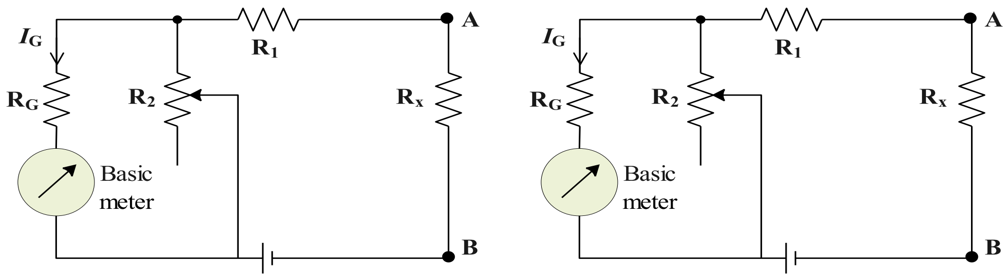
shunt-type ohmmeter

1) Series type Ohmmeter

The Series Type Ohmmeter is illustrated in Figure (III.6(a)). When the resistance R_X is zero, maximum current flows through the circuit, including the shunt resistance. R_X is adjusted until the indicator shows a full-scale current reading. A convenient quantity to use in the design of a series type Ohmmeter is the value of $R_x = R_h$ which causes half-scale deflection of the motor. $R_h = (R_1 + R_2)//R_G$

Then, total resistance for the battery is $2R_n$

$R_{Tot} = S.V$, $R_2 = \frac{I_{fsd}.R_G R_h}{E - I_{fsd}.R_h}$ and $R_1 = R_h - \frac{I_{fsd}.R_G R_h}{E}$



(a) *Series type Ohmmeter.* **(b)** *Shunt type Ohmmeter.*

Fig. III.6 *Basic Ohmmeter Circuit*

2) Shunt Type Ohmmeter

The Shunt Type Ohmmeter is depicted in Figure (III.6(b)). When RX is zero, the current through the meter is zero. If R_X is infinite, the current finds a path through the meter. By choosing an appropriate value for R_1, the pointer can be adjusted to read full scale. When R_X is zero, the meter will display full-scale current. When R_X is zero, the full-scale meter current will be

$$I_{fsd} = \frac{V}{R_1 + R_G} \tag{III.6}$$

For any value of R_x,

$$I_G = \frac{E.R_x}{R_1 R_G + R_x(R_1 + R_G)} \tag{III.7}$$

and

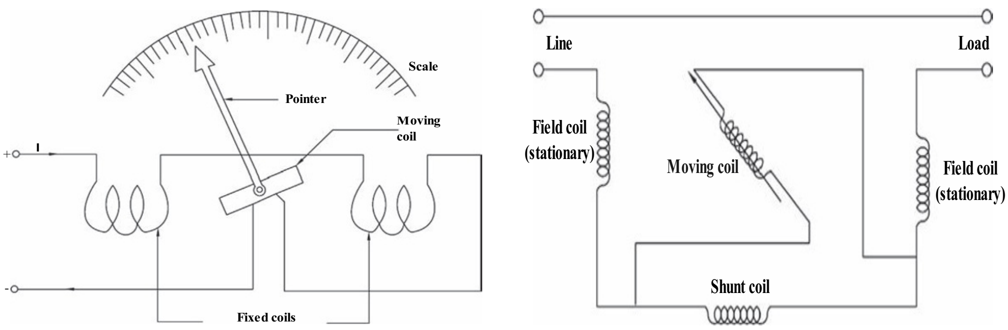
$$S = \frac{I_G}{I_{fsd}} \tag{III.8}$$

2.3.2 AC Indicating Instruments

PMMC meters are unsuitable for measuring AC signals since they register the average value of the AC, resulting in zero deflection.

A. Electrodynamometer

The Electrodynamometer is an apparatus utilized for measuring electric voltage, current, and power. It comprises two types of coils : a fixed coil and a moving coil. The fixed coil generates the magnetic field, which is then distributed into a more uniform field, as depicted in Figure (III.7(a)).



(a) *Electrodynamometer schematic.* **(b)** *Shunt type Ohmmeter.*

Fig. III.7 *construction Electrodynamometer*

$DeflectingtorqueT_d = BANI$

Where,

B is Flux Density, A is Area of cross section of coil, I is Current, and N is Number of turns of coil

B. Electrodynamometer in Current and Voltage Measurement :

The measured current generates a magnetic field flux in an electrodynamometer, causing the movable coil to rotate. The circuit diagram for an electrodynamometer ammeter is depicted in Figure (III.7(b)). The fixed coil (FC) divides into equal halves and generates the magnetic field within which the movable coil (MC) rotates.

In an ammeter, the fixed and moveable coils are connected in series, ensuring they carry the same current. Consequently, $I_1 = I_2, \phi = 0$

and

$$\theta = \left(\frac{I^2}{k}\right)\frac{dm}{d\theta} \tag{III.9}$$

In voltmeter, the fixed and moveable coils are connected in series with a high non-inductive resistance. Hence, $I_1 = I_2 = \frac{V}{Z}, \phi = 0$ or $\theta = \left(\frac{V^2}{Z^2 k}\right)\frac{dm}{d\theta}$

C. Electrodynamometer Wattmeter

he Electrodynamometer Wattmeter relies on the interaction between the magnetic fields produced by moving and fixed coils. It serves the purpose of measuring power in both AC and DC circuits, as depicted in Figure (18). Within the electrodynamometer Wattmeter, a current-carrying conductor is positioned within a magnetic field, resulting in the experience of a mechanical force. This force causes the pointer mounted on the calibrated scale to deflect. Notably, an electrodynamometer Wattmeter comprises both current and pressure coils, illustrated in Figure Figure(18). The expression for instantaneous power is as follows :

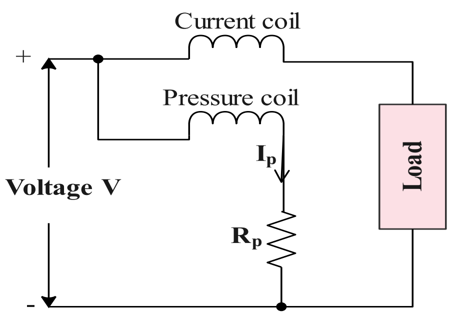


Fig. III.8 *Electrodynamometer Wattmeter.*

$$T_i = \left(\frac{V i_c}{R_p}\right)\cos\phi\frac{dm}{d\theta} \tag{III.10}$$

Where ϕ is phase angle, i_c is the current coil current and R_p is the series resistance of pressure coil circuits.

therefore

$$\theta_i = \left(\frac{Vi_c}{KR_p}\right) \cos\phi \frac{dm}{d\theta} \tag{III.11}$$

2.4 Oscilloscope

Typically, an oscilloscope is equipped with 2 or 4 channels (CH1 to CH4), facilitating the observation of two or four voltages, denoted as $V_1, V_2, ...andV_4$, respectively, over a period of time. The Vx voltage functions as the horizontal sweep voltage, which is directly proportional to time, and is provided by the 'time base' component to monitor voltage fluctuations over time. The signal being analyzed is showcased on a cathode ray tube, typically colored green, with the oscilloscope trace being determined by two primary components : one horizontal and one vertical. The cathode ray tube (Figure (III.9)) is the pivotal element of the Cathode Ray Oscilloscope (CRO), responsible for generating, accelerating, and deflecting an electron beam onto a phosphor-coated screen where it becomes visible. Various electrical signals and voltages are necessary to achieve these functions, supplied by the oscilloscope's power supply circuit. The electron gun's heater requires low voltage, while the CRT requires high voltage for beam acceleration, alongside normal voltage for control circuits. Horizontal and vertical deflecting plates regulate the beam according to input signals, creating a visible spot on the screen that moves horizontally with a constant time-dependent rate, regulated by a time base circuit. The signal under examination is fed to the vertical deflection plates via a vertical amplifier, enabling the electron beam to deflect both horizontally (X-axis) and vertically (Y-axis). A triggering circuit synchronizes these deflections, ensuring that the horizontal deflection initiates at the same point of the input vertical signal during each sweep.

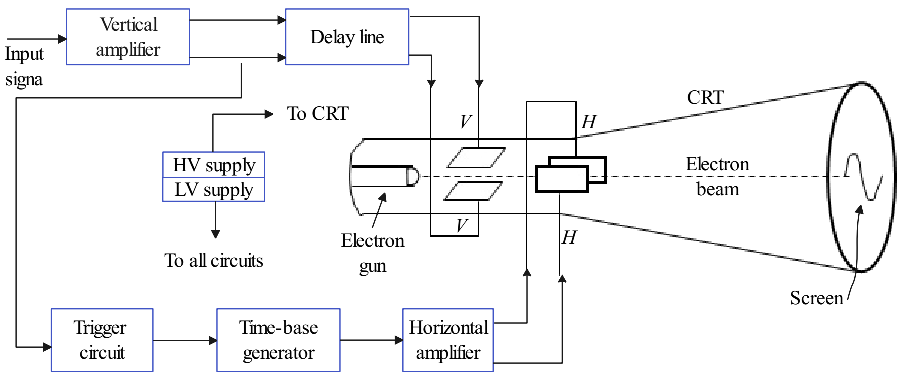


Fig. III.9 *Basic block diagram of a general-purpose oscilloscope .*

The oscilloscope is a versatile tool for measuring electrical voltages, frequencies, and phase differences.

— **To measure voltages**, you connect the oscilloscope probe to the circuit of interest, and the oscilloscope displays the waveform of the voltage on its screen. This allows for instant visualization of the voltage and precise measurements such as amplitude, period, and frequency.

— To **measure frequencies**, you can use the oscilloscope's automatic measurement function or manually count the number of waveform cycles on the screen during a given period of time. The frequency is then calculated by dividing the number of cycles by the measurement duration.

— **Lissajous figures** are graphical patterns formed by two slightly different frequency sinusoidal signals displayed simultaneously on the oscilloscope. They are used to visualize and measure phase differences between two signals. By analyzing the shape of the Lissajous figure, you can accurately determine the phase difference between the two signals, which is useful in various applications such as signal synchronization or characterization of electrical circuits.

3 Digital Measuring Devices

Digital measuring devices are increasingly used due to their accuracy, precision, and ease of reading. It is essential for users of digital devices to understand the language adopted by the manufacturers of these devices. The principle is to convert an analog quantity into a numerical value that can be displayed (see Figure (III.10)). To achieve this, electronic circuits must be used, the main ones being : analog-to-digital converters, the oscillator, the counter, and the display.

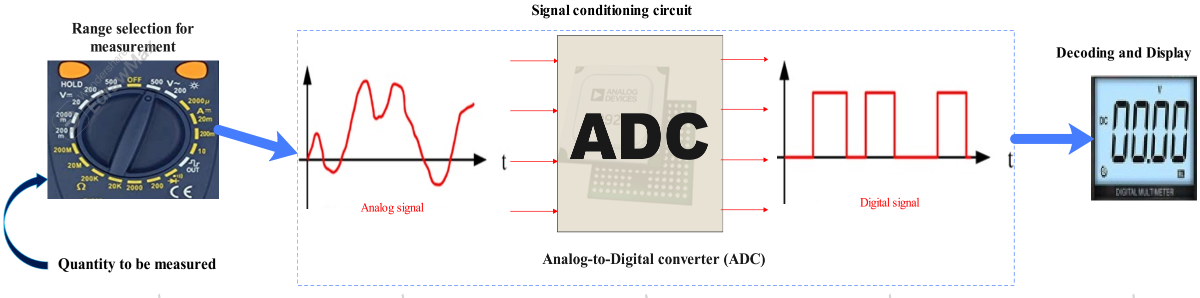


Fig. III.10 *schematic representation of a digital measuring device.*

In this diagram :

✓ The input receives the analog signal to be measured.

✓ Conditioning may involve filters or amplifiers to prepare the signal for conversion.

✓ The ADC converts the analog signal into a digital representation.

✓ Processing may include various operations such as digital filtering, calculation, etc.

✓ The display finally shows the digital results

3.0.1 Analog-to-Digital Converters (ADC)

Analog-to-Digital Converters (ADC) There are two types of analog-to-digital converters :

☞ Single-slope ADC

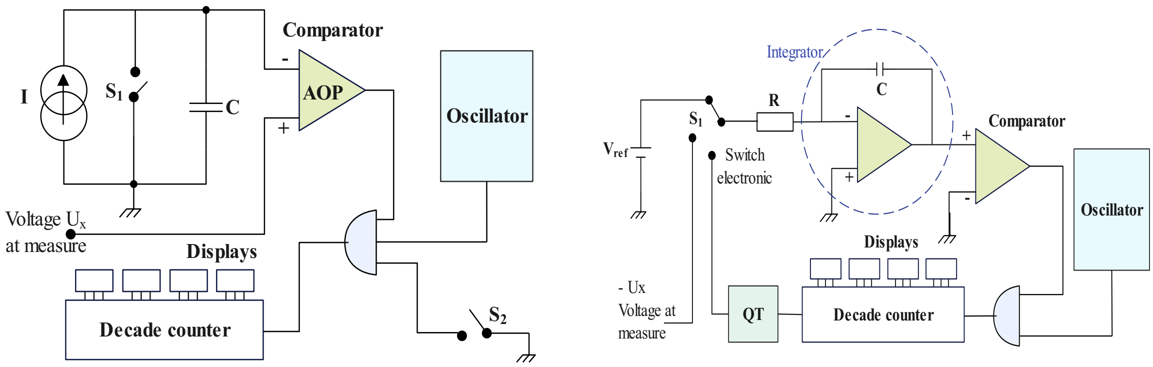
☞ Dual-slope ADC

The former are used for applications where high precision is not required.

Note : The operating principle of ADC is based on the charging and discharging of a capacitor with a constant current.

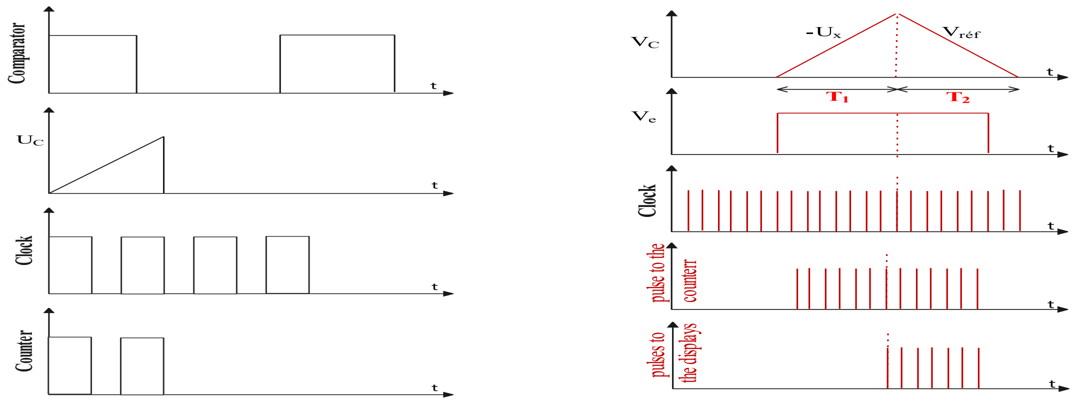
I) Single-slope Analog-to-Digital Converter

The ADC includes a comparator where the voltage across the capacitor is applied to its inverting input, and the unknown voltage to be measured is applied to its non-inverting input. A logic gate receives the oscillator signal, whose frequency can be adjusted based on the selected range, along with the comparator output signal. Finally, there's a decade counter and display units (figure (III.11(a))).



(a) *Single-slope* **(b)** *Two-ramp.*

Fig. III.11 *Analog digital converter circuit*



(a) *Single-slope* **(b)** *Two-ramp.*

Fig. III.12 *Analog to digital converter oscillogram*

II) Two-ramp digital analog circuit

The role of the integrator is to charge the capacitor linearly. Comparing it with the single-slope ADC (figure (III.11(b))), we observe that it includes an electronic switch and a flip-flop to control this switch. Thus, we notice the presence of an integrator used to charge the capacitor linearly. The input of the integrator is alternately subjected to two constant voltages, one being a fixed reference voltage, and the other, an unknown voltage to be measured, is of negative polarity.

3.0.2 Characteristics of a digital device :

Information : This term refers to the physical data at the input of the device.

Signal : It is the electrical quantity (current or voltage) representing the information.

Sensor : This is the device that captures the information and converts it into a signal usable by the measuring device.

(**Example :** the microphone is a sensor that converts sound into an electrical signal)

Number of points : (N) It corresponds to the number of different values that the device can display within a measurement range (example : for a device with 4 displays, the number of measurement points is $N = 104$).

Quantization step : (q) the smallest value different from 0 in the measurement range (example : for a measuring device with 4 displays, used in the range of $10V$, the quantization step is $q = 10/N = 1mV$).

Digit : Refers to the device that displays all digits from $0to9$ of equal weight in a number.

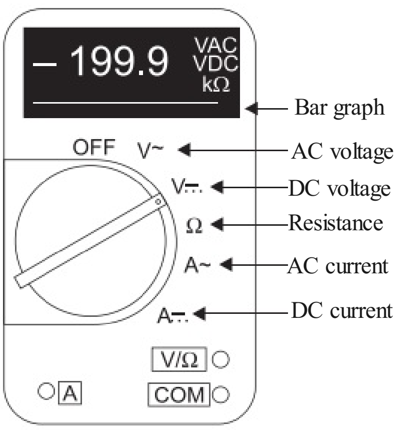
Resolution : It is the value of the quantization step in the range. It corresponds to the smallest variation in the value of the quantity that the device can detect within a range. *Resolution = measurement range / number of points N .*

(Example : the resolution of a device with 100000 points in the range of $1V$ is equal to $10\mu V$).

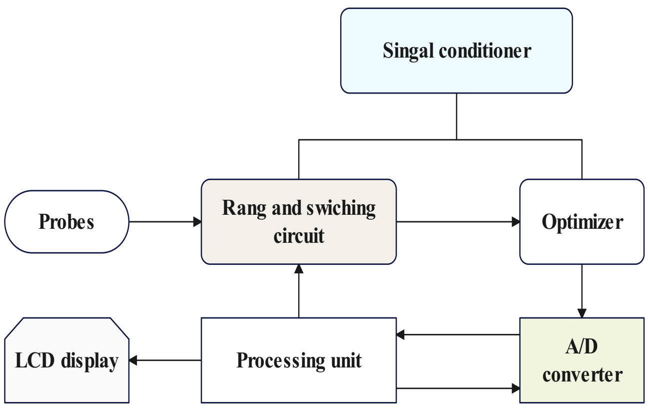
Precision : The precision of a device depends on the device's resolution, the quality of the components, the precision of voltage and time references, etc. The precision of a digital device is generally given as a percentage of the reading for each range. This precision can be very high for some devices. Common portable devices have precisions ranging from 0.1% to 1% of the reading depending on the range and the measured quantity, and in most cases, to one or two units (or digits).

A) Digital Multimeter

The digital Multimeter is built around a digital voltmeter and includes at least a current-to-voltage converter to operate as an ammeter and a constant current generator to function as an ohmmeter (figure (III.13(b))). The selection of the type of measurement (of the instrument), the range or scale of measurement is usually done using a rotary switch, and push buttons may control additional functions. The newest multimeters, often the easiest to use, automatically select the correct mode and range. The main components used in a digital Multimeter are shown in Figure (figure (III.13(a))).



(a) *Basic Digital Multimeter*



(b) *Block Diagram.*

Fig. III.13 *Digital Multimeter.*

4 Comparison of analog and digital instruments

Table (III.3) shows the comparison between analog and digital instruments in terms of resolution, portability, power requirements, observation errors, environmental factors, accuracy, range, and polarity.

Tableau III.3 – Symbology Found on Analog Measuring Device Dials.

Characteristic	Analog Instruments	Digital Instruments
Resolution	One part in several hundreds	One part in several thousands
Portability	Easily portable, no external power source needed	Require external power, less portable
Power Requirements	High power requirement, may load circuit	Negligible power requirement
Observational Errors	Exist due to deflection of pointers on scale	Direct indication in decimal numbers
Environmental Factors	Operate in wide range of environments	Affected by humidity, temperature
Accuracy	Usually within ±0.1% of full scale	Very high accuracy
Range and Polarity	Manual range and polarity selection	Automatic range indication and polarity selection

Exercises and solutions

Exercises and Solutions

1 Exercises

Exercise 01

Provide a precise and clear definition of the following terms : **measurement range, resolution, fidelity, accuracy, precision.**

Exercise 02

1. The ammeter measures ....
2. The voltmeter measures ....
3. The ohmmeter measures ....
4. The wattmeter measures ....
5. The multimeter measures ....
6. The oscilloscope measures ....
7. The frequency meter measures ....
8. The period meter measures ....
9. The phase meter measures ....

Exercise 3

Answer with "Yes" or "No"

1. **Direct Method :** This method involves using multiple instruments to measure a quantity, utilizing one or more relationships between the measured quantities.

◯ Yes, ◯ No

2. **Indirect Method :** This method involves directly reading the value of the quantity to be measured from the measuring instrument.

   ◯ Yes, ◯ No

3. **Substitution Method :** The unknown quantity is replaced by a known standard quantity.

   ◯ Yes, ◯ No

4. **Resonance Method :** The resonance method is used to measure capacitance and inductance.

   ◯ Yes, ◯ No

5. **MAXWELL Bridge :** Used for measuring resistance.

   ◯ Yes, ◯ No

6. **OWEN Bridge :** Used for measuring inductance.

   ◯ Yes, ◯ No

7. **Wheatstone Bridge :** Used for measuring resistance.

   ◯ Yes, ◯ No

Exercise 04

Express the relative and Absolute Uncertainty in the following cases :

1). For the power $P = RI^2$;

2). For the voltage $U = \frac{R_1}{R_1+R_2}E$;

3). For the reactive power $Q = \sqrt{S^2 - P^2}$.

Exercise 05

We measured the same resistance R using the 4000Ω range of a digital ohmmeter with 4000 points. The displayed value was 1000Ω, with the indicated accuracy :

$$\Delta R = (2\% \text{ of reading} + 5 \text{ counts})$$

1. Calculate the absolute and relative uncertainty
2. Express the result in two ways.

Exercise 06

Consider the electrical circuit shown in Figure 1, where :

$$E = 48\,\mathrm{V} \pm 1\%, \quad R_1 = (38 \pm 0.3)\,\Omega, \quad R_2 = (20 \pm 0.2)\,\Omega.$$

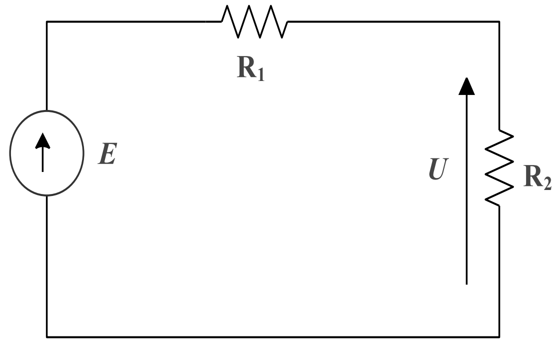


Fig. 14

1. Derive the expression for the voltage U as a function of E, R_1, and R_2.
2. Compute U.
3. Derive the expression for the relative uncertainty $\frac{\Delta U}{U}$.
4. Calculate $\frac{\Delta U}{U}$, then deduce ΔU.
5. Present the result in both forms.

Exercise 07

We aim to determine the resistance R using two different measurement techniques.

I. Volt-Ampere Method

The resistance is measured using a voltmeter with internal resistance R_V and an ammeter with resistance R_A, as shown in Figure 1. The experimental data are provided in the table below :

Instrument	Internal Resistance (Ω)	Range	Reading	Scale	Class
Ammeter	0.1	30mA	20	30	1.5
Voltmeter	$100k\Omega$	30V	20	30	1.0

Tasks

a). What is the name of this circuit ?

b). Compute the measured voltage U_{mes}, the measured current I_{mes}, and determine the corresponding resistance R_{mes}.

c). Determine the absolute instrumental error ΔR_{inst}.

d). Calculate the error ΔR_{meth} caused by the measurement method.

e). Compute the total measurement error ΔR.

f). Evaluate the power P dissipated in the resistance R.

g). Express the uncertainty ΔP in power, given that $P = RI^2$, and determine its numerical value.

II. Direct Measurement Method

A digital ohmmeter set to the 4000Ω range (with 4000 display counts) was used to measure the resistance. The recorded value was 1000Ω. The accuracy of the instrument is given by :

$$\Delta R = (2\% \text{ of reading} + 5 \text{ counts})$$

Tasks

1. Compute the absolute and relative uncertainty.
2. Express the final measurement result in two different formats.

Exercise 08

We have a Wheatstone bridge with a proportional ratio equal to $\frac{R_1}{R_2}$, where $R_1 = 100\,\Omega$ and $R_2 = 1000\,\Omega$, with a precision of 0.2% on the decades. The variable resistance R is $2520\,\Omega \pm 0.2\%$.

A). Represent the setup illustrating this measurement method.

B). At the balance of the bridge, give the expression and the value of R_x.

C). Calculate ΔR_1, ΔR_2, and ΔR.

D). Show that the relative uncertainty $\frac{\Delta R_x}{R_x}$ is given by the expression :

$$\frac{\Delta R_x}{R_x} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R}{R}$$

Determine the relative uncertainty $\frac{\Delta R_x}{R_x}$ and the absolute uncertainty ΔR_x.

E). Write the result in two ways..

Exercise 09

Figure (15) shows the schematic diagram of a measurement bridge.

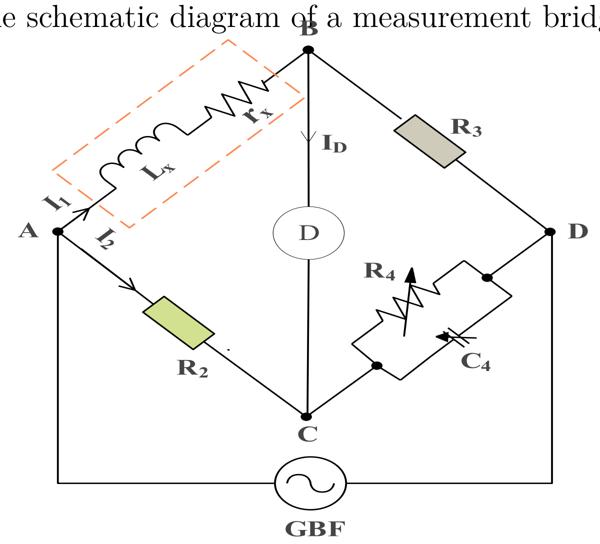


Fig. 15

1. Give the name of this bridge ;
2. Under what condition is the bridge said to be balanced ?
3. Show that when the bridge is balanced, we can write :

$$r_x = \frac{PQ}{R} \quad \text{and} \quad L_x = CPQ,$$

where Q and P are fixed pure resistances ;

4. For $Q = 1.0\,\text{k}\Omega \pm 5\%$, $P = 0.2\,\text{k}\Omega \pm 1\%$, $R = 1.5\,\text{k}\Omega \pm 5\%$, $C = 2\,\mu\text{F} \pm 5\%$, and $\omega = 2\pi f$, calculate the values of r_x and L_x ;
5. Determine the relative uncertainties $\frac{\Delta r_x}{r_x}$ and $\frac{\Delta L_x}{L_x}$, then find Δr_x and ΔL_x.

Exercise 10

In the case of single-phase sinusoidal alternating current, we aim to measure the power of a load using the so-called three-voltmeter method (see diagram, figure(II.10). The measurements are summarized in the following table :

	Scale	*Range*	*Reading*	*Class*(%)
$V_1(U_1)$	1000 V	100	74	2.5
$V_2(U_2)$	30 V	30	18	2.5
$V(U)$	10 V	100	50	2.5

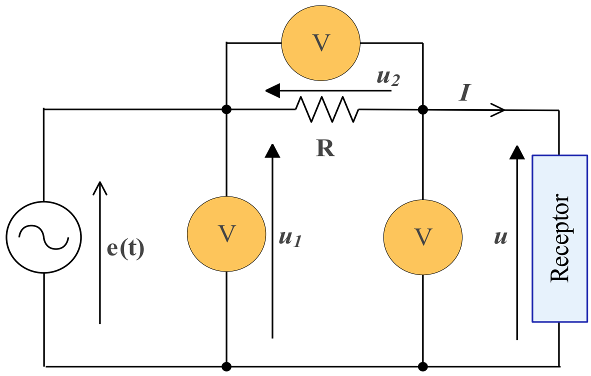


Fig. 16 *Measurement of Power Using Voltmeters Method.*

1. Determine the measured voltages U_1, U_2, and U, and the absolute uncertainties of each voltmeter ΔU_1, ΔU_2, and ΔU).
2. Calculate the power using the following relation :

$$p = \frac{1}{2R}\left(U_1^2 - U_2^2 - U^2\right),$$

   with $R = (110 \pm 2.2)\,\Omega$.
3. Calculate the absolute uncertainty and then the relative uncertainty $\frac{\Delta p}{p}$.
4. Provide the result in two ways.

Exercise 11

The objective is to measure the powers P, Q, and S of a three-phase asynchronous motor using appropriate measuring instruments.

— The apparent power S is determined using an ammeter (A) and a voltmeter (V).

— The active power P and reactive power Q are obtained using two wattmeters (W_1) and (W_2).

— Based on the characteristics of the measuring instruments listed below, perform the required calculations :

Instrument	Technology	Measurement Range	Value Read	Scale	Accuracy Class
V	Magnetoelectric with rectifier	500 V	86	100	1.5
A	Magnetoelectric with rectifier	20 A	78	100	1
W_1	Electrodynamic	600 V - 25 A	66	150	1.5
W_2	Electrodynamic	600 V - 25 A	26	150	1.5

Tableau 4 – Characteristics of measuring instruments

a) Determine the absolute uncertainty ΔI and the relative uncertainty ΔV, then deduce the uncertainties related to the apparent power : $\frac{\Delta S}{S}$ and ΔS.

b) Calculate the absolute uncertainties associated with powers ΔP_1 and ΔP_2, then deduce those of the global quantities ΔP and ΔQ, as well as the relative uncertainties $\frac{\Delta P}{P}$ and $\frac{\Delta Q}{Q}$.

c) Present the results in two different formats.

Exercise 12

We have measured the voltage ($V = 25V$) of an electrical circuit using :

1) An analog voltmeter with a class 1.5 deflection on a $3V$ range and a scale of 30 divisions. The reading is estimated to half a division.

2) A digital voltmeter with 2000 counts on a 20 V range, with a specified accuracy of $\pm(0.1\%$ of the reading $+ 0.01\%$ of the range).

1. Determine the absolute and relative uncertainties in percentages for the voltage measurement using the analog meter.

2. Determine the absolute and relative uncertainties in percentages for the voltage measurement using the digital meter.

Exercise 13

The figure (17) provided below represents the face of an analog voltmeter with a needle, as shown in the figure below : Give the meaning of each symbol displayed on the voltmeter face.

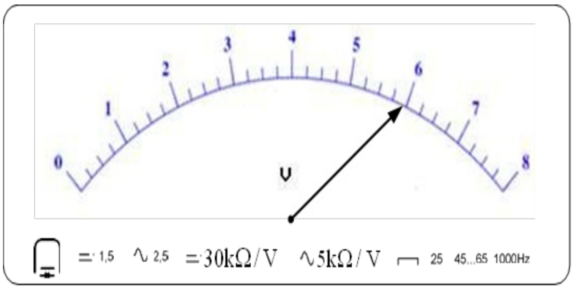


Fig. 17

Exercise 14

A voltmeter is built using a moving coil mechanism with an internal resistance of $R_G = 1000\,\Omega$ and a sensitivity of $I_G = 50\,\mu A$, with available ranges of 1V, 3V, 10V, and 30V.

1. Draw the schematic diagram of this voltmeter.
2. Determine the additional resistances required.
3. Calculate the characteristic resistance of the voltmeter.
4. Determine the total resistance of the voltmeter.

Exercise 15

The characteristics of a moving coil mechanism are : $R_G = 50\,\Omega$, $I_G = 0.5\,mA$. The objective is to design an ammeter with three different ranges : 0.5A, 0.2A, and 0.05A, using two different methods.

1. Multi-range Ammeter

a) Draw the basic circuit diagram.

b) Determine the multiplication factors for each range.

c) Calculate the shunt resistances R_1, R_2, and R_3.

2. Universal Shunt Ammeter

a) Draw the basic circuit diagram.

b) Express the multiplication factors (m_1, m_2, m_3) in terms of R_{S1}, R_{S2}, R_{S3}, and R_G.

c) Compute the values of R_{S1}, R_{S2}, and R_{S3}.

Exercise 16

Find the voltage reading and % error of each reading obtained with a voltmeter on (i) $5V$range, (ii) $10V$ range and (iii) $30V$range, if the instrument has a $20kW/V$ sensitivity and is connected across Rb of Figure (18)

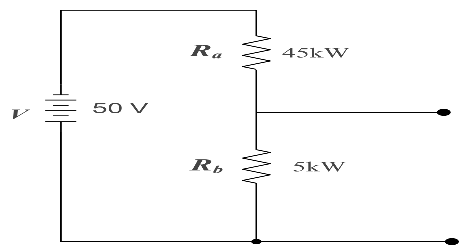


Fig. 18

2 Correction of exercises

Exercise 01

1. **Measurement range :** It is the range of possible variations of the quantity to be measured.
2. **Resolution :** The resolution of a device is the smallest change in the measured quantity that results in a perceptible change in the instrument's reading.
3. **Fidelity :** Fidelity refers to the ability of a device to deliver consistent measurements without errors, even when repeated.
4. **Accuracy :** Accuracy is the ability of a measuring device to provide a result close to the true value. It is related to the average value obtained from a large number of measurements in comparison with the actual value.
5. **Precision :** A measuring instrument is considered precise when it demonstrates both high fidelity and accuracy.

Exercise 02

1. The ammeter measures **electric current**.
2. The voltmeter measures **voltage**.
3. The ohmmeter measures **resistance**.
4. The wattmeter measures **power**.
5. The multimeter measures c**urrent, voltage, resistance, and other electrical properties**.
6. The oscilloscope **displays waveforms of electrical signals over time and frequency.**
7. The frequency meter measures **frequency**.
8. The period meter measures **the period of a signal**.
9. The phase meter determines **the phase difference between signals**.

Exercise 03

1) ,2) and 5) $\otimes$ No

3), 4), 6) and 7) $\otimes$ Yes

Exercise 04

1) Calculation of $\frac{\Delta P}{P}$ and ΔP for the following expression : $P = RI^2$

$$\ln(P) = \ln(R) + 2\ln(I)$$

The differential form :

$$\frac{dP}{P} = \frac{dR}{R} + 2\frac{dI}{I}$$

Thus :

$$\frac{\Delta P}{P} = \frac{\Delta R}{R} + 2\frac{\Delta I}{I}$$

Therefore :

$$\Delta P = I^2\Delta R + 2RI\Delta I$$

2) Calculation of $\frac{\Delta U}{U}$ and ΔU for the following expression :

$$U = \frac{R_1}{R_2 + R_1} E$$

The differential form :

$$\frac{\Delta U}{U} = \left|\frac{\partial U}{\partial R_1}\right|_{R_2, E=\text{const}} \frac{\Delta R_1}{U} + \left|\frac{\partial U}{\partial R_2}\right|_{R_1, E=\text{const}} \frac{\Delta R_2}{U} + \left|\frac{\partial U}{\partial E}\right|_{R_1, R_2=\text{const}} \frac{\Delta E}{U}$$

Thus :

$$\Delta U = \frac{R_1 E}{(R_1 + R_2)^2} \Delta R_1 + \frac{R_1 R_2 E}{(R_1 + R_2)^2} \Delta R_2 + \frac{R_1}{R_1 + R_2} \Delta E$$

Therefore :

$$\frac{\Delta U}{U} = \frac{R_1}{R_1 + R_2} \frac{\Delta R_1}{R_1} + \frac{R_1}{R_1 + R_2} \frac{\Delta R_2}{R_2} + \frac{\Delta E}{E}$$

3) Calculation of $\frac{\Delta Q}{Q}$ and ΔQ for the following expression : $Q = \sqrt{S^2 - P^2}$

$$\ln(Q) = \frac{1}{2} \ln(S^2 - P^2)$$

The differential form :

$$\frac{dQ}{Q} = \frac{S\,dS - P\,dP}{S^2 - P^2}$$

Thus :

$$\frac{\Delta Q}{Q} = \frac{\Delta S}{S} \left(\frac{S^2}{S^2 - P^2}\right) + \frac{\Delta P}{P} \left(\frac{P^2}{S^2 - P^2}\right)$$

Therefore :

$$\Delta Q = \Delta S \left(\frac{S}{\sqrt{S^2 - P^2}}\right) + \Delta P \left(\frac{P}{\sqrt{S^2 - P^2}}\right)$$

Exercise 05

Data : $C = G = 4000\,\Omega\,;\ N = 4000\,\text{points}\,;\ L = 1000\,\Omega$

The uncertainty formula is :

$$\Delta R = \pm\,(2\% \times L + 5\,\text{points})$$

Calculation of Absolute Uncertainty : Substituting the given values, we have :

$$\Delta R = 0.02 \times L + 5 \times \text{points}$$

$$\Delta R = 0.02 \times 1000 + 5 = 20 + 5 = 25\,\Omega$$

Relative Uncertainty : The relative uncertainty is :

$$\Delta R_{\%} = \frac{\Delta R}{L} \times 100 = \frac{25}{1000} \times 100 = 2.5\%$$

Two Ways to Present the Result :

— $R = 1000\,\Omega \pm 2.5\%$

— $R = 1000 \pm 25\,\Omega$

Exercise 06

Data : $E = (24 \pm 1)$ [**V**], $R_1 = 38\Omega \pm 1\%$, $R_2 = 20\Omega \pm 1\%$.

1. Expression of voltage U as a function of E, R_1, and R_2 Applying the voltage divider theorem, we obtain :
$$U = \frac{R_2}{2R_1 + R_2}E$$
2. Calculation of U
$$U = \frac{20}{2 \times 38 + 20} \times 24 = 5V$$
3. Expression of the relative uncertainty $\frac{\Delta U}{U}$
$$\Delta U = \left|\frac{\partial U}{\partial R_1}\right|_{R_2,E=cst} \Delta R_1 + \left|\frac{\partial U}{\partial R_2}\right|_{R_1,E=cst} \Delta R_2 + \left|\frac{\partial U}{\partial E}\right|_{R_1,R_2=cst} \Delta E$$
$$\frac{\Delta U}{U} = \frac{2}{2R_1 + R_2}\Delta R_1 + \frac{R_1}{2R_1 + R_2}\frac{\Delta R_2}{R_2} + \frac{\Delta E}{E}$$
4. Calculation of $\frac{\Delta U}{U}$ With $\Delta R_1 = \frac{1}{100}R_1 = 0.38\Omega$ and $\Delta R_2 = \frac{1}{100}R_2 = 0.2\Omega$

$$\frac{\Delta U}{U} = \frac{2}{96} \times 0.38 + \frac{2 \times 38}{96} \times 0.01 + \frac{1}{24}$$

$$\Rightarrow \left(\frac{\Delta U}{U}\right)\% = 5.78\%$$

5. We deduce the absolute error :

$$\Delta U = \frac{5.78}{100} \times U = 0.289V$$

Exercise 07

I. Volt-Ampere Method

a). The name of this circuit is : *Aval.*

b). The measured voltage and current are obtained as :

$$U_{\text{mes}} = \left(\frac{\text{Range} \times \text{Reading}}{\text{Scale}}\right) = \left(\frac{30V \times 20}{30}\right) = 20V$$

$$I_{\text{mes}} = \left(\frac{30mA \times 20}{30}\right) = 20mA = 0.02A$$

The resistance is then calculated using Ohm's Law :

$$R_{\text{mes}} = \frac{U_{\text{mes}}}{I_{\text{mes}}} = \frac{20V}{0.02A} = 1000\Omega$$

c). The absolute instrumental error is determined as follows :

$$\Delta U = \frac{\text{Class} \times \text{Range}}{100} = \frac{1.0 \times 30V}{100} = 0.3V$$

$$\Delta I = \frac{1.5 \times 30mA}{100} = 0.45mA = 0.00045A$$

The total instrumental error in resistance is given by :

$$\frac{\Delta R_{\text{inst}}}{R} = \frac{\Delta U}{U} + \frac{\Delta I}{I} = \frac{0.3}{20} + \frac{0.00045}{0.02} = 0.015 + 0.0225 = 0.0375$$

$$\Delta R_{\text{inst}} = R_{\text{mes}} \times 0.0375 = 1000 \times 0.0375 = 37.5\Omega$$

d). The method error is evaluated by considering the influence of the voltmeter's internal resistance :

$$R_{\text{actual}} = \frac{R_V \times R}{R_V + R}$$

Solving for the error,

$$\Delta R_{\text{meth}} = |R_{\text{mes}} - R_{\text{actual}}|$$

Substituting values,

$$\Delta R_{\text{meth}} = \left| \frac{-R_{\text{mes}}^2}{R_V - R_{\text{mes}}} \right| \approx 1\Omega$$

e). The total uncertainty in resistance is :

$$\Delta R = \Delta R_{\text{inst}} + \Delta R_{\text{meth}} = 37.5\Omega + 1\Omega = 38.5\Omega$$

The relative uncertainty is :

$$\frac{\Delta R}{R_{\text{mes}}} \times 100 = \frac{38.5}{1000} \times 100 = 3.85\%$$

II. Direct Measurement Method

1. Given :

$$C = 4000\Omega, \quad N = 4000 \text{ counts}, \quad L = 1000\Omega$$

The absolute uncertainty is calculated as :

$$\Delta R = \left(\frac{2}{100} \times L + \frac{5 \times C}{N} \right)$$

$$\Delta R = \left(\frac{2}{100} \times 1000 + \frac{5 \times 4000}{4000} \right) = (20 + 5) = 25\Omega$$

The relative uncertainty is :

$$\frac{\Delta R}{R} \times 100 = \frac{25}{1000} \times 100 = 2.5\%$$

2. The final result is expressed in two formats :

(a) $R = 1000\Omega \pm 2.5\%$

(b) $R = 1000 \pm 25\Omega$

Exercise 08

A). Wheatstone Bridge Schematic Representation

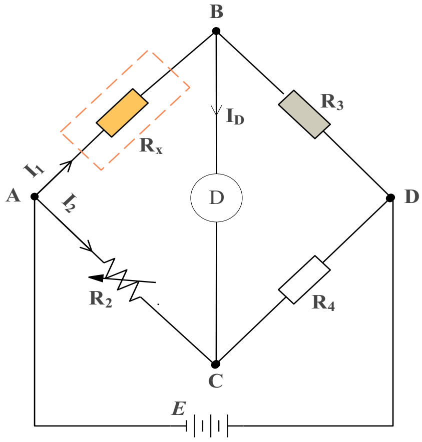


Fig. 19 *Configuration of the Wheatstone Bridge.*

B). At equilibrium, we have :

$$R_1 R = R_2 R_x \quad \Rightarrow \quad R_x = \frac{R_1 R}{R_2} \quad \Rightarrow \quad R_x = 252.1\,\Omega$$

C). Calculation of uncertainties :

$$\Delta R_1 = \frac{0.2}{100} \times 100 = 0.2\,\Omega; \quad \Delta R_2 = \frac{0.2}{100} \times 1000 = 2\,\Omega$$

$$\Delta R = \frac{0.2}{100} \times 2521 = 5.042\,\Omega$$

D). Demonstration of the relative uncertainty : From the expression in response 1, we have :

$$R_x = \frac{R_1 R}{R_2}$$

Applying the natural logarithm to this expression :

$$\ln(R_x) = \ln\left(\frac{R_1 R}{R_2}\right)$$

This simplifies to :

$$\ln(R_x) = \ln(R_1) + \ln(R) - \ln(R_2)$$

Differentiating this equation, we obtain :

$$\frac{dR_x}{R_x} = \frac{dR_1}{R_1} + \frac{dR}{R} - \frac{dR_2}{R_2}$$

Therefore, the relative uncertainty is :

$$\frac{\Delta R_x}{R_x} = \frac{\Delta R_1}{R_1} + \frac{\Delta R}{R} + \frac{\Delta R_2}{R_2}$$

E). Determination of the relative uncertainty $\frac{\Delta R_x}{R_x}$:

$$\frac{\Delta R_x}{R_x} = \frac{0.2}{100} + \frac{2}{1000} + \frac{5.042}{2521}$$

$$\frac{\Delta R_x}{R_x} = 0.002 + 0.002 + 0.002 = 0.006 \quad \Rightarrow \quad \frac{\Delta R_x}{R_x} = 0.6\%$$

F). Absolute uncertainty calculation :

$$\Delta R_x = 0.006 \times R_x = 0.006 \times \frac{R_1 \times R}{R_2} \quad \Rightarrow \quad \Delta R_x = 15.126\,\Omega$$

Exercise 09

P, Q : fixed pure resistances

$z_x = R_x + jL_x\omega$: unknown impedance

$\frac{1}{Z_4} = \frac{1}{R} + jC\omega$: known and variable impedance

1. MAXWELL Bridge
2. The condition for the bridge to be balanced is when $I_G = 0$, which means $U_{CD} = 0 \Leftrightarrow U_C = U_D$.

   The expression for Z_x is obtained by applying the voltage divider :

$$U_{AD} = \frac{P}{Z_4 + P}U(t) \quad \text{and} \quad U_{AC} = \frac{Z_x}{Q + Z_x}U(t)$$

   At equilibrium, we have :

$$U_{CD} = U_{CA} + U_{AD} = -\frac{Z_x}{Q + Z_x}U(t) + \frac{P}{Z_4 + P}U(t) = 0$$

From which :

$$Z_x Z_4 = PQ \quad \Rightarrow \quad Z_x = \frac{PQ}{Z_4} \quad \Rightarrow \quad R_x + jL_x\omega = PQ\left(\frac{1}{R} + jC\omega\right)$$

Thus, we get :

$$R_x = \frac{PQ}{R} \quad \text{and} \quad L_x = PQC$$

Calculating the values of R_x and L_x :

$$R_x = \frac{1.5 \times 0.2}{1.5} = 0.2\,\mathrm{k\Omega}$$

$$L_x = 1.5 \times 0.2 \times 10^6 \times 2 \times 10^{-6} = 0.6\,\mathrm{H} \quad \Rightarrow \quad L_x = 600\,\mathrm{mH}$$

3. Determining the relative uncertainty $\Delta R_x/R_x$ and $\Delta L_x/L_x$:

$$\Delta R_x/R_x = \frac{\Delta P}{P} + \frac{\Delta Q}{Q} + \frac{\Delta R}{R} \quad \Rightarrow \quad \Delta R_x/R_x = 2\% + 1\% + 2\% = 5\%$$

$$\Delta L_x/L_x = \frac{\Delta P}{P} + \frac{\Delta Q}{Q} + \frac{\Delta C}{C} \quad \Rightarrow \quad \Delta L_x/L_x = 2\% + 1\% + 5\% = 8\%$$

4. Calculating the values of ΔR_x and ΔL_x :

$$\Delta R_x = \frac{5}{100} R_x = 10\,\Omega$$

$$\Delta L_x = \frac{8}{100} L_x = 0.048\,\mathrm{H} = 48\,\mathrm{mH}$$

Exercise 10

1. a Calculation of the Measured Voltages

   The measured voltages are as follows :

$$U = \text{Reading} \times \frac{\text{Range}}{\text{Scale}}$$

$$U_1 = 74 \times \frac{1000}{100} = 740\,\mathrm{V}$$

$$U_2 = 18 \times \frac{30}{30} = 18\,\mathrm{V}$$

$$U = 50 \times \frac{10}{10} = 50\,\mathrm{V}$$

1.b Calculating the Absolute Uncertainties The absolute uncertainties are given by :

$$\Delta U = \text{Reading} \times \frac{\text{Class}}{100}$$

$$\Delta U_1 = 74 \times \frac{2.5}{100} = 1.85\,\mathrm{V}$$

$$\Delta U_2 = 18 \times \frac{2.5}{100} = 0.45\,\mathrm{V}$$

$$\Delta U = 50 \times \frac{2.5}{100} = 1.25\,\mathrm{V}$$

2. Calculating the Power

   The power equation is given by :

$$P = \frac{1}{2R}(U_1^2 - U_2^2 - U^2)$$

   Substituting the values :

$$P = \frac{1}{2 \times 110}(740^2 - 18^2 - 50^2)$$

$$P = \frac{1}{220}(547600 - 324 - 2500) = \frac{1}{220}(544776) = 2472.6\,\mathrm{W}$$

   The partial derivatives of power with respect to the variables are :

$$\Delta P = \left|\frac{\partial P}{\partial U_1}\right|_{U_2,U,R=cst} \Delta U_1 + \left|\frac{\partial P}{\partial U_2}\right|_{U_1,U,R=cst} \Delta U_2 + \left|\frac{\partial P}{\partial U}\right|_{U_1,U_2,R=cst} \Delta U + \left|\frac{\partial P}{\partial R}\right|_{U_1,U_2,U=cst} \Delta R$$

3. a. Calculating the Absolute Uncertainty of Power

$$\Delta p = 2R\left(U_1\Delta U_1 + U_2\Delta U_2 + U\Delta U + \left(U_1^2 - U_2^2 - U_2\right)\frac{2}{R^2}\Delta R\right) = 49.481 \text{ Watt}$$

   b. The relative uncertainty is given by :

$$\frac{\Delta p}{p} \times 100 = \frac{49.481}{2472.6} \times 100 = 2.00\%$$

4. Final Results

The measured power is :

$$p = 2472.6\,\text{W} \pm 11.75\,\text{W}$$

$$p = 2472.6 \pm 49.481\text{W}$$

Exercise 11

a) Uncertainty in Apparent Power S

Uncertainty in Current ΔI

The absolute uncertainty in the current measured by the ammeter is given by :

$$\Delta I = \frac{1 \times 20}{100} = 0.2\,\text{A}$$

The relative uncertainty in current is :

$$\frac{\Delta I}{I} = \frac{0.2}{20} = 1\%$$

Uncertainty in Voltage ΔV :

The absolute uncertainty in the voltage measured by the voltmeter is given by :

$$\Delta V = \frac{1.5 \times 500}{100} = 7.5\,\text{V}$$

The relative uncertainty in voltage is :

$$\frac{\Delta V}{V} = \frac{7.5}{500} = 1.5\%$$

Uncertainty in Apparent Power S

The apparent power S is given by $S = V \times I = 10kVAR$. The relative uncertainty in S is the sum of the relative uncertainties in V and I :

$$\frac{\Delta S}{S} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = 1.5\% + 1\% = 2.5\%$$

The absolute uncertainty in the apparent power is :

$$\Delta S = 2.5\% \times (V \times I) = 2.5\% \times 500 \times 20 = 250\,\text{VA}$$

b) Calculation of Total Active Power P

Calculation of P_1 and P_2 :

For P_1 :

$$P_1 = \frac{600 \times 25 \times 66}{150} = 6600\,\mathrm{W}$$

For P_2 :

$$P_2 = \frac{600 \times 25 \times 26}{150} = 2600\,\mathrm{W}$$

Calculation of Total Active Power P :

The total active power is :

$$P = P_1 + P_2 = 6600 + 2600 = 9200\,\mathrm{W}$$

Calculation of Reactive Power Q

The reactive power Q is calculated by the difference between P_1 and P_2, multiplied by $\sqrt{3}$:

$$Q = (P_1 - P_2) \times \sqrt{3} = (6600 - 2600) \times \sqrt{3} = 4000 \times \sqrt{3}$$

Calculating Q :

$$Q \approx 4000 \times 1.732 = 6930\,\mathrm{VAR}$$

Calculation of Uncertainties in Active Power P and Reactive Power Q

The total uncertainty of P and Q depends on the uncertainties of P1 and P2 : For P_1 :

$$\frac{\Delta P_1}{P_1} = \frac{225}{6600} \times 100 = 3.41\%$$

For P_2 :

$$\frac{\Delta P_2}{P_2} = \frac{225}{2600} \times 100 = 8.65\%$$

Relative Uncertainty in P

The absolute uncertainty in P is $\Delta P = \Delta P_1 + \Delta P_2 = 450\,\mathrm{W}$.

The relative uncertainty in P is :

$$\varepsilon_{\%} = \frac{\Delta P}{P} = \frac{450}{9200} \times 100 = 4.89\%$$

Relative Uncertainty in Q :

$$\frac{\Delta Q}{Q} = 3.41\% + 8.65\% = 12.06\%$$

Calculation of the absolute uncertainty of Q.

$$\Delta Q = 6930 * 0.12 = 831.6VAR$$

c) The results presented in two distinct formats.

1. **Result of apparent power** S :
   (a) $S = S \pm \Delta S$, VA $= 10 \pm 0.25$ kVA
   (b) $S = S$ VA $\pm \frac{\Delta S}{S} = 10$ kVA $\pm 2.5\%$
2. **Result of active power** P :
   (a) $P = P \pm \Delta P$, W $= 9200 \pm 450$ W
   (b) $P = P$ W $\pm \frac{\Delta P}{P} = 9200$ W $\pm 4.89\%$
3. **Result of reactive power** Q :
   (a) $Q = Q \pm \Delta Q$, VAR $= 6930 \pm 450$ VAR
   (b) $Q = Q$ VAR $\pm \frac{\Delta Q}{Q} = 6930$ VAR $\pm 12.06\%$

Exercise 12

Given Data

We have measured a voltage of $V = 25V$ using :

1. **Analog Voltmeter :** Class 1.5, Range $3V$, 30-division scale, estimation error of half a division.
2. **Digital Voltmeter :** 2000 counts on a $20V$ range, accuracy of $\pm$(0.1% of the reading +0.01% of the range).

Uncertainty Calculation for the Analog Voltmeter

Absolute Uncertainty Calculation

The absolute uncertainty consists of two components :

1. Instrument Class Contribution :

$$\Delta V_{\text{class}} = \frac{1.5}{100} \times 3V = 0.045V$$

2. Reading Estimation Contribution :

$$\Delta V_{\text{reading}} = \frac{0.5}{30} \times 3V = 0.05V$$

3. Total Absolute Uncertainty :

$$\Delta V_{\text{analog}} = \Delta V_{\text{class}} + \Delta V_{\text{reading}} = 0.045V + 0.05V = 0.095V$$

Relative Uncertainty Calculation

$$\Delta V_{\%} = \frac{\Delta V_{\text{analog}}}{V} \times 100 = \frac{0.095V}{25V} \times 100 = 0.38\%$$

Uncertainty Calculation for the Digital Voltmeter

Absolute Uncertainty Calculation

1. Contribution from the Reading :

$$0.1\% \times 25V = 0.001 \times 25 = 0.025V$$

2. Contribution from the Range :

$$0.01\% \times 20V = 0.0001 \times 20 = 0.002V$$

3. Total Absolute Uncertainty :

$$\Delta V_{\text{digital}} = 0.025V + 0.002V = 0.027V$$

Relative Uncertainty Calculation

$$\Delta V_{\%} = \frac{\Delta V_{\text{digital}}}{V} \times 100 = \frac{0.027V}{25V} \times 100 = 0.108\%$$

Exercise 13

The figure (20) represents the face of an analog voltmeter with a needle. Below is the meaning of each symbol typically displayed on this device.

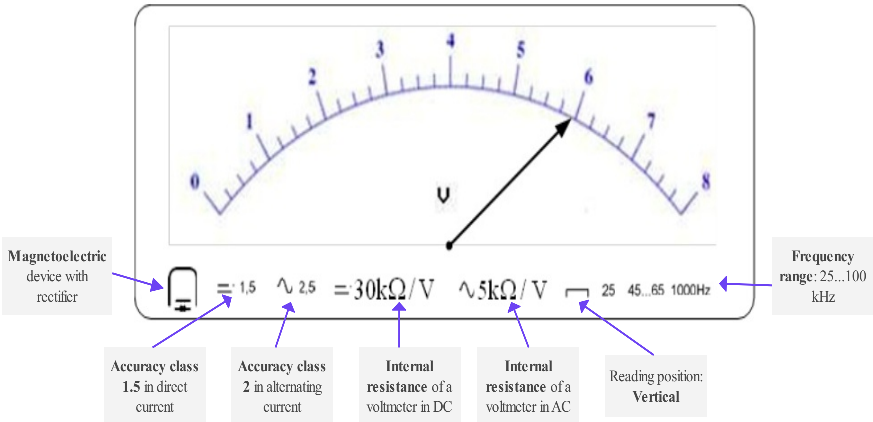


Fig. 20

Exercise 15

1) Voltmeter Diagram

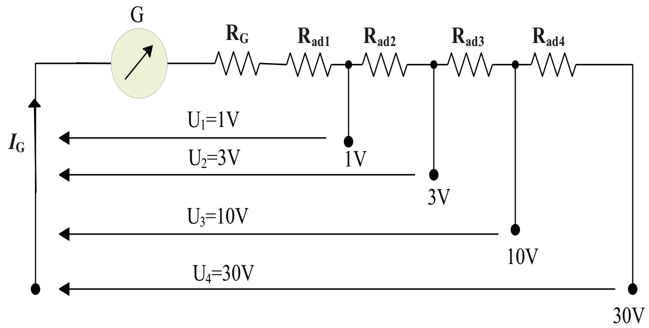


Fig. 21

2) Calculation of Additional Resistances

a). Calculation of R_{a1} for $U_1 = 1\,\mathrm{V}$

$$U_1 = R_G I_G + R_{a1} I_G \Rightarrow R_{a1} = \frac{U_1}{I_G} - R_G$$

$$R_{a1} = \frac{1}{50 \times 10^{-6}} - 10^3$$

$$R_{a1} = 19\,k\Omega$$

b). Calculation of R_{a2} for $U_2 = 3\,\mathrm{V}$

$$U_2 = (R_G + R_{a1} + R_{a2}) I_G \Rightarrow R_{a2} = \frac{U_2}{I_G} - (R_G + R_{a1})$$

$$R_{a2} = \frac{3}{50 \times 10^{-6}} - (1 + 19) \times 10^3$$

$$R_{a2} = 40\,k\Omega$$

c).Calculation of R_{a3} for $U_3 = 10\,\mathrm{V}$

$$U_3 = (R_G + R_{a1} + R_{a2} + R_{a3}) I_G \Rightarrow R_{a3} = \frac{U_3}{I_G} - (R_G + R_{a1} + R_{a2})$$

$$R_{a3} = \frac{10}{50 \times 10^{-6}} - (1 + 19 + 40) \times 10^3$$

$$R_{a3} = 140\,k\Omega$$

d) Calculation of R_{a4} for $U_4 = 30\,\mathrm{V}$

$$U_4 = (R_G + R_{a1} + R_{a2} + R_{a3} + R_{a4})I_G \Rightarrow R_{a4} = \frac{U_4}{I_G} - (R_G + R_{a1} + R_{a2} + R_{a3})$$

$$R_{a4} = \frac{30}{50 \times 10^{-6}} - (1 + 19 + 40 + 140) \times 10^3$$

$$R_{a4} = 400\,k\Omega$$

3) Characteristic Resistance

$$R_c = \frac{1}{I_G}$$

$$R_c = \frac{1}{50 \times 10^{-6}} = 2000\,\Omega/V$$

4) Total Resistance

$$R_T = R_c \times \text{Highest Measurement Range}$$

$$R_T = 2000 \times 30 = 60000\,\Omega$$

Exercise 16

1) Multi-range Ammeter :

a) Schematic Diagram :

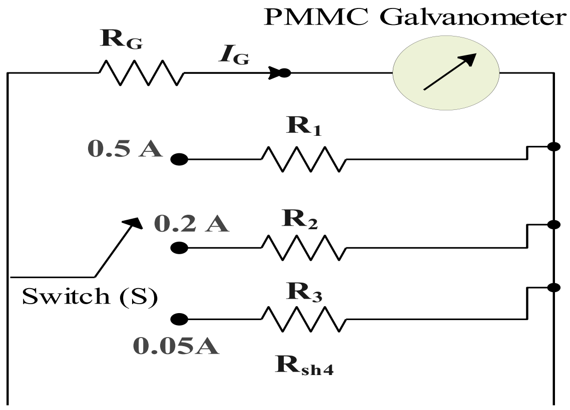


Fig. 22

b) Determining the Multiplier Factors :

We have the following relation for the multiplier :

$$m_i = \frac{I_i}{I_G}$$

Where I_i is the current corresponding to each range. Therefore :

$$m_1 = \frac{0.5}{0.5 \times 10^{-3}} = 1000$$

$$m_2 = \frac{0.2}{0.5 \times 10^{-3}} = 400$$

$$m_3 = \frac{0.05}{0.5 \times 10^{-3}} = 100$$

c) Determining the Shunt Resistances :

Using the formula :

$$R_i = \frac{R_G}{m_i - 1}$$

We calculate the shunt resistances :

$$R_1 = \frac{50}{1000 - 1} = 0.05\,\Omega$$

$$R_2 = \frac{50}{400 - 1} = 0.125\,\Omega$$

$$R_3 = \frac{50}{100 - 1} = 0.5\,\Omega$$

2) Universal Ammeter :

a) Schematic Diagram :

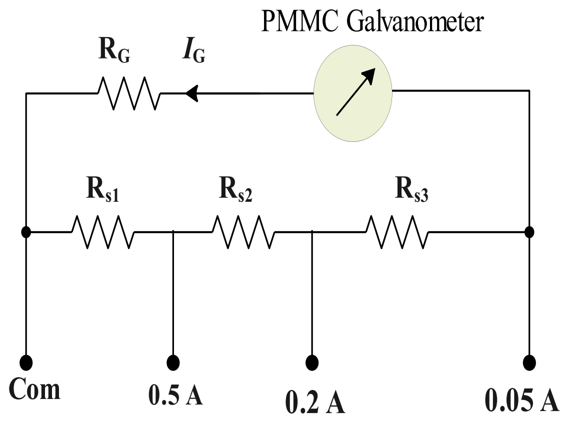


Fig. 23

b) Determining the Multiplier Factors m_1, m_2, and m_3 in Terms of R_{S1}, R_{S2}, and R_{S3} :

We use the following relationships to determine the multiplier factors :

$$R_{S1}(I_1 - I_G) = (R_{S2} + R_{S3} + R_G)I_G$$

$$R_{S1}(1 - m_1) = (R_{S2} + R_{S3} + R_G)$$

$$m_1 = \frac{R_{S1}}{R_{S1} + R_{S2} + R_{S3} + R_G}$$

For the second range :

$$(R_{S1} + R_{S2})(I_2 - I_G) = (R_{S3} + R_G)I_G$$

$$(R_{S1} + R_{S2})(1 - m_2) = (R_{S3} + R_G)$$

$$m_2 = \frac{R_{S1} + R_{S2}}{R_{S1} + R_{S2} + R_{S3} + R_G}$$

For the third range :

$$(R_{S1} + R_{S2} + R_{S3})(I_3 - I_G) = R_G I_G$$

$$(R_{S1} + R_{S2} + R_{S3})(1 - m_3) = R_G I_G$$

$$m_3 = \frac{R_{S1} + R_{S2} + R_{S3}}{R_{S1} + R_{S2} + R_{S3} + R_G}$$

c) Calculation of R_{S1}, R_{S2}, and R_{S3} :

Now, using the relations for the shunt resistances :

$$R_{S2} = R_{S1} \times \left(\frac{m_1 - m_2}{m_2}\right)$$

$$R_{S2} = 1.5 \times R_{S1}$$

Similarly, for R_{S3} :

$$R_{S3} = 7.5 \times R_{S1}$$

Finally, by using the resistance formula for all three ranges :

$$R_{S1}(m_1 - 1 - 1.5 - 7.5) = R_G$$

$$R_{S1} = 0.05\,\Omega$$

Thus, we obtain :

$$R_{S2} = 0.075\,\Omega$$

$$R_{S3} = 0.375\,\Omega$$

Exercise 17

The voltage drop across R_b without the voltmeter connected is calculated using the voltage equation

$$V_b = V \times \frac{R_b}{R_a + R_b}$$

Substituting the values :

$$V_b = 50 \times \frac{5}{45 + 5} = 50 \times \frac{5}{50} = 5\,V$$

Thus, the true voltage across R_b is $5\,V$.

(i) On the 5 V range :

$$R_m = S \times range = 20k\Omega \times 5V = 100k\Omega$$

$$R_{eq} = \frac{R_b \times R_m}{R_b + R_m} = 4.762k\Omega$$

The voltmeter reading is

$$V_R b = 50 \times \frac{4.762}{45 + 4.762} = 50 \times \frac{4.762}{49.762} \approx 4.79\,V$$

The % error on the $5V$ range is

$$\%\,\text{Error} = \frac{\text{Actual voltage} - \text{Voltage reading in meter}}{\text{Actual voltage}} \times 100 = \left(\frac{5 - 4.79}{5}\right) \times 100 \approx 4.2\%$$

(ii) On 10 V range

$$R_m = 200\,k\Omega, \quad R_{eq} = \frac{5 \times 200}{5 + 200} = \frac{1000}{205} \approx 4.878\,k\Omega$$

$$V_m = 50 \times \frac{4.878}{45 + 4.878} = 50 \times \frac{4.878}{49.878} \approx 4.89\,V$$

$$\text{Percentage Error} = \left(\frac{5 - 4.89}{5}\right) \times 100 \approx 2.2\%$$

(iii) For the 30 V Range :

$$R_m = 600\,k\Omega, \quad R_{eq} = \frac{5 \times 600}{5 + 600} = \frac{3000}{605} \approx 4.959\,k\Omega$$

$$V_m = 50 \times \frac{4.959}{45 + 4.959} = 50 \times \frac{4.959}{49.959} \approx 4.96\,V$$

$$\text{Percentage Error} = \left(\frac{5 - 4.96}{5}\right) \times 100 \approx 0.8\%$$

3 PROPOSED EXERCISES

Exercise 01

To measure R_x, the resistance of a motor winding, we used an ammeter with an internal resistance R_A for the selected range, and a voltmeter with resistance R_V. The readings on the devices are as follows :

Device	*Range*	*Scale*	*Reading*	*Class*(%)	*ReadingError*
Ammeter	1 A	100	75	1.5	0.5
Voltmeter	3 V	30	18	1.5	0.5

1) Determine the measured voltage U, the measured current I, and the power P.

2) Deduce the measured value of the resistance R_x.

3) Express $\frac{\Delta R_x}{R_x}$ in terms of $\frac{\Delta U}{U}$ and $\frac{\Delta I}{I}$.

4) Calculate the relative error $\frac{\Delta R_x}{R_x}$ and the absolute error ΔR_x.

5) Write the result of the resistance R_x in two different ways.

Exercise 02

We want to measure the power of a receiver using the so-called **three-ammeter method** (see schematic). The measurements are provided in the following table :

Ammeter	Range (A)	Reading	Scale	Class (%)	Reading Error
A_1	10A	74	100	2.5	0.5
A_2	3A	20	30	2.5	0.25
A	10A	57	100	2.5	0.5

Tableau 5 – Measured values from the ammeters

Tasks

1. Complete the Following Table

I_1	ΔI_1	I_2	ΔI_2	I	ΔI

Tableau 6 – Table to be completed

2. Compute the Power P

The power P is given by the formula :

$$P = \frac{R}{2}\left(I^2 - I_1^2 - I_2^2\right)$$

3. Show that the Relative Uncertainty is Given by :

$$\frac{\Delta P}{P} = \frac{\Delta R}{R} + \frac{2I_1\Delta I_1 + 2I_2\Delta I_2 + 2I\Delta I}{I^2 - I_1^2 - I_2^2}$$

4. Calculate the Uncertainty

Compute the relative uncertainty $\frac{\Delta P}{P}$ and then determine the absolute uncertainty ΔP.

5. Present the Result in Two Formats

Express the final result in two different formats.

Given Data :

$$R = 110\Omega \pm 2\%$$

We aim to design an ammeter with three measurement ranges (3A, 1A, 0.1A) using two different methods. The given moving-coil meter has the following characteristics :

— Internal resistance : $R_g = 50\Omega$

— Full-scale deflection current : $I_g = 50mA$

1. Multi-Range Ammeter

1. Draw the circuit diagram of a multi-range ammeter.
2. Determine the multiplication factors for each range.
3. Calculate the shunt resistances R_1, R_2, and R_3.

Exercise 03

Part A : Universal Shunt Ammeter

1. Draw the circuit diagram of a universal shunt ammeter.
2. Express the multiplication factors m_1, m_2, m_3 in terms of R_1, R_2, R_3, and R_g.
3. Determine the ratios $\frac{m_1}{m_2}$ and $\frac{m_2}{m_3}$, then express R_2 and R_3 as functions of R_1.
4. Compute the values of the shunt resistances R_1, R_2, R_3.

Part B : Designing a Voltmeter

We aim to design a voltmeter with four measurement ranges (5V, 1V, 0.5V, 0.1V). The given moving-coil meter has the following characteristics :

— Internal resistance : $R_g = 50\Omega$

— Full-scale deflection current : $I_g = 50mA$

Tasks

1. Draw the circuit diagram of the voltmeter.
2. Calculate the required series resistances.
3. Determine the characteristic resistance of the voltmeter.
4. Compute the total resistance of the voltmeter.

ANNEXES A

a). color method

Table (A.1) illustrates how to read resistors based on each band. This method is particularly suitable for low-power resistors. However, it is important to note that the color method may be limited in terms of accuracy and tolerance and is more suitable for standardized resistance values.

Tableau A.1 – Color Code Table of a Resistor :

Colors	Bands on the left side of the resistor			Bands on the right side of the resistor		
	Significant figures			Multiplier	Tolerance	Temperature coefficient (ppm/°C)
	1st band	**2nd band**	**3rd band**	**4th band**	**5th band**	**6th band**
Black	**0**	**0**	**0**	10^0		
Brown	**1**	**1**	**1**	10^1	±1.00%	**100**
Red	**2**	**2**	**2**	10^2	±2.00%	**50**
Orange	**3**	**3**	**3**	10^3		**15**
Yellow	**4**	**4**	**4**	10^4		**25**
Green	**5**	**5**	**5**	10^5	±0.5%	
Blue	**6**	**6**	**6**	10^6	±0.25%	**10**
Purple	**7**	**7**	**7**	10^7	±0.1%	**5**
Gray	**8**	**8**	**8**	10^8	±0.05%	
White	**9**	**9**	**9**	10^9		**1**
Gold				10^{-1}	±5%	
Silver				10^{-2}	±10%	

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