

IBN KHALDOUN UNIVERSITY OF TIARET

FACULTY OF APPLIED SCIENCES
Department of Science and Technology

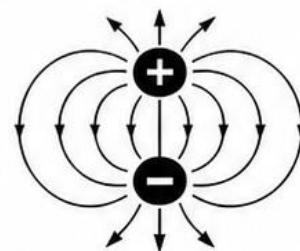
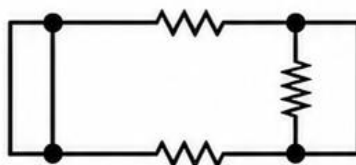
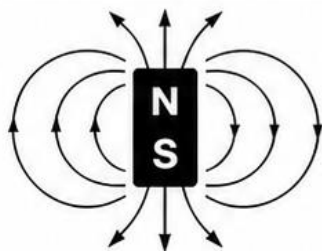
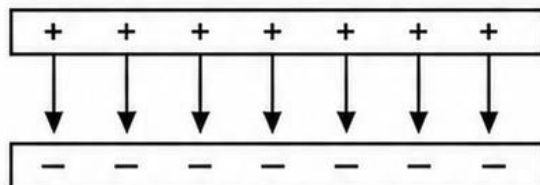


PEDAGOGICAL HANDOUT

COURSES AND EXERCISES WITH SOLUTIONS

PHYSICS 2

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Program: **1st Year Science and
Technology (LMD/ENG)**



Semester: **2**



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Foreword:

Physics 2 is one of the fundamental modules designed for first-year students in the field of Science and Technology, including both LMD students and Engineering preparatory students. This module aims to provide students with the basic concepts of electricity and magnetism while developing their analytical skills, mathematical reasoning, and ability to apply physical laws to different electrical and electromagnetic systems.

The module is taught during the second semester with a weekly schedule consisting of one hour and a half of lectures and three hours of tutorial sessions, allowing students to combine theoretical understanding with practical problem-solving approaches.

This handbook is organized into four main parts. The first part presents the essential mathematical tools required for the study of physics, including elements of length, surface, and volume in different coordinate systems, as well as multiple derivatives and integrals. The second part is devoted to electrostatics, covering electric charges, electric fields, electric potential, electric dipoles, Gauss's law, conductors in electrostatic equilibrium, and capacitors.

The third part focuses on electrokinetics, where electric currents and electrical circuits are studied through Ohm's law, Joule's law, circuit analysis, and Kirchhoff's laws. The fourth part deals with electromagnetism and introduces the magnetic field, Lorentz force, Laplace force, Faraday's law of induction, Biot-Savart law, and magnetic dipoles.

Special attention has been given to the pedagogical aspect of this handbook. Numerous worked examples and application exercises covering the different topics of the module are included, together with simplified and progressive explanations intended to help students better understand the physical concepts and acquire rigorous problem-solving methods. Emphasis has also been placed on developing scientific reasoning and methodological skills that are essential for undergraduate students in science and engineering.



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**MATHEMATICAL
REMINDEERS**

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**ELEMENTS OF LENGTH, SURFACE, AND VOLUME
IN CARTESIAN, CYLINDRICAL, SPHERICAL COORDINATE SYSTEMS**

DIFFERENTIAL OPERATORS

MULTIPLE DERIVATIVES AND INTEGRALS

— • —

Chapter I: Mathematical Reminders

1. Introduction :

In physics and applied mathematics, choosing an appropriate coordinate system is essential for describing positions and simplifying the study of physical phenomena. The most commonly used coordinate systems are the Cartesian, cylindrical, and spherical coordinate systems.

In these coordinate systems, we have already studied the differential elements of length, surface, and volume, whose mathematical expressions vary according to the nature of each system.

Vector analysis also makes use of important differential operators such as the Nabla operator, the gradient, the divergence, and the curl (rotation), which play a fundamental role in many areas of physics and engineering.

In this chapter, we will review these operators and express them in Cartesian, cylindrical, and spherical coordinates.

2. Elements of Length, Surface, and Volume in Different Coordinate Systems:

2.1. General Definitions

- Element of length dl :

A differential vector representing a very small displacement in space (direction + magnitude).

- Element of surface dS :

A very small surface element representing a portion of a surface with a normal direction.

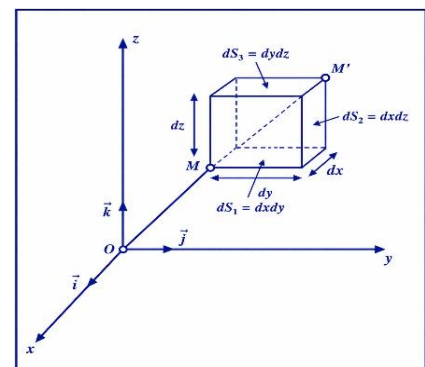
- Element of volume dV :

A very small volume element representing a portion of space.

2.2. Cartesian Coordinate System

Consider a Cartesian coordinate system $R(Oxyz)$ equipped with the basis $(\vec{i}, \vec{j}, \vec{k})$

M is a point in space in this coordinate system, its coordinates are (x, y, z)



➤ Element of length:

$$dl_1 = dx, dl_2 = dy, dl_3 = dz$$

$$\overrightarrow{dl} = dx \vec{i} + dy \vec{j} + dz \vec{k}$$

$$dl = \sqrt{(dx^2 + dy^2 + dz^2)}$$

(represents a small straight displacement along the three axes)

➤ Element of surface:

$$dS_1 = dx dy$$

small rectangle in the
 Oxy plane)

$$dS_2 = dx dz$$

(surface in Oxz)

$$dS_3 = dy dz$$

(surface in Oyz)

➤ Element of volume:

$$dV = dx dy dz$$

(represents a very small rectangular parallelepiped)

Exercise:

- Calculate the area of a rectangle with sides a and b.
- Calculate the volume of a rectangular box with dimensions a, b, c.

Solution:

$$S = \int_0^b \int_0^a dx dy \rightarrow$$

$$S = x \Big|_0^a y \Big|_0^b \rightarrow$$

$$S = a \times b$$

$$V = \int_0^c \int_0^b \int_0^a dx dy dz \rightarrow$$

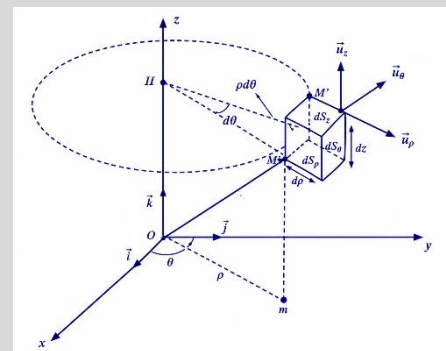
$$V = x \Big|_0^a y \Big|_0^b z \Big|_0^c \rightarrow$$

$$V = a \times b \times c$$

2.3. Cylindrical Coordinate System

Consider a Cylindrical coordinate system $R(O, \rho, \theta, z)$ equipped with the basis $(\vec{u}_\rho, \vec{u}_\theta, \vec{u}_z)$.

M is a point in space in this coordinate system, its coordinates are (ρ, θ, z) .



➤ Element of length:

$$dl = \underbrace{d\rho}_{dl_1} \vec{u}_\rho + \underbrace{\rho d\theta}_{dl_2} \vec{u}_\theta + \underbrace{dz}_{dl_3} \vec{u}_z$$

$$dl = \sqrt{(d\rho^2 + \rho^2 d\theta^2 + dz^2)}$$

$dl_1 = d\rho$	$dl_2 = \rho d\theta$	$dl_3 = dz$
radial displacement	circular displacement	vertical displacement

➤ Element of surface :

$dS = \rho d\rho d\theta$ (small disk element) (z = constant)	$dS = \rho d\theta dz$ (lateral cylindrical surface) ($\rho = \text{constant}$)	$dS = d\rho dz$ (radial plane surface) ($\theta = \text{constant}$)
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➤ Element of volume:

$$dV = \rho d\rho d\theta dz$$

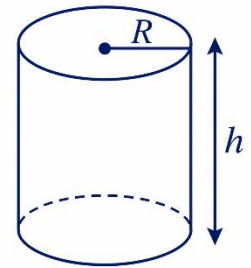
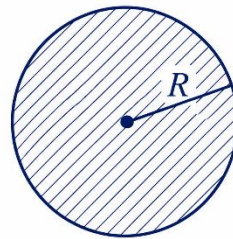
(represents a small cylindrical volume)

Note :

$0 \leq r \leq R$	$0 \leq \theta \leq 2\pi$	$0 \leq z \leq h$
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Exercise:

- 1) Calculate the circumference of a circle of radius R.
- 2) Calculate the area of a disk of radius R.
- 3) Calculate the lateral surface area of a cylinder of radius R and height h.
- 4) Calculate the volume of a cylinder.



Solution:

- 1) Calculate the circumference of a circle of radius R:

$$L = \int r d\theta = R\theta \Big|_0^{2\pi} = R \times (2\pi - 0)$$

$$\boxed{L = 2\pi R}$$

- 2) Calculate the area of a disk of radius R

$$S = \int_0^{2\pi} \int_0^R r dr d\theta \quad S = \left(\frac{1}{2}R^2 - 0\right) \times (2\pi - 0)$$

$$\boxed{S = \pi R^2}$$

3) Calculate the lateral surface area of a cylinder of radius R and height h

$$S = \int_0^h \int_0^{2\pi} R d\theta dz \quad ; \quad S = R (2\pi - 0) \times (h - 0)$$

$$\boxed{S = 2\pi Rh}$$

4) Calculate the volume of a cylinder

$$V = \iiint r dr d\theta dz = \int_0^h \int_0^{2\pi} \int_0^R r dr d\theta dz$$

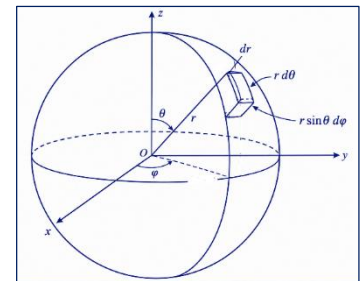
$$V = \left(\frac{1}{2} R^2 - 0\right) \times (2\pi - 0) \times (h - 0)$$

$$\boxed{V = \pi R^2 h}$$

2.4. Spherical Coordinate System

Consider Cylindrical coordinate system $R(O, r, \theta, \varphi)$ equipped with the basis $(\vec{u}_r, \vec{u}_\theta, \vec{u}_\varphi)$.

M is a point in space in this coordinate system, its coordinates are (r, θ, φ) .



➤ Element of length:

$$\vec{dl} = \underbrace{dr}_{dl_1} \vec{u}_r + \underbrace{r d\theta}_{dl_2} \vec{u}_\theta + \underbrace{r \sin\theta d\varphi}_{dl_3} \vec{u}_\varphi$$

$dl_1 = dr$
radial displacement

$dl_2 = r d\theta$
polar displacement

$dl_3 = r \sin\theta d\varphi$
azimuthal displacement

➤ Element of surface:

$dS_1 = r^2 \sin\theta d\theta d\varphi$
(sphere surface element)
($r = \text{constant}$)

$dS_2 = r \sin\theta dr d\varphi$
(conical surface element)
($\theta = \text{constant}$)

$dS_3 = r dr d\theta$
(meridian plane surface element)
($\varphi = \text{constant}$)

➤ Element of volume:

$$dV = r^2 \sin\theta dr d\theta d\varphi$$

(represents a small spherical volume)

Note :

$$0 \leq r \leq R$$

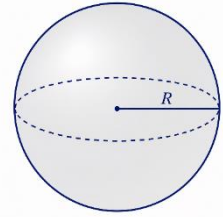
$$0 \leq \theta \leq \pi$$

$$0 \leq \varphi \leq 2\pi$$

Exercise:

Consider a sphere of radius R described in spherical coordinates (r, θ, ϕ) .

1. Compute the surface area S of the sphere starting from the surface element and using integration.
2. Compute the volume V of the sphere starting from the volume element and using integration.



Solution:

1. Surface Area :

Surface element (for $r = R$ (constant)):

$$dS = r^2 \sin\theta \, d\theta \, d\phi$$

$$S = \int_0^{2\pi} \int_0^{\pi} r^2 \sin\theta \, d\theta \, d\phi \quad \Bigg| \quad S = R^2 \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi \quad \Bigg| \quad S = R^2 (-\cos\theta) \Big|_0^{\pi} \Big|_0^{2\pi} \phi$$

$$S = R^2 \underbrace{(-\cos\pi + \cos 0)}_{1+1=2} \times (2\pi - 0)$$

$$S = 4\pi R^2$$

Compute the volume V of the sphere starting from the volume element and using integration.

2. Volume of the sphere :

$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

Express the volume as a triple integral using dV , then compute it.

Use the limits

$$0 \leq r \leq R \quad ; \quad 0 \leq \theta \leq \pi$$

$$0 \leq \phi \leq 2\pi$$

$$V = \int_0^{2\pi} \int_0^{\pi} \int_0^R r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$V = \left(\frac{1}{3}r^3\right) \Big|_0^R (-\cos\theta) \Big|_0^{\pi} (\phi) \Big|_0^{2\pi}$$

$$V = \left(\frac{1}{3}R^3 - 0\right) \underbrace{(-\cos\pi + \cos 0)}_{=2} (2\pi - 0) \Big|_0^{2\pi}$$

$$V = \frac{4}{3}\pi R^3$$

3. Differential Operators

3.1. Nabla Operator

The nabla operator is a vector differential operator used to represent spatial derivatives in vector calculus.

- ✓ Cartesian Coordinates (x, y, z)

$$\vec{\nabla} = \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k}$$

- ✓ Cylindrical Coordinates (ρ, θ, z)

$$\vec{\nabla} = \frac{\partial}{\partial \rho} \vec{u}_\rho + \frac{1}{\rho} \frac{\partial}{\partial \theta} \vec{u}_\theta + \frac{\partial}{\partial z} \vec{k}$$

- ✓ Spherical Coordinates (r, θ, ϕ)

$$\vec{\nabla} = \frac{\partial}{\partial r} \vec{u}_r + \frac{1}{r} \frac{\partial}{\partial \theta} \vec{u}_\theta + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \vec{u}_\phi$$

3.2. Gradient

The gradient of a scalar field f is a vector field defined as the action of the nabla operator on f :

$$\vec{\nabla} f = \overrightarrow{\text{grad} f}$$

It represents the direction and rate of the maximum increase of the function.

- ✓ Cartesian Coordinates (x, y, z)

Let $f = f(x, y, z)$, then:

$$\overrightarrow{\text{grad} f} = \vec{\nabla} f = \frac{\partial f}{\partial x} \vec{i} + \frac{\partial f}{\partial y} \vec{j} + \frac{\partial f}{\partial z} \vec{k}$$

- ✓ Cylindrical Coordinates (ρ, θ, z)

Let $f = f(\rho, \theta, z)$ then:

$$\overrightarrow{\text{grad} f} = \vec{\nabla} f = \frac{\partial f}{\partial \rho} \vec{u}_\rho + \frac{1}{\rho} \frac{\partial f}{\partial \theta} \vec{u}_\theta + \frac{\partial f}{\partial z} \vec{k}$$

- ✓ Spherical Coordinates (r, θ, ϕ)

Let $f = f(r, \theta, \phi)$ then:

$$\overrightarrow{\text{grad}}f = \vec{\nabla}f = \frac{\partial f}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{u}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\vec{u}_\varphi$$

Exercise1:

Let the scalar field be defined by:

$$f(x, y, z) = x^2y + yz^2$$

1. Compute the gradient $\vec{\nabla}f$.
2. Evaluate $\vec{\nabla}f$ at the point $M(1,1,1)$.

Solution:

1) Compute the gradient

$$\overrightarrow{\text{grad}}f = \vec{\nabla}f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

$$\frac{\partial f}{\partial x} = 2xy$$

$$\frac{\partial f}{\partial y} = x^2 + z^2$$

$$\frac{\partial f}{\partial z} = 2yz$$

$$\vec{\nabla}f = 2xy\vec{i} + (x^2 + z^2)\vec{j} + 2yz\vec{k}$$

2) At the point $M(1, 1, 1)$

$$\vec{\nabla}f(1, 1, 1) = 2\vec{i} + 2\vec{j} + 2\vec{k}$$

Exercise2:

Let:

$$f(\rho, \theta, z) = \rho^2 + z^2$$

1. Compute $\vec{\nabla}f$ in cylindrical coordinates.
2. Evaluate it at $(\rho, z) = (1,1)$.

Solution:

$$\overrightarrow{\text{grad}}f = \vec{\nabla}f = \frac{\partial f}{\partial \rho}\vec{u}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \theta}\vec{u}_\theta + \frac{\partial f}{\partial z}\vec{k}$$

$$\frac{\partial f}{\partial \rho} = 2\rho$$

$$\frac{\partial f}{\partial \theta} = 0$$

$$\frac{\partial f}{\partial z} = 2z$$

$$\vec{\nabla}f = 2\rho\vec{u}_\rho + 2z\vec{k}$$

At $(\rho, z) = (1,1)$:

$$\vec{\nabla}f = 2\vec{u}_\rho + 2\vec{k}$$

3.3. Divergence

The divergence of a vector field \vec{A} is a scalar quantity defined as:

$$\text{Div } \vec{A} = \vec{\nabla} \cdot \vec{A}$$

✓ Cartesian Coordinates (x, y, z)

$$\text{Let } \vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$

$$\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

✓ Cylindrical Coordinates (ρ, θ, z)

$$\text{Let } \vec{A} = A_\rho\vec{u}_\rho + A_\theta\vec{u}_\theta + A_z\vec{k}$$

$$\text{div } \vec{A} = \vec{\nabla} \cdot \vec{A} = \left(\frac{\partial}{\partial \rho} \vec{u}_\rho + \frac{1}{\rho} \frac{\partial}{\partial \theta} \vec{u}_\theta + \frac{\partial}{\partial z} \vec{k} \right) \cdot (A_\rho\vec{u}_\rho + A_\theta\vec{u}_\theta + A_z\vec{k})$$

$$\text{div } \vec{A} = \frac{1}{\rho} \frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho} \frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$$

✓ Spherical Coordinates (r, θ, φ)

$$\text{Let } \vec{A} = A_r\vec{u}_r + A_\theta\vec{u}_\theta + A_\varphi\vec{u}_\varphi$$

$$\text{div } \vec{A} = \vec{\nabla} \cdot \vec{A} = \left(\frac{\partial}{\partial r} \vec{u}_r + \frac{1}{r} \frac{\partial}{\partial \theta} \vec{u}_\theta + \frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi} \vec{u}_\varphi \right) \cdot (A_r\vec{u}_r + A_\theta\vec{u}_\theta + A_\varphi\vec{u}_\varphi)$$

$$\text{div } \vec{A} = \frac{1}{r^2} \frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\sin \theta A_\theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial A_\varphi}{\partial \varphi}$$

Exercise1:

Let:

$$\vec{A}(x, y, z) = (x^2, y^2, z^2)$$

1. Compute the divergence $\vec{\nabla} \cdot \vec{A}$
2. Evaluate it at the point $M(1,1,1)$

Solution:

$$\vec{\nabla} \cdot \vec{A} = \frac{\partial(x^2)}{\partial x} + \frac{\partial(y^2)}{\partial y} + \frac{\partial(z^2)}{\partial z}$$

$$\vec{\nabla} \cdot \vec{A} = 2x + 2y + 2z$$

At $M(1,1,1)$:

$$\mathit{div}\vec{A}_{(1,1,1)} = 2 + 2 + 2 = 6$$

Exercise2:

Let:

$$A(\rho, \theta, z) = \rho^2 \vec{u}_\rho + 0 \vec{u}_\theta + z^2 \vec{k}$$

1. Compute the divergence $\vec{\nabla} \cdot \vec{A}$
2. Evaluate it at $(\rho, z) = (1,1)$

Solution:

$$\begin{aligned} \mathit{div}\vec{A} = \vec{\nabla} \cdot \vec{A} &= \frac{1}{\rho} \frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho} \frac{\partial A_\theta}{\partial \theta} \\ &+ \frac{\partial A_z}{\partial z} \end{aligned} = \frac{1}{\rho} \frac{\partial(\rho^3)}{\partial \rho} + 0 + \frac{\partial z^2}{\partial z} = \frac{1}{\rho} (3\rho^2) + 1 = 3\rho + 2z$$

At $\rho = 1 ; z = 1$:

$$\mathit{div}\vec{A} = 3 + 2 = 5$$

3.4. Curl (Rotation)

The curl (or rotationnel) of a vector field \vec{A} is a vector quantity defined as:

$$\overrightarrow{\text{Rot}} \vec{A} = \vec{\nabla} \wedge \vec{A}$$

It measures the **local rotation** or circulation of the vector field.

✓ Cartesian Coordinates (x, y, z)

Let:

$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

$$\overrightarrow{\text{rot}} \vec{A} = \vec{\nabla} \wedge \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \vec{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \vec{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \vec{k}$$

✓ Cylindrical Coordinates (ρ, θ, z)

Let:

$$\vec{A} = A_\rho \vec{u}_\rho + A_\theta \vec{u}_\theta + A_z \vec{k}$$

$$\overrightarrow{\text{rot}} \vec{A} = \frac{1}{\rho} \begin{vmatrix} \vec{u}_\rho & \rho \vec{u}_\theta & \vec{k} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\theta & A_z \end{vmatrix} = \frac{1}{\rho} \left(\frac{\partial A_z}{\partial \theta} - \frac{\partial \rho A_\theta}{\partial z} \right) \vec{u}_\rho + \frac{1}{\rho} \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right) \rho \vec{u}_\theta + \frac{1}{\rho} \left(\frac{\partial \rho A_\theta}{\partial \rho} - \frac{\partial A_\rho}{\partial \theta} \right) \vec{k}$$

$$\overrightarrow{\text{rot}} \vec{A} = \left(\frac{1}{\rho} \frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z} \right) \vec{u}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right) \vec{u}_\theta + \frac{1}{\rho} \left(\frac{\partial \rho A_\theta}{\partial \rho} - \frac{\partial A_\rho}{\partial \theta} \right) \vec{k}$$

✓ Spherical Coordinates (r, θ, φ)

Let:

$$\vec{A} = A_r \vec{u}_r + A_\theta \vec{u}_\theta + A_\varphi \vec{u}_\varphi$$

$$\overrightarrow{\text{rot}} \vec{A} = \vec{\nabla} \wedge \vec{A} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \vec{u}_r & r \vec{u}_\theta & r \sin \theta \vec{u}_\varphi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ A_r & r A_\theta & r \sin \theta A_\varphi \end{vmatrix}$$

$$= \frac{1}{r^2 \sin \theta} \left[\left(\frac{\partial r \sin \theta A_\varphi}{\partial \theta} - \frac{\partial r A_\theta}{\partial \varphi} \right) \vec{u}_r + \left(\frac{\partial A_r}{\partial \varphi} - \frac{\partial r \sin \theta A_\varphi}{\partial r} \right) r \vec{u}_\theta + \left(\frac{\partial r A_\theta}{\partial r} - \frac{\partial A_r}{\partial \theta} \right) r \sin \theta \vec{u}_\varphi \right]$$

$$\overrightarrow{\text{rot}} \vec{A} = \frac{1}{r \sin \theta} \left(\frac{\partial (\sin \theta A_\varphi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \varphi} \right) \vec{u}_r + \left(\frac{1}{r \sin \theta} \frac{\partial A_r}{\partial \varphi} - \frac{1}{r} \frac{\partial r A_\varphi}{\partial r} \right) \vec{u}_\theta + \frac{1}{r} \left(\frac{\partial r A_\theta}{\partial r} - \frac{\partial A_r}{\partial \theta} \right) \vec{u}_\varphi$$

Exercise 1:

Let the vector field be defined by:

$$\vec{A}_{(x,y,z)} = xy \vec{i} + yz \vec{j} + zx \vec{k}$$

- Compute the curl of the vector field $A(\overrightarrow{\text{rot}} \vec{A})$

Solution:

$$\overrightarrow{\text{rot}} \vec{A} = \vec{\nabla} \wedge \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & zx \end{vmatrix} = \left(\frac{\partial zx}{\partial y} - \frac{\partial yz}{\partial z} \right) \vec{i} + \left(\frac{\partial xy}{\partial z} - \frac{\partial zx}{\partial x} \right) \vec{j} + \left(\frac{\partial yz}{\partial x} - \frac{\partial xy}{\partial y} \right) \vec{k}$$

$$\overrightarrow{\text{rot}} \vec{A} = (0 - y)\vec{i} + (0 - z)\vec{j} + (0 - x)\vec{k} = -y\vec{i} - z\vec{j} - x\vec{k}$$

Exercise2:

Given the vector field:

$$\vec{A}_{(\rho,\theta,z)} = \rho z \vec{u}_\rho + \rho^2 \vec{u}_\theta + z^2 \vec{u}_z$$

- Determine its curl in cylindrical coordinates.

Solution:

$$\begin{aligned} \overrightarrow{\text{rot}} \vec{A} &= \frac{1}{\rho} \begin{vmatrix} \vec{u}_\rho & \rho \vec{u}_\theta & \vec{k} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ \rho z & \rho^3 & z^2 \end{vmatrix} = \frac{1}{\rho} \left(\frac{\partial z^2}{\partial \theta} - \frac{\partial \rho^3}{\partial z} \right) \vec{u}_\rho + \left(\frac{\partial \rho z}{\partial z} - \frac{\partial z^2}{\partial \rho} \right) \vec{u}_\theta + \frac{1}{\rho} \left(\frac{\partial \rho^3}{\partial \rho} - \frac{\partial \rho z}{\partial \theta} \right) \vec{k} \\ \overrightarrow{\text{rot}} \vec{A} &= \frac{1}{\rho} (0 - 0) \vec{u}_\rho + (\rho - 0) \vec{u}_\theta + \frac{1}{\rho} (3\rho^2 - 0) \vec{k} \end{aligned}$$

$$\overrightarrow{\text{rot}} \vec{A} = \rho \vec{u}_\theta + 3\rho \vec{k}$$

Glossary Tables for Chapter I: Mathematical Reminders

مصطلحات المحور الأول: التذكير الرياضي

English	العربية
Angle	زاوية
Arc length	طول القوس
Area element	عنصر مساحة / عنصر سطح
Cartesian coordinates	إحداثيات ديكارتية
Coordinate system	جملة إحداثيات / نظام إحداثيات
Cross product	جداء شعاعي
Curl / Rotation	الدوران / مؤثر الدوران
Curvilinear coordinates	إحداثيات منحنية
Cylindrical coordinates	إحداثيات أسطوانية
Differential	تفاضل
Differential element	عنصر تفاضلي
Differential operators	مؤثرات تفاضلية
Direction	اتجاه
Directional derivative	مشتقة اتجاهية
Divergence	التباعد
Dot product	جداء سلمي
Gradient	التدرج
Gradient vector	شعاع التدرج
Laplace operator / Laplacian	مؤثر لابلاس
Length element	عنصر طول / عنصر تفاضلي للطول
Line integral	تكامل خطي
Magnitude	الطويلة
Mathematical reminders	تذكير رياضي
Nabla operator	مؤثر نابلا

Normal vector	شعاع عمودي / شعاع ناظمي
Orthogonal coordinates	إحداثيات متعامدة
Partial derivative	مشتقة جزئية
Position vector	شعاع الموضع
Scalar field	حقل سلمي
Scalar quantity	مقدار سلمي
Spherical coordinates	إحداثيات كروية
Surface element	عنصر سطح / عنصر تفاضلي للسطح
Surface integral	تكامل سطحي
Tangent vector	شعاع مماسي
Unit vector	شعاع وحدة
Vector	شعاع
Vector field	شعاع الحقل
Vector quantity	مقدار شعاعي
Volume element	عنصر حجم / عنصر تفاضلي للحجم
Volume integral	تكامل حجمي

— • **CHAPTER** • —

II

— ◆ —

ELECTROSTATICS

← • →

ELECTRIC CHARGES AND ELECTROSTATIC FIELDS

ELECTROSTATIC POTENTIAL

ELECTRIC DIPOLE

ELECTRIC FLUX

GAUSS'S LAW

CONDUCTORS IN EQUILIBRIUM

ELECTROSTATIC PRESSURE

CAPACITANCE OF A CONDUCTOR AND A CAPACITOR

Chapter II: Electrostatics

1. Introduction :

Electrostatics is the study of electrical phenomena (electric field and electrostatic potential) produced by static electric charges. It is a branch of physics concerned with charges at rest, meaning charges that do not vary with time. Electric charge is a fundamental property of matter, which can be positive or negative, and is responsible for the attractive or repulsive forces between bodies.

In this chapter, we will study charge distributions (discrete and continuous), the electric field, and Gauss's law used to determine the electric field in symmetric systems.

2. Discrete Charge Distribution

A discrete charge distribution consists of a set of separate point charges located at well-defined positions in space.

We consider charges: $q_1, q_2, q_3, \dots, q_n$

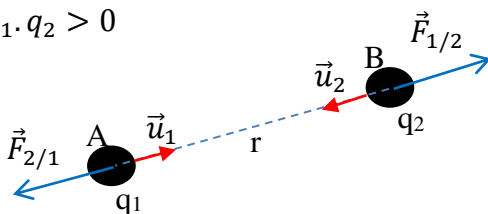
located at points: $M_1, M_2, M_3, \dots, M_n$

Each charge is assumed a point charge, meaning its physical size is negligible compared to the distances involved.

2.1. Coulomb's Law

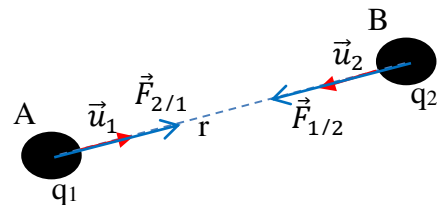
Coulomb's law states that two point charges q_1 and q_2 , located at points A and B and separated by a distance r , exert on each other an electrostatic force along the line joining them.

If: $q_1 \cdot q_2 > 0$



If $q_1 q_2 > 0$, the force is **repulsive**

If: $q_1 \cdot q_2 < 0$



If $q_1 q_2 < 0$, the force is **attractive**

The force exerted by charge q_1 on charge q_2 is given in vector form by:

$$\vec{F}_{1/2} = k \frac{q_1 q_2}{r^2} \vec{u}_1$$

$$\vec{F}_{1/2} = k \frac{q_1 q_2}{AB^3} \overrightarrow{AB}$$

$$\vec{u}_1 = \frac{\overrightarrow{AB}}{AB}$$

where:

- $k = \frac{1}{4\pi\epsilon_0} = 9 \cdot 10^9 \text{ Nm}^2/\text{C}^2$
- $\epsilon_0 = 8.85 \cdot 10^{-12} \text{ N}^{-1}\text{m}^{-2}\text{C}^2$
- r is the distance between the charges

The forces $\vec{F}_{1/2}$ and $\vec{F}_{2/1}$ are equal in magnitude and opposite in direction, in accordance with Newton's third law.

$$\vec{F}_{1/2} = -\vec{F}_{2/1} \text{ and } \|\vec{F}_{1/2}\| = \|\vec{F}_{2/1}\|$$

➤ Superposition Principle of the electric force:

The principle of superposition states that when several point charges are present, the total electrostatic force exerted on a given charge is equal to the vector sum of the forces exerted by each of the other charges taken separately.

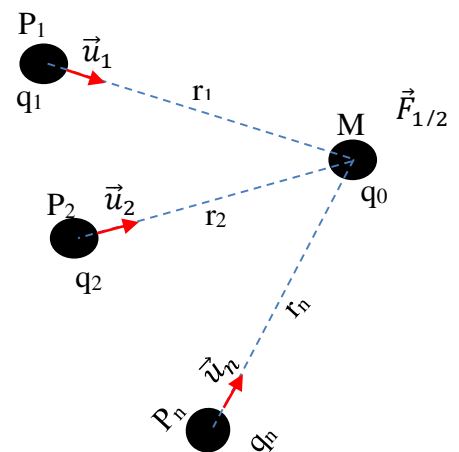
Consider charges $q_1, q_2, q_3, \dots, q_n$ located at points $P_1, P_2, P_3, \dots, P_n$ in space.

If a charge q_0 is placed at a point M , the total force acting on it is:

$$\vec{F}_{total}(M) = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n = \sum_{i=1}^n \vec{F}_i$$

$$\vec{F}_{total}(M) = kq_0 \sum_{i=1}^n \frac{q_i}{r_i^2} \vec{u}_i$$

$$\vec{F}_{total}(M) = kq_0 \sum_{i=1}^n \frac{q_i}{P_i M^3} \overrightarrow{P_i M}$$



Where:

\vec{F}_i is the force exerted on the charge q at point M by the charge q_i located at P_i .

Example:

Consider four identical charges ($q_1 = q_2 = q_3 = q_4 = q$) placed at the vertices of a square $ABCD$ of side a , i.e., located at points **A**, **B**, **C**, and **D**.

We aim to determine the resultant force acting on the charge at point **A**.

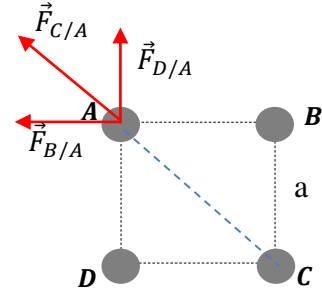
According to the superposition principle:

$$\vec{F}(A) = \vec{F}_{B/A} + \vec{F}_{C/A} + \vec{F}_{D/A}$$

$$\vec{F}_{B/A} = k \frac{q_1 q_2}{r_1^2} \vec{u}_1 = k \frac{q^2}{a^2} (-\vec{i}) = -\frac{kq^2}{a^2} \vec{i}$$

$$\begin{aligned} \vec{F}_{C/A} &= k \frac{q_1 q_2}{CA^3} \vec{CA} = \frac{kq^2}{(\sqrt{2}a)^3} (-a\vec{i} + a\vec{j}) \\ &= \frac{kq^2}{2\sqrt{2}a^2} (-\vec{i} + \vec{j}) \end{aligned}$$

$$\vec{F}_{D/A} = k \frac{q_1 q_3}{r_3^2} \vec{u}_3 = \frac{kq^2}{a^2} \vec{j}$$



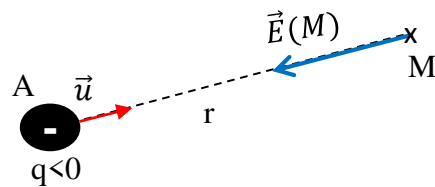
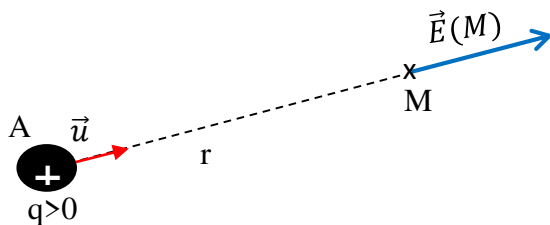
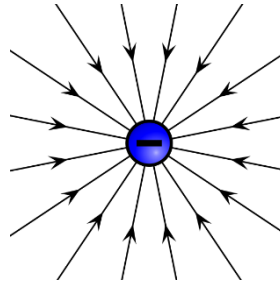
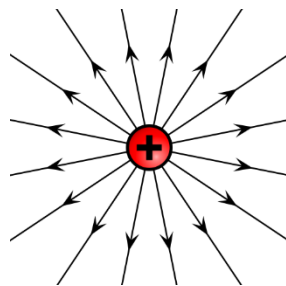
$$\vec{F}(A) = \vec{F}_{B/A} + \vec{F}_{C/A} + \vec{F}_{D/A} = -\frac{kq^2}{a^2} \vec{i} + \frac{kq^2}{2\sqrt{2}a^2} (-\vec{i} + \vec{j}) + \frac{kq^2}{a^2} \vec{j} = \frac{kq^2}{a^2} \left(-\vec{i} + \frac{1}{2\sqrt{2}} (-\vec{i} + \vec{j}) + \vec{j} \right)$$

$$\vec{F}(A) = \frac{kq^2}{a^2} \left(\left(-1 - \frac{1}{2\sqrt{2}} \right) \vec{i} + \left(1 + \frac{1}{2\sqrt{2}} \right) \vec{j} \right) \text{ (N)}$$

2.2. Electrostatic Field (Electric Field):

The electrostatic field is a physical quantity that describes the effect of electric charges in space.

It is defined as the force per unit positive test charge placed at a given point.



The electric field created by charge **q** at a point **M** is given by:

$$\vec{E}(M) = \frac{Kq}{r^2} \vec{u}$$

(V/m) ; (N/C)

$$\vec{E}(M) = \frac{Kq}{\|\vec{AM}\|^3} \vec{AM}$$

$$k = \frac{1}{4\pi\epsilon_0} = 9 \cdot 10^9 \text{ Nm}^2/\text{C}^2$$

The electric field is directly relate to the electrostatic force. It is define as the force per unit charge.

$$\vec{F} = q\vec{E}$$

➤ Superposition Principle of the Electric Field:

The superposition principle of the electric field states that the total electric field at a given point is equal to the vector sum of the electric fields produced by each charge taken individually.

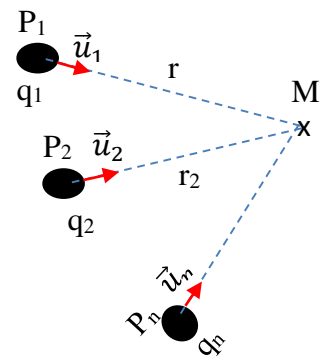
Consider charges q_1, q_2, \dots, q_n located at points P_1, P_2, \dots, P_n .

The total electric field at point M is given by:

$$\vec{E}(M) = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_n = \sum_{i=1}^n \vec{E}_i$$

$$\vec{E}(M) = k \sum_{i=1}^n \frac{q_i}{r_i^2} \vec{u}_i$$

$$\vec{E}(M) = k \sum_{i=1}^n \frac{q_i}{P_i M^3} \vec{P_i M}$$



Exercise:

Consider a square ABCD of side a . Three charges ($q_1=q_3=-q$, and $q_2=q$) are placed at the vertices B, C, and D.

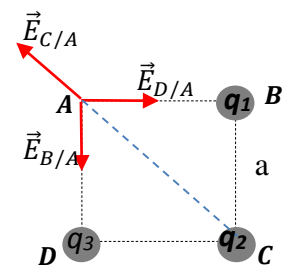
- Determine the electric field at point A.

Solution:

The electric field at point A is the vector sum of the fields created by the three charges located at B, C, and D:

$$\vec{E}(A) = \vec{E}_B(A) + \vec{E}_C(A) + \vec{E}_D(A)$$

$$\vec{E}_B = k \frac{q_1}{r_1^2} \vec{u}_1 = k \frac{(-q)}{a^2} (-\vec{i}) = \frac{kq}{a^2} \vec{i}$$



$$\vec{E}_C = k \frac{q_2}{CA^3} \vec{CA} = \frac{kq}{(\sqrt{2}a)^3} (-a\vec{i} + a\vec{j}) = \frac{kq}{2\sqrt{2}a^2} (-\vec{i} + \vec{j})$$

$$\vec{E}_D = k \frac{q_3}{r_3^2} \vec{u}_3 = \frac{k(-q)}{a^2} \vec{j} = -\frac{kq}{a^2} \vec{j}$$

$$\vec{E}(A) = \vec{E}_B + \vec{E}_C + \vec{E}_D = \frac{kq}{a^2} \vec{i} + \frac{kq}{2\sqrt{2}a^2} (-\vec{i} + \vec{j}) - \frac{kq}{a^2} \vec{j} = \frac{kq}{a^2} \left(\vec{i} + \frac{1}{2\sqrt{2}} (-\vec{i} + \vec{j}) - \vec{j} \right)$$

$$\vec{E}(A) = \frac{kq}{a^2} \left(\left(1 - \frac{1}{2\sqrt{2}}\right) \vec{i} - \left(1 + \frac{1}{2\sqrt{2}}\right) \vec{j} \right) \text{ (N/C)}$$

2.3. The Electrostatic Potential:

The electrostatic potential at a point M is a scalar quantity that represents the electric potential energy per unit charge at that point.

It is defined by:

$$V(M) = \frac{Kq}{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad (V)$$

ϵ_0 is the permittivity of free space

Properties:

- If $q > 0$, then $V(M) > 0$
- If $q < 0$, then $V(M) < 0$

➤ Superposition principle:

If several point charges q_1, q_2, \dots, q_n are present, the total potential at point M is the algebraic sum of the individual potentials:

$$V(M) = V_1 + V_2 + \dots + V_n = \sum_{i=1}^n V_i(M) = k \sum_{i=1}^n \frac{q_i}{r_i}$$

where r_i is the distance between the charge q_i and the point M.

Exercise:

Consider a square ABCD of side $2a$, with charges located at the vertices in the following order: **A(+q), B(+2q), C(-2q), D(+q)**

Determine the electrostatic potential at the center **O** of the square.

Solution:

The distance from the center O to each vertex is:

$$AO = BO = CO = DO = \frac{2a}{\sqrt{2}} = \sqrt{2}a$$

$$V(O) = V_A + V_B + V_C \quad \left| \quad V(O) = \frac{Kq_1}{AO} + \frac{Kq_2}{BO} + \frac{Kq_3}{CO} + \frac{Kq_4}{DO}$$

$$V(O) = \frac{Kq}{AO} + \frac{2Kq}{BO} - \frac{2Kq}{CO} + \frac{Kq}{DO} \rightarrow V(O) = \frac{Kq}{AO} + \frac{2Kq}{BO} - \frac{2Kq}{CO} + \frac{Kq}{DO}$$

$$V(O) = \frac{Kq}{\sqrt{2}a} + \frac{2Kq}{\sqrt{2}a} - \frac{2Kq}{\sqrt{2}a} + \frac{Kq}{\sqrt{2}a} = \frac{Kq}{\sqrt{2}a} (1 + 2 - 2 + 1) = \frac{2Kq}{\sqrt{2}a} = \frac{\sqrt{2}Kq}{a}$$

2.4. Relation between the Electric Field and the Electrostatic Potential:

The electric field is directly relate to the spatial variation of the electrostatic potential.

The electric field is the negative gradient of the electrostatic potential:

$$\vec{E} = -\overrightarrow{\text{grad}} V$$

Between two points A and B:

$$V(B) - V(A) = - \int_A^B \vec{E} \cdot \vec{dl}$$

2.5. Work of the Electrostatic Force:

The work done by the electrostatic force when a charge moves from one point to another is relate to the electric field and the potential difference.

Consider a fixed point charge q_0 , and a test charge q moving radially from point A to point B, at distances r_A and r_B from q_0 .

The work done by the electrostatic force from A to B is:

The element work of F is:

$$dW(\vec{F}) = \vec{F} \cdot \vec{dr}$$

$$W_{A \rightarrow B}(\vec{F}) = \int_{r_A}^{r_B} \vec{F} \cdot \vec{dr} = \int_{r_A}^{r_B} \frac{kq_0q}{r^2} dr = kq_0q \int_{r_A}^{r_B} \frac{1}{r^2} dr = kq_0q \left[\frac{-1}{r} \right]_{r_A}^{r_B}$$

$$= kq_0q \left(\frac{1}{r_A} - \frac{1}{r_B} \right) = q \left(\frac{kq_0}{r_A} - \frac{kq_0}{r_B} \right)$$

$$= q(V_A - V_B)$$

$$W_{A \rightarrow B}(\vec{F}) = q(V_A - V_B)$$

2.6. Electrostatic Potential Energy

The electrostatic potential energy is defined from the elementary relation:

$$dE_p = -dW(\vec{F}) = -\vec{F} \cdot \vec{dr}$$

This means that the variation of potential energy is equal to the negative of the work done by the electrostatic force.

For a charge q_0 in the field of a point charge q :

$$\vec{F} = k \frac{q_0 q}{r^2} \vec{u}$$

For a radial displacement:

$$\vec{dr} = dr \vec{u}_r$$

Note: (about radial motion)

In this derivation, we assume a **radial displacement** (along the line joining the charges) in order to simplify the calculation, since the force is central and directed along \vec{u}_r .

thus:

$$dE_p = -F dr = -k \frac{q_0 q}{r^2} dr$$

$$E_p(r) - E_p(\infty) = - \int_{\infty}^r \frac{k q_0 q}{r^2} dr = -k q_0 q \int_{\infty}^r \frac{1}{r^2} dr = -k q_0 q \left[\frac{-1}{r} \Big|_{\infty}^r \right]$$

$$\boxed{E_p(r) = k \frac{q_0 q}{r}}$$

Note:

The integration is performed from ∞ to r because it represents bringing the charge from infinity to the point r .

Important remark

The electrostatic force is a conservative force, therefore:

- The work does not depend on the path followed
- It depends only on:
 - the charge
 - the initial position
 - the final position

Interpretation

- ✓ If $q_0q > 0 \rightarrow$ repulsion $\rightarrow E_p > 0$
- ✓ If $q_0q < 0 \rightarrow$ attraction $\rightarrow E_p < 0$

Exercise:

In an orthonormal coordinate system (O, \vec{i}, \vec{j}) , consider the following point charges:

- $q_1 = +q$ located at point $A(a, 0)$
- $q_2 = -2$ located at point $B(0, a)$
- $q_3 = +q$ located at point $C(-a, 0)$

We study the electrostatic quantities at the origin $O(0,0)$.

Given:

$$k = 9 \cdot 10^9, \quad a > 0, \quad q > 0$$

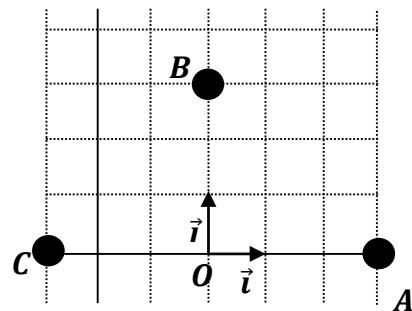
1. Plot the points A, B, and C in the coordinate system.
2. Determine the unit vectors along the directions AO, BO, and CO.
3. Determine the electric field at point O created by each charge.
4. Using the superposition principle, deduce the resultant electric field $\vec{E}(O)$.
5. Calculate the electrostatic potential $V(O)$.
6. A test charge q_0 placed at point O. Determine the electrostatic force $\vec{F}(O)$ acting on it.
7. Determine the electrostatic potential energy of the charge q_0 at point O.
8. The charge q_0 is moved from O to a point at infinity. Calculate the work done by the electrostatic force during this displacement.
9. Does this work depend on the path followed? Justify your answer.

Solution:

1. Plot the points A, B, and C:
2. Unit vectors :

The relevant directions are:

- from A to O: along $-\vec{i}$
- from B to O: along $-\vec{j}$
- from C to O: along $+\vec{i}$



Hence,

$$\vec{u}_{AO} = -\vec{i} \quad , \quad \vec{u}_{BO} = -\vec{j} \quad , \quad \vec{u}_{CO} = +\vec{i}$$

3. Electric field created by each charge at **O**:

Field due to $q_1 = +q$ at $A(a, 0)$

$$\vec{E}_1 = k \frac{q_1}{r_1^2} \vec{u}_1 = k \frac{q}{a^2} (-\vec{i}) = \frac{-kq}{a^2} \vec{i}$$

Field due to $q_2 = -2q$ at $B(0, a)$

$$\vec{E}_2 = k \frac{q_2}{r_2^2} \vec{u}_2 = k \frac{(-2q)}{a^2} (-\vec{j}) = \frac{2kq}{a^2} \vec{j}$$

Field due to $q_3 = +q$ at $C(-a, 0)$

$$\vec{E}_3 = k \frac{q_3}{r_3^2} \vec{u}_3 = \frac{kq}{a^2} \vec{i}$$

4. The resultant electric field $\vec{E}(O)$:

$$\vec{E}(O) = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{-kq}{a^2} \vec{i} + \frac{2kq}{a^2} \vec{j} + \frac{kq}{a^2} \vec{i} = \frac{kq}{a^2} (-\vec{i} + 2\vec{j} + \vec{i})$$

$$\vec{E}(O) = \frac{2kq}{a^2} \vec{j}$$

5. The electrostatic potential $V(O)$:

$$V(O) = V_1 + V_2 + V_3 = \frac{Kq_1}{AO} + \frac{Kq_2}{BO} + \frac{Kq_3}{CO} = \frac{Kq}{a} - \frac{2Kq}{a} + \frac{Kq}{a} = 0 \text{ (V)}$$

6. The electrostatic force $\vec{F}(O)$:

$$\vec{F}(O) = q_0 \vec{E}(O) = q_0 \frac{2kq}{a^2} \vec{j} = \frac{2kq_0q}{a^2} \vec{j}$$

7. The electrostatic potential energy of the charge q_0 at point **O**

$$E_p(O) = q_0 V(O) = 0 \text{ Joule}$$

8. The work:

$$W_{O \rightarrow \infty}(\vec{F}) = q_0(V_O - V_\infty) = 0 \text{ Joule}$$

2.7. Electric Dipole:

An electric dipole is a physical system consisting of two electric charges (electric charge: a physical quantity measured in coulombs that is the source of electric fields) of equal magnitude $+q$ and $-q$, but opposite signs, separated by a small fixed distance d .

a) Dipole Moment

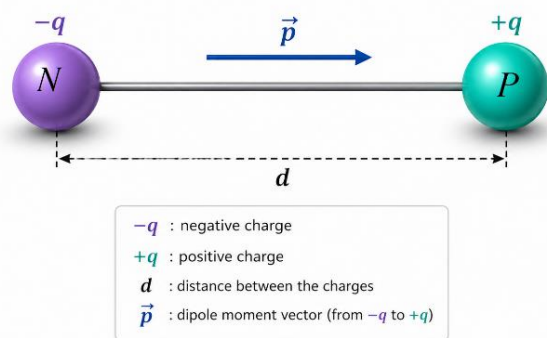
The dipole moment is a vector quantity (vector quantity: a physical quantity having both magnitude and direction) that characterizes the dipole:

$$\vec{p} = q \cdot \overrightarrow{NP}$$

q : magnitude of the charge

\overrightarrow{NP} : distance vector (vector connecting the two charges)

Direction of \vec{p} : from negative charge to positive charge



b) Electric Potential Energy

The dipole has an electric potential energy (potential energy: energy related to position or configuration in a field) given by:

$$E_p = -\vec{p} \cdot \vec{E}$$

- ✓ E_p : potential energy
- ✓ Depends on the angle between \vec{p} and \vec{E} . This energy represents the system's ability to do work due to its position in the field.

3. Continuous Charge Distribution

3.1. Introduction

In electrostatics, electric charge is often not concentrated at discrete points, but rather distributed continuously over a:

- Line (wire)
- Surface (sheet)
- Volume (solid object)

In such cases, summation is replaced by integration:

The idea is to divide the charge into infinitesimal elements dq , then integrate their contributions.

3.2. Charge Density:

Charge density describes how charge is distributed in space. There are three main types:

a. Linear Charge Density λ

$$\lambda = \frac{dq}{dl} \rightarrow dq = \lambda dl$$

Where:

- **dq**: infinitesimal charge
- **λ** : linear charge density
- **dl**: infinitesimal line element (or differential length element)

✓ Cartesian:

$$dl = dx, \text{ or } dy \text{ or } dz$$

✓ Polar (Circular Arc):

$$dl = R d\theta$$

b. Surface Charge Density σ

$$\sigma = \frac{dq}{dS} \rightarrow dq = \sigma dS$$

Where:

- **dq**: infinitesimal charge
- **σ** : surface charge density
- **dS**: infinitesimal surface element (or differential surface element)

✓ Cartesian

$$dS = dx dy,$$

$$dS = dy dz,$$

$$dS = dx dz$$

- $dx, dy \rightarrow$ surface parallel to the xy-plane
- $dy, dz \rightarrow$ surface parallel to the yz-plane
- $dx, dz \rightarrow$ surface parallel to the xz-plane

✓ Polar (Disk)

$$dS = \rho d\rho d\theta$$

(surface element of a disk)

- $0 \leq r \leq R$
- $0 \leq \theta \leq 2\pi$

✓ Cylindrical (Curved Surface)

$$dS = R d\theta dz$$

(lateral surface element of a cylinder)

- $0 \leq \theta \leq 2\pi$
- $0 \leq z \leq h$

✓ Spherical (Sphere)

$$dS = r^2 \sin \theta \, d\theta \, d\varphi$$

(surface element of a sphere)

- $0 \leq \theta \leq \pi$
- $0 \leq \varphi \leq 2\pi$

c. Volume Charge Density ρ :

$$\rho = \frac{dq}{dV} \rightarrow dq = \rho dV$$

where:

- dq : infinitesimal charge
- ρ : volume charge density
- dV : infinitesimal volume element (or differential volume element)

✓ Cartesian

$$dV = dx \, dy \, dz$$

infinitesimal rectangular box

✓ Cylindrical

$$dV = \rho \, d\rho \, d\theta \, dz$$

(volume element of a cylinder)

- $0 \leq \rho \leq R$
- $0 \leq \theta \leq 2\pi$
- $0 \leq z \leq h$

✓ Spherical

$$dV = r^2 \sin \theta \, dr \, d\theta \, d\varphi$$

(volume element of a sphere)

- $0 \leq r \leq R$
- $0 \leq \theta \leq \pi$
- $0 \leq \varphi \leq 2\pi$

3.3. Electric Field and Potential in Continuous Charge Distributions:

3.3.1 Electric Field \vec{E} :

Electric field due to a continuous distribution:

$$d\vec{E}(M) = \frac{Kdq}{r^2} \vec{u}$$

$$\vec{E}(M) = K \int \frac{dq}{r^2} \vec{u}$$

$$d\vec{E}(M) = \frac{Kdq}{\|\vec{AM}\|^3} \vec{AM}$$

$$\vec{E}(M) = K \int \frac{dq}{\|\vec{AM}\|^3} \vec{AM}$$

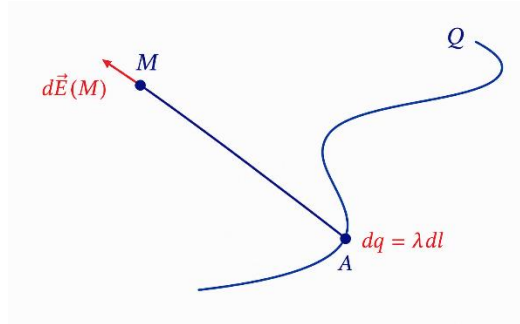
◆ **Linear Charge Distribution :**

Let a line charge be distributed along a curve with linear charge density λ .

$$dq = \lambda dl$$

$$\vec{E}(M) = K \int \frac{\lambda dl}{r^2} \vec{u}$$

$$\vec{E}(M) = K \int \frac{\lambda dl}{\|\vec{AM}\|^3} \vec{AM}$$



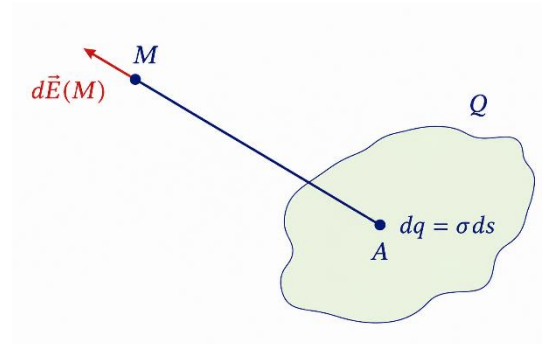
◆ **Surface Charge Distribution :**

Let a surface charge distribution be characterized by a surface charge density σ .

$$dq = \sigma dS$$

$$\vec{E}(M) = K \iint \frac{\sigma dS}{r^2} \vec{u}$$

$$\vec{E}(M) = K \iint \frac{\sigma dS}{\|\vec{AM}\|^3} \vec{AM}$$



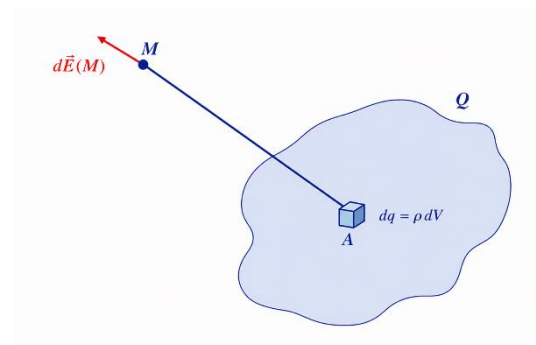
◆ **Volume Charge Distribution**

Let a volume charge distribution be characterized by a volume charge density ρ .

$$dq = \rho dV$$

$$\vec{E}(M) = K \iiint \frac{\rho dV}{r^2} \vec{u}$$

$$\vec{E}(M) = K \iiint \frac{\rho dV}{\|\vec{AM}\|^3} \vec{AM}$$



3. 3.2 Electric Potential V :

$$dV(M) = \frac{Kdq}{r}$$

$$V(M) = K \int \frac{dq}{r}$$

◆ **Linear Charge Distribution :**

$$V(M) = K \int \frac{\lambda dl}{r}$$

◆ **Surface Charge Distribution :**

$$V(M) = K \iint \frac{\sigma dS}{r}$$

◆ Volume Charge Distribution :

$$V(M) = K \iiint \frac{\rho dV}{r}$$

3. 3.3 Relation Between Field and Potential:

$$\vec{E} = -\vec{\nabla}V$$

The electric field is the negative gradient of the electric potential.

In Cartesian Coordinates:

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right)$$

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

$$V(B) - V(A) = -\int_A^B \vec{E} \cdot d\vec{l}$$

where in Cartesian:

$$d\vec{l} = dx\vec{i} + dy\vec{j} + dz\vec{k}$$

Exercise1:

A finite uniformly charged wire of linear charge density λ lies along the y-axis from $y = -b$ to $y = L$. A point $M(a, 0)$ is located on the x-axis.

- Determine the electric field $\vec{E}(M)$

Given:

$$L = 60 \text{ cm}; \lambda = 2 \times 10^{-4} \text{ C/m}; a = 50 \text{ cm}; b = 20 \text{ cm}$$

Solution:

Electric field at M(a,0):

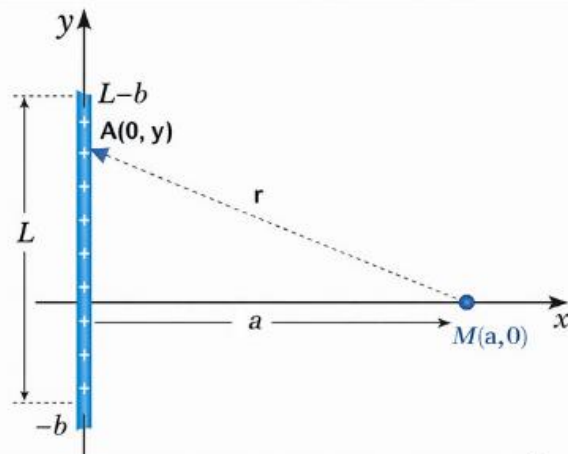
Take a small element of charge:

$$dq = \lambda dy$$

The vector from dq to M is

$$\vec{AM} = a\vec{i} - y\vec{j}$$

$$\|\vec{AM}\| = \sqrt{a^2 + y^2}$$



So the elemental field is

$$d\vec{E}(M) = \frac{Kdq}{\|\vec{AM}\|^3} \vec{AM} = K\lambda \frac{(a\vec{i} - y\vec{j})}{(a^2 + y^2)^{3/2}} dy$$

$$\vec{E}(M) = K\lambda \int_{-b}^{L-b} \frac{(a\vec{i} - y\vec{j})}{(a^2 + y^2)^{3/2}} dy$$

x-component :

$$E_x = K\lambda \int_{-b}^{L-b} \frac{a}{(a^2 + y^2)^{3/2}} dy = K\lambda \left(\frac{y}{a\sqrt{a^2 + y^2}} \right) \Big|_{-b}^{L-b}$$

$$\int \frac{a}{(a^2 + y^2)^{3/2}} dy = \frac{y}{a\sqrt{a^2 + y^2}}$$

$$E_x = \frac{K\lambda}{a} \left(\frac{L-b}{\sqrt{a^2 + (L-b)^2}} + \frac{b}{\sqrt{a^2 + b^2}} \right)$$

y-component :

$$E_y = -K\lambda \int_{-b}^{L-b} \frac{y}{(a^2 + y^2)^{3/2}} dy = K\lambda \left(\frac{1}{\sqrt{a^2 + y^2}} \right) \Big|_{-b}^{L-b}$$

$$\int \frac{y}{(a^2 + y^2)^{3/2}} dy = \frac{-1}{\sqrt{a^2 + y^2}}$$

$$E_y = K\lambda \left(\frac{1}{\sqrt{a^2 + (L-b)^2}} - \frac{1}{\sqrt{a^2 + b^2}} \right)$$

$$\vec{E}(M) = \frac{K\lambda}{a} \left(\frac{L-b}{\sqrt{a^2 + (L-b)^2}} + \frac{b}{\sqrt{a^2 + b^2}} \right) \vec{i} + K\lambda \left(\frac{1}{\sqrt{a^2 + (L-b)^2}} - \frac{1}{\sqrt{a^2 + b^2}} \right) \vec{j}$$

$$\vec{E}(M) = 35.86 \times 10^5 \vec{i} - 5.31 \times 10^5 \vec{j}$$

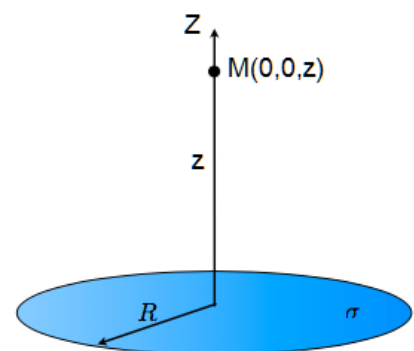
Exercise2 :

A uniformly charged disk of radius R lies in the (xy) plane, centered at the origin.

The surface charge density is constant and equal to $\sigma > 0$.

A point $M(0,0,z)$ is located on the axis perpendicular to the disk.

- 1) Determine the electric potential $V(M)$ at the same point.
- 2) Calculate the electric field $\vec{E}(M)$ at point M.
- 3) Verify that:
 - When $z \rightarrow 0$, the field approaches that of an infinite plane.
 - When $z \gg R$, the disk behaves like a point charge.



Solution:

1) Electric potential $V(M)$:

Element of charge

A ring of radius r and thickness dr has area

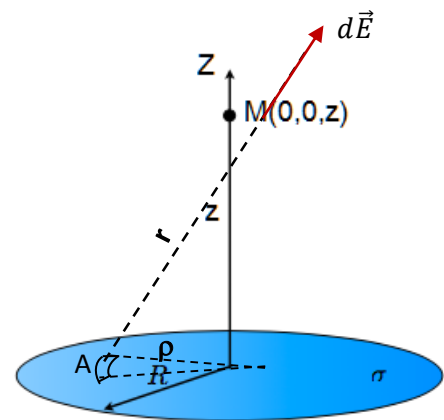
$$dS = \rho d\rho d\theta$$

so its charge is

$$dq = \sigma dS = \sigma \rho d\rho d\theta$$

Every point of this ring is at the same distance from M :

$$r = \sqrt{\rho^2 + z^2}$$



Thus the elemental potential is

$$dV(M) = \frac{Kdq}{r} = \frac{K dq}{\sqrt{\rho^2 + z^2}}$$

Substitute dq :

$$dV(M) = \frac{Kdq}{r} = \frac{K\sigma dS}{\sqrt{\rho^2 + z^2}} = \frac{K\sigma \rho d\rho d\theta}{\sqrt{\rho^2 + z^2}}$$

$$V(M) = K\sigma \int_0^{2\pi} \int_0^R \frac{\rho d\rho d\theta}{\sqrt{\rho^2 + z^2}} = K\sigma \int_0^R \frac{\rho d\rho}{\sqrt{\rho^2 + z^2}} \int_0^{2\pi} d\theta$$

$$V(M) = K\sigma \left[\sqrt{\rho^2 + z^2} \right]_0^R \int_0^{2\pi} d\theta$$

$$V(M) = 2\pi K\sigma \left(\sqrt{R^2 + z^2} - |z| \right)$$

$$V(M) = \frac{\sigma}{2\epsilon_0} \left(\sqrt{R^2 + z^2} - |z| \right)$$

Since usually we take $z > 0$ on the upper side of the disk:

$$V(M) = \frac{\sigma}{2\epsilon_0} \left(\sqrt{R^2 + z^2} - z \right)$$

2) Electric field $\vec{E}(M)$:

Because of symmetry, the field has only a z -component:

$$E_z = -\frac{dV}{dz}$$

For $z > 0$,

$$V = \frac{\sigma}{2\epsilon_0} (\sqrt{R^2 + z^2} - z)$$

Differentiate:

$$\frac{dV}{dz} = \frac{\sigma}{2\epsilon_0} \left(\frac{z}{\sqrt{R^2 + z^2}} - 1 \right)$$

Therefore

$$E_x = -\frac{dV}{dz} = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{R^2 + z^2}} \right)$$

So for $z > 0$,

$$\vec{E}(M) = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{R^2 + z^2}} \right) \vec{k}$$

4. Gauss's Law and Electric Flux

4.1. Introduction:

Gauss's Law is a fundamental result in electrostatics.

It relates the electric flux through a closed surface to the charge enclosed within that surface.

This theorem greatly simplifies the calculation of electric fields, especially in cases with high symmetry.

4.2. Vector representation of a surface:

To study flux, we associate to each surface element a vector called the surface vector.

$$\vec{dS} = dS \vec{u}_n$$

where:

- dS : Surface element.
- \vec{u}_n : Unit vector normal (perpendicular) to the surface.
- For a closed surface, it is oriented outward.

Interpretation:

- The direction of \vec{dS} defines the orientation of the surface.
- Its magnitude equals the area of the surface element.

4.3. Electric flux:

The electric flux through a surface S is defined as:

$$\Phi = \oiint \vec{E} \cdot \vec{dS}$$

Using the dot product:

$$d\Phi = E \cdot dS = E \cdot dS \cdot \cos\theta$$

where:

- E : magnitude of the electric field
- θ : angle between \vec{E} and \vec{dS}

Thus:

$$\Phi = \oiint E \cdot dS \cdot \cos\theta$$

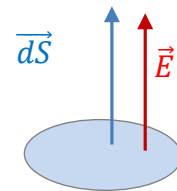
The flux represents the way in which the vector field \vec{E} crosses the surface.

special cases :

1. $\theta = 0$

$$\cos 0 = 1 \Rightarrow d\Phi = E dS$$

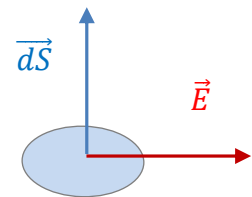
- Maximum positive flux.
- Field is perpendicular to the surface and pointing outward.



2. $\theta = \frac{\pi}{2}$

$$\cos \frac{\pi}{2} = 0 \Rightarrow d\Phi = 0$$

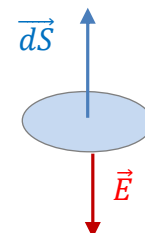
- No flux.
- Field is parallel to the surface.



3. $\theta = \pi$

$$\cos \pi = -1 \Rightarrow d\Phi = -E dS$$

- maximum (in magnitude) but negative flux
- field is entering the surface



4.4. Statement of Gauss's Law:

The total electric flux through any closed surface is equal to the enclosed charge divided by the permittivity of free space.

$$\oiint \vec{E} \cdot d\vec{S} = \frac{\Sigma Q_{enc}}{\epsilon_0}$$

Note:

- ✓ The surface must be closed.
- ✓ Only the enclosed charge is considered.
- ✓ External charges do not affect the total flux.

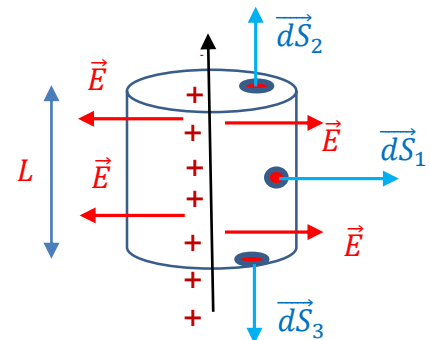
4.5. Applications of Gauss's Law:

1. Infinite line charge :

We consider an infinite straight line uniformly charged with a linear charge density $\lambda > 0$

a) Choice of Gaussian surface

We choose as a Gaussian surface a cylinder of radius r and length L , coaxial with the line charge, due to cylindrical symmetry.



The electric field is radial and depends only on the distance r

The closed cylindrical surface consists of three parts:

1. Lateral surface S_1
2. Top base S_2
3. Bottom base S_3

b) Total Flux:

$$d\Phi = d\Phi_{lateral} + d\Phi_{top} + d\Phi_{bottom} \quad \left| \quad \Phi = \oiint_{S_1} \vec{E} \cdot d\vec{S}_1 + \oiint_{S_2} \vec{E} \cdot d\vec{S}_2 + \oiint_{S_3} \vec{E} \cdot d\vec{S}_3$$

$$\Phi = \underbrace{\oiint_{S_1} \vec{E} \cdot d\vec{S}_1}_{\vec{E} \parallel d\vec{S}_1} + \underbrace{\oiint_{S_2} \vec{E} \cdot d\vec{S}_2}_{=0 \text{ } (\vec{E} \perp d\vec{S}_2)} + \underbrace{\oiint_{S_3} \vec{E} \cdot d\vec{S}_3}_{=0 \text{ } (\vec{E} \perp d\vec{S}_2)}$$

$$\Phi = \oiint_{S_1} \vec{E} \cdot \vec{dS}_1 = \oiint_{S_1} E \cdot dS_1 \cos(\theta) = E \cdot S_1 = E \cdot 2\pi r L$$

$$S_1 = S_{lateral(G)} = 2\pi r L$$

c) **Enclosed charge :**

$$Q_{enc} = \lambda L$$

d) **Application of Gauss's Law :**

$$\oiint \vec{E} \cdot \vec{dS} = \frac{\Sigma Q_{enc}}{\epsilon_0}$$

$$E \cdot 2\pi r L = \frac{\lambda L}{\epsilon_0}$$

Electric field

$$E(r) = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\vec{E}(r) = \frac{\lambda}{2\pi\epsilon_0 r} \vec{u}_r$$

Note:

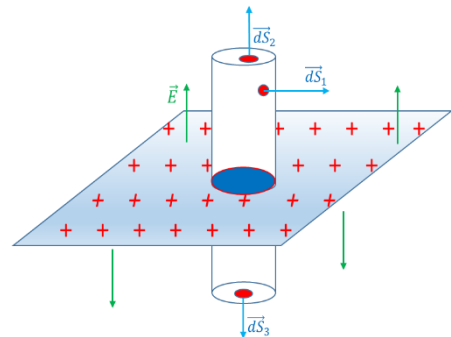
- ✓ If $\lambda > 0$, the electric field is directed **radially outward** from the line charge.
- ✓ If $\lambda < 0$, the electric field is directed **radially inward**, toward the line charge.

2. Infinite Plane :

We consider an infinite plane uniformly charged with a surface charge density $\sigma > 0$.

a) **Choice of Gaussian surface**

We choose as a Gaussian surface a cylindrical pillbox of cross-sectional area S , whose axis is perpendicular to the plane, due to planar symmetry.



The surface consists of:

1. Lateral surface S_1
2. Top face S_2
3. Bottom face S_3

b) **Total Flux:**

$$d\Phi = d\Phi_{lateral} + d\Phi_{top} + d\Phi_{bottom}$$

$$\Phi = \oiint_{S_1} \vec{E} \cdot \vec{dS}_1 + \oiint_{S_2} \vec{E} \cdot \vec{dS}_2 + \oiint_{S_3} \vec{E} \cdot \vec{dS}_3$$

$$\Phi = \underbrace{\iint_{S_1} \vec{E} \cdot \vec{dS}_1}_{=0} + \underbrace{\iint_{S_2} \vec{E} \cdot \vec{dS}_2}_{(\vec{E} \parallel \vec{dS}_2)} + \underbrace{\iint_{S_3} \vec{E} \cdot \vec{dS}_3}_{(\vec{E} \parallel \vec{dS}_2)}$$

$$\Phi = \iint_{S_2} \vec{E} \cdot \vec{dS}_2 + \iint_{S_3} \vec{E} \cdot \vec{dS}_3 = \iint_{S_1} E \cdot dS_1 \cos(o) + \iint_{S_2} E \cdot dS_2 \cos(o) = E \cdot S_1 + E \cdot S_2$$

$$\Phi = 2E \cdot S$$

$$(S_1 = S_2 = S_{lateral(G)} = \pi r^2)$$

c) Enclosed charge :

$$Q_{enc} = \sigma S_{lateral(G)} = \sigma \pi r^2$$

d) Application of Gauss's Law :

$$\iint \vec{E} \cdot \vec{dS} = \frac{\sum Q_{enc}}{\epsilon_0}$$

$$2E \cdot S = \frac{\sigma S}{\epsilon_0}$$

$$E(r) = \frac{\sigma}{2\epsilon_0}$$

$$\begin{cases} \vec{E}(r) = \frac{\sigma}{2\epsilon_0} \vec{k} & z > 0 \\ \vec{E}(r) = \frac{-\sigma}{2\epsilon_0} \vec{k} & z < 0 \end{cases}$$

3. Cylindrical Surface Charge :

We consider an infinite cylindrical shell of radius R , uniformly charged on its surface with a surface charge density $\sigma > 0$.

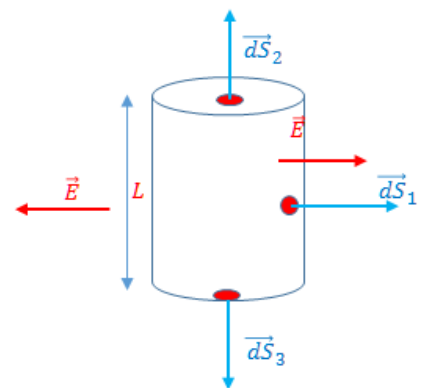
For an infinite charged cylinder, the electric field has **cylindrical symmetry**.

a) Choice of Gaussian surface

We choose as a Gaussian surface a cylinder of radius r and length L , coaxial with the charged cylinder, due to cylindrical symmetry.

The closed cylindrical Gaussian surface consists of:

1. the lateral surface S_1
2. the top base S_2
3. the bottom base S_3



b) Total Flux:

$$d\Phi = d\Phi_{lateral} + d\Phi_{top} + d\Phi_{bottom}$$

$$\Phi = \oiint_{S_1} \vec{E} \cdot d\vec{S}_1 + \oiint_{S_2} \vec{E} \cdot d\vec{S}_2 + \oiint_{S_3} \vec{E} \cdot d\vec{S}_3$$

$$\Phi = \underbrace{\oiint_{S_1} \vec{E} \cdot d\vec{S}_1}_{\vec{E} \parallel d\vec{S}_1} + \underbrace{\oiint_{S_2} \vec{E} \cdot d\vec{S}_2}_{=0 \text{ } (\vec{E} \perp d\vec{S}_2)} + \underbrace{\oiint_{S_3} \vec{E} \cdot d\vec{S}_3}_{=0 \text{ } (\vec{E} \perp d\vec{S}_2)}$$

$$\Phi = \oiint_{S_1} \vec{E} \cdot d\vec{S}_1 = \oiint_{S_1} E \cdot dS_1 \cos(\theta) = E \times S_1$$

$$S_1 = S_{lateral(G)} = 2\pi rL$$

$$\Phi = E \cdot 2\pi rL$$

a) Enclosed charge:

If $r < R$:

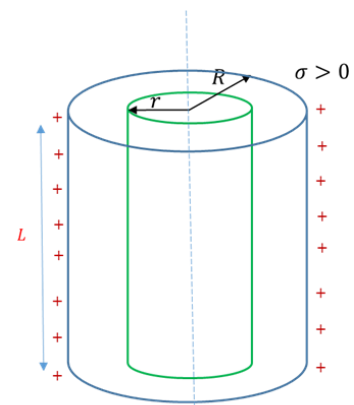
$$Q_{enc} = 0$$

b) Application of Gauss's Law:

$$\oiint \vec{E} \cdot d\vec{S}_1 = \frac{\Sigma Q_{enc}}{\epsilon_0}$$

$$E \cdot 2\pi rL = \frac{0}{\epsilon_0}$$

$$\boxed{E(r) = 0}$$



If $r > R$:

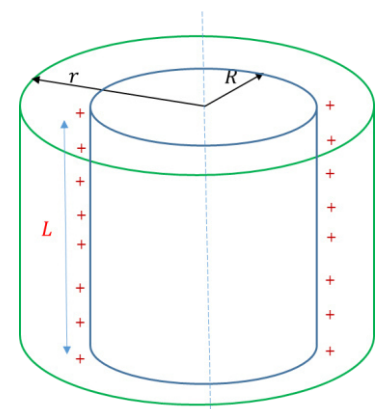
$$Q_{enc} = \sigma S_{(R)} = \sigma 2\pi RL$$

c) Application of Gauss's Law:

$$\oiint \vec{E} \cdot d\vec{S} = \frac{\Sigma Q_{enc}}{\epsilon_0}$$

$$E \cdot 2\pi rL = \frac{\sigma 2\pi RL}{\epsilon_0}$$

$$\boxed{E(r) = \frac{\sigma R}{\epsilon_0 r}}$$



$$\begin{cases} \vec{E}(r) = 0 \vec{u}_r & r < R \\ \vec{E}(r) = \frac{\sigma R}{\epsilon_0 r} \vec{u}_r & r > R \end{cases}$$

Note:

- If $\sigma > 0$, the electric field is directed **radially outward**
- If $\sigma < 0$, the electric field is directed **radially inward**

4. Cylindrical Volume Charge :

We consider an infinite solid cylinder of radius R , uniformly charged in its volume with a volume charge density $\rho > 0$.

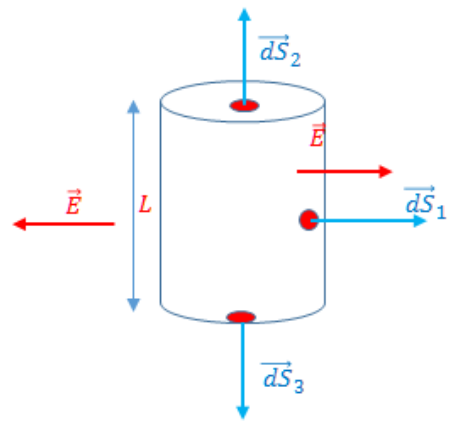
For an infinite charged cylinder, the electric field has cylindrical symmetry.

a) Choice of Gaussian surface

We choose as a Gaussian surface a cylinder of radius r and length L , coaxial with the uniformly charged cylinder, due to cylindrical symmetry.

The closed cylindrical Gaussian surface consists of:

4. the lateral surface S_1
5. the top base S_2
6. the bottom base S_3



b) Total Flux:

$$d\Phi = d\Phi_{lateral} + d\Phi_{top} + d\Phi_{bottom}$$

$$\Phi = \iint_{S_1} \vec{E} \cdot \vec{dS}_1 + \iint_{S_2} \vec{E} \cdot \vec{dS}_2 + \iint_{S_3} \vec{E} \cdot \vec{dS}_3$$

$$\Phi = \underbrace{\iint_{S_1} \vec{E} \cdot \vec{dS}_1}_{\vec{E} \parallel \vec{dS}_1} + \underbrace{\iint_{S_2} \vec{E} \cdot \vec{dS}_2}_{=0 \text{ } (\vec{E} \perp \vec{dS}_2)} + \underbrace{\iint_{S_3} \vec{E} \cdot \vec{dS}_3}_{=0 \text{ } (\vec{E} \perp \vec{dS}_2)}$$

$$\Phi = \iint_{S_1} \vec{E} \cdot \vec{dS}_1 = \iint_{S_1} E \cdot dS_1 \cos(0) = E \times S_1$$

$$S_1 = S_{lateral(G)} = 2\pi rL$$

$$\Phi = E \cdot 2\pi rL$$

c) Enclosed charge :

If $r < R$:

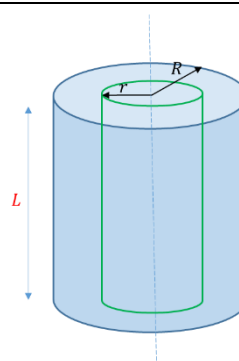
$$Q_{enc} = \rho V_{(r)} = \rho \pi r^2 L$$

Application of Gauss's Law :

$$\oiint \vec{E} \cdot d\vec{S} = \frac{\Sigma Q_{enc}}{\epsilon_0}$$

$$E \cdot 2\pi r L = \frac{\rho \pi r^2 L}{\epsilon_0}$$

$$E(r) = \frac{\rho r}{2\epsilon_0}$$



If $r > R$:

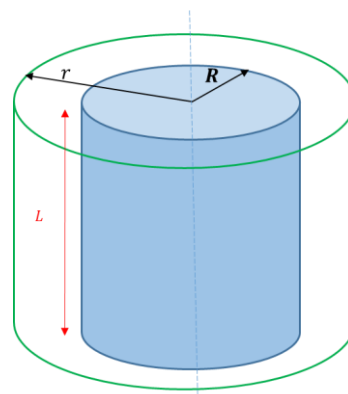
$$Q_{enc} = \rho V_{(R)} = \rho \pi R^2 L$$

Application of Gauss's Law :

$$\oiint \vec{E} \cdot d\vec{S} = \frac{\Sigma Q_{enc}}{\epsilon_0}$$

$$E \cdot 2\pi r L = \frac{\rho \pi R^2 L}{\epsilon_0}$$

$$E(r) = \frac{\rho R^2}{2\epsilon_0 r}$$



$$\begin{cases} \vec{E}(r) = \frac{\rho r}{2\epsilon_0} \vec{u}_r & r < R \\ \vec{E}(r) = \frac{\rho R^2}{2\epsilon_0 r} \vec{u}_r & r > R \end{cases}$$

Note:

- If $\rho > 0$, the electric field is directed **radially outward**
- If $\rho < 0$, the electric field is directed **radially inward**

5. Spherical Surface Charge :

We consider a spherical shell of radius R , uniformly charged on its surface with a surface charge density $\sigma > 0$.

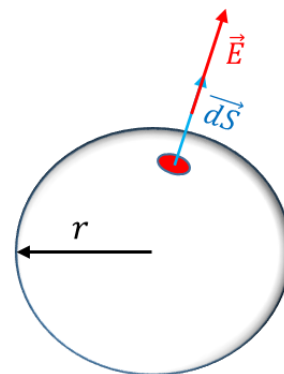
Because of spherical symmetry, the electric field is radial.

a) Choice of Gaussian surface

We choose as a Gaussian surface a sphere of radius r , concentric with the charged sphere, due to spherical symmetry.

The electric field is radial and depends only on the distance r

The Gaussian surface is a sphere of radius r .



At every point on this surface:

- ✓ \vec{E} is radial
- ✓ $d\vec{S}$ is also radial
- ✓ therefore $\vec{E} \parallel d\vec{S}$

b) Total Flux:

$$\Phi = \oiint \vec{E} \cdot d\vec{S} = \oiint_S E \cdot dS$$

$$\Phi = E \times S_G$$

$$\Phi = E \times 4\pi r^2$$

c) Enclosed charge :

If $r < R$:

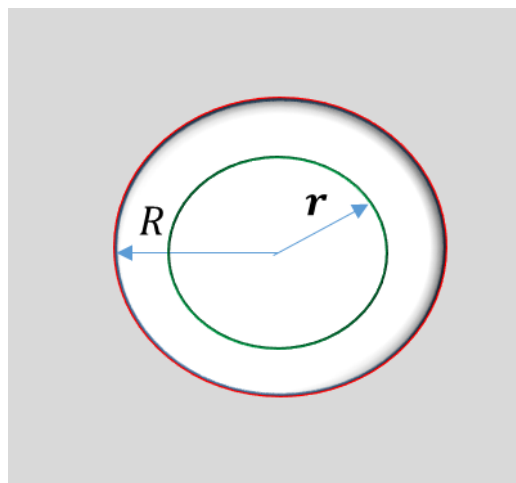
$$Q_{enc} = 0$$

Application of Gauss's Law :

$$\oiint \vec{E} \cdot d\vec{S} = \frac{\sum Q_{enc}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{0}{\epsilon_0}$$

$$\boxed{E(r) = 0}$$



If $r > R$:

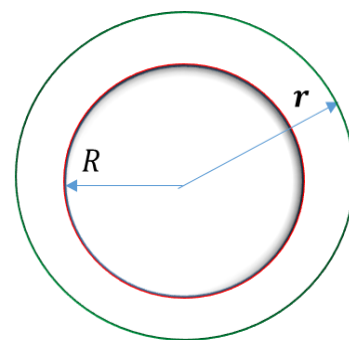
$$Q_{enc} = \sigma S_{sph} = \sigma 4\pi R^2$$

Application of Gauss's Law :

$$\oiint \vec{E} \cdot d\vec{S} = \frac{\sum Q_{enc}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{\sigma 4\pi R^2}{\epsilon_0}$$

$$\boxed{E(r) = \frac{\sigma R^2}{\epsilon_0 r^2}}$$



$$\begin{cases} \vec{E}(r) = 0 \vec{u}_r & r < R \\ \vec{E}(r) = \frac{\sigma R^2}{2\epsilon_0 r^2} \vec{u}_r & r > R \end{cases}$$

Note:

- If $\sigma > 0$, the electric field is directed **radially outward**
- If $\sigma < 0$, the electric field is directed **radially inward**

6. Spherical Volume Charge :

We consider a solid sphere of radius R , uniformly charged throughout its volume with a volume charge density $\rho > 0$.

Because of spherical symmetry, the electric field is radial.

The Gaussian surface is a sphere of radius r .

At every point:

- ✓ \vec{E} is radial
- ✓ $d\vec{S}$ is radial
- ✓ therefore $\theta = 0$

Total Flux:

$$\Phi = \oiint \vec{E} \cdot d\vec{S} = \oiint_S E \cdot dS$$

$$\Phi = E \times S$$

$$\Phi = E \times 4\pi r^2$$

Enclosed charge :

If $r < R$:

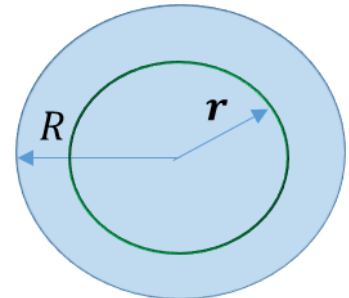
$$Q_{enc} = \rho V_{sph}(r) = \rho \frac{4}{3} \pi r^3$$

Application of Gauss's Law :

$$\oiint \vec{E} \cdot d\vec{S} = \frac{\Sigma Q_{enc}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{\rho \frac{4}{3} \pi r^3}{\epsilon_0}$$

$$E(r) = \frac{\rho r}{3\epsilon_0}$$



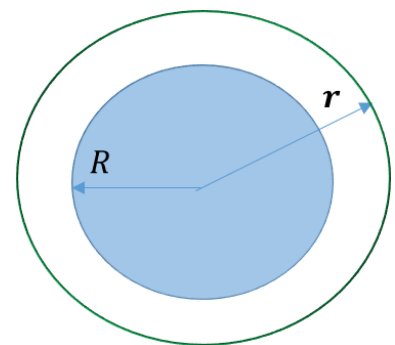
If $r > R$:

$$Q_{enc} = \rho V_{sph}(R) = \rho \frac{4}{3} \pi R^3$$

Application of Gauss's Law :

$$\oiint \vec{E} \cdot d\vec{S} = \frac{\Sigma Q_{enc}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{\rho \frac{4}{3} \pi R^3}{\epsilon_0}$$



$$\boxed{E(r) = \frac{\rho R^3}{3\epsilon_0 r^2}}$$

$$\begin{cases} \vec{E}(r) = \frac{\rho r}{3\epsilon_0} \vec{u}_r & r < R \\ \vec{E}(r) = \frac{\rho R^3}{3\epsilon_0 r^2} \vec{u}_r & r > R \end{cases}$$

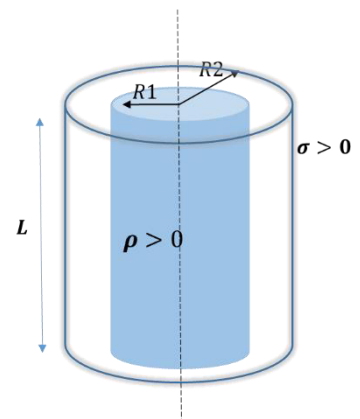
Note:

- ✓ If $\rho > 0$, the electric field is directed **radially outward**
- ✓ If $\rho < 0$, the electric field is directed **radially inward**

Exercise:

Consider two infinitely long coaxial cylinders. The inner solid cylinder of radius R_1 is uniformly charged with a volume charge density ρ . The outer cylindrical shell of radius R_2 is uniformly charged with a surface charge density σ , with $R_2 > R_1$.

- 1- Using Gauss's law, determine the electric field $\vec{E}(r)$ in all regions of space.
- 2- Deduce the electric potential $V(r)$ up to an additive constant.

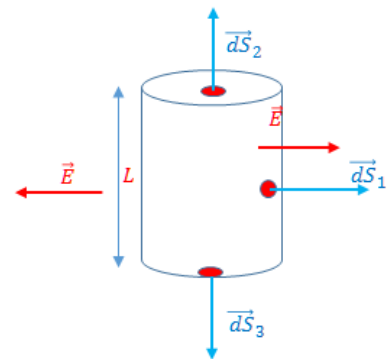


Solution:

1. Electric field:

Choice of Gaussian surface

We choose as a Gaussian surface a cylinder of radius r and length L , coaxial with the uniformly charged cylinder, due to cylindrical symmetry.



Total Flux:

$$d\Phi = d\Phi_{lateral} + d\Phi_{top} + d\Phi_{bottom}$$

$$\begin{aligned} \Phi &= \iint_{S_1} \vec{E} \cdot \vec{dS}_1 + \iint_{S_2} \vec{E} \cdot \vec{dS}_2 \\ &+ \iint_{S_3} \vec{E} \cdot \vec{dS}_3 \end{aligned}$$

$$\Phi = \underbrace{\iint_{S_1} \vec{E} \cdot \vec{dS}_1}_{\vec{E} \parallel \vec{dS}_1} + \underbrace{\iint_{S_2} \vec{E} \cdot \vec{dS}_2}_{=0 \text{ } (\vec{E} \perp \vec{dS}_2)} + \underbrace{\iint_{S_3} \vec{E} \cdot \vec{dS}_3}_{=0 \text{ } (\vec{E} \perp \vec{dS}_2)}$$

$$\Phi = \iint_{S_1} \vec{E} \cdot \vec{dS}_1 = \iint_{S_1} E \cdot dS_1 \cos(0) = E \times S_1$$

$$S_1 = S_{\text{lateral}(G)} = 2\pi r L$$

$$\Phi = E \cdot 2\pi r L$$

Enclosed charge :

If $r < R_1$:

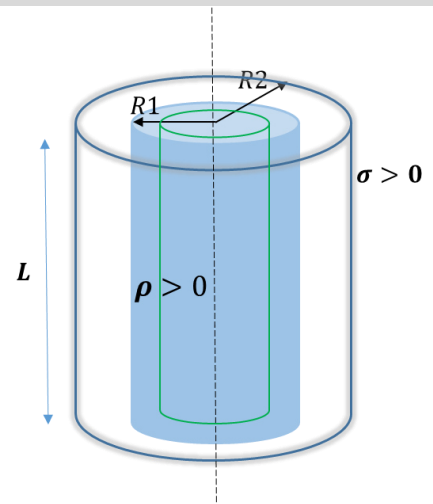
$$Q_{\text{enc}} = \rho V_{\text{cy}(r)} = \rho \pi r^2 L$$

Application of Gauss's Law :

$$\iint \vec{E} \cdot \vec{dS} = \frac{\sum Q_{\text{enc}}}{\epsilon_0}$$

$$E \cdot 2\pi r L = \frac{\rho \pi r^2 L}{\epsilon_0}$$

$$\boxed{E(r) = \frac{\rho r}{2\epsilon_0}}$$



If $R_1 < r < R_2$:

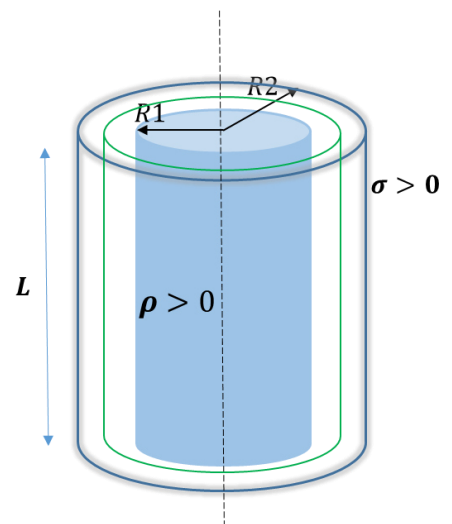
$$Q_{\text{enc}} = \rho V_{\text{cy}(R_1)} = \rho \pi R_1^2 L$$

Application of Gauss's Law :

$$\iint \vec{E} \cdot \vec{dS} = \frac{\sum Q_{\text{enc}}}{\epsilon_0}$$

$$E \cdot 2\pi r L = \frac{\rho \pi R_1^2 L}{\epsilon_0}$$

$$\boxed{E(r) = \frac{\rho R_1^2}{2\epsilon_0 r}}$$



If $r > R_2$:

$$Q_{enc} = \sigma S_{cy}(R_2) + \rho V_{cy}(R_1)$$

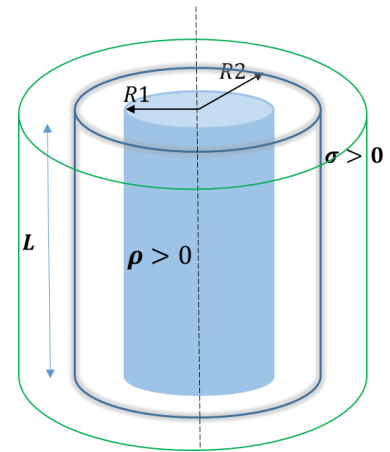
$$Q_{enc} = \sigma 2\pi R_2 L + \rho \pi R_1^2 L$$

Application of Gauss's Law :

$$\oiint \vec{E} \cdot d\vec{S} = \frac{\Sigma Q_{enc}}{\epsilon_0}$$

$$E \cdot 2\pi r L = \frac{\sigma 2\pi R_2 L + \rho \pi R_1^2 L}{\epsilon_0}$$

$$\boxed{E(r) = \frac{2\sigma R_2 + \rho R_1^2}{2\epsilon_0 r}}$$



$$\begin{cases} \vec{E}(r) = \frac{\rho r}{2\epsilon_0} \vec{u}_r & r < R_1 \\ \vec{E}(r) = \frac{\rho R_1^2}{2\epsilon_0 r} \vec{u}_r & R_1 < r < R_2 \\ \vec{E}(r) = \frac{2\sigma R_2 + \rho R_1^2}{2\epsilon_0 r} \vec{u}_r & r > R_2 \end{cases}$$

2. Electric potential V(r):

We use:

$$\vec{E} = -\overrightarrow{\text{grad}} V$$

Since the field is radial:

$$E(r) = -\frac{dV}{dr}$$

So:

$$dV = -E(r) dr$$

Region 1: $r < R_1$

$$\vec{E}_1(r) = \frac{\rho r}{2\epsilon_0}$$

$$V_1(r) = -\int \frac{\rho r}{2\epsilon_0} dr$$

$$dV_1 = -\frac{\rho r}{2\epsilon_0} dr$$

$$V_1(r) = -\frac{\rho r^2}{4\epsilon_0} + C_1$$

Region 2: $R_1 < r < R_2$

$$E_2(r) = \frac{\rho R_1^2}{2\epsilon_0 r}$$

$$V_2(r) = - \int \frac{\rho R_1^2}{2\epsilon_0 r} dr$$

$$dV_2 = - \frac{\rho R_1^2}{2\epsilon_0 r} dr$$

$$V_2(r) = - \frac{\rho R_1^2}{2\epsilon_0} \ln r + C_2$$

Region 3: $r > R_2$

$$E_3(r) = \frac{2\sigma R_2 + \rho R_1^2}{2\epsilon_0 r}$$

$$V_3(r) = - \int \left(\frac{2\sigma R_2 + \rho R_1^2}{2\epsilon_0 r} \right) dr$$

$$dV_3 = - \left(\frac{2\sigma R_2 + \rho R_1^2}{2\epsilon_0 r} \right) dr$$

$$V_3(r) = - \left(\frac{2\sigma R_2 + \rho R_1^2}{2\epsilon_0} \right) \ln r + C_3$$

$$V(r) = \begin{cases} -\frac{\rho r^2}{4\epsilon_0} + C_1 \\ -\frac{\rho R_1^2}{2\epsilon_0} \ln r + C_2 \\ -\left(\frac{2\sigma R_2 + \rho R_1^2}{2\epsilon_0} \right) \ln r + C_3 \end{cases}$$

Exercise2:

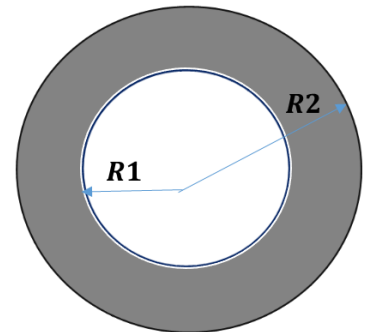
Consider a spherical shell centered at point O, with inner radius R_1 and outer radius R_2 , where $R_2 > R_1$. The region between the two spheres is uniformly charged with a volume charge density $\rho > 0$, while the inner region ($r < R_1$) contains no charge.

- Using Gauss's theorem, determine the electric field $\vec{E}(r)$ in all regions of space:

$$r < R_1 ; R_1 < r < R_2 ; r > R_2$$

- Deduce the expression of the electric potential $V(r)$ in all regions, up to an additive constant, using the relation:

$$\vec{E} = -\vec{\nabla}V$$



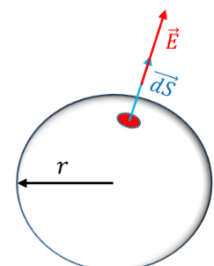
Solution:

1. Electric field:

Choice of Gaussian surface

We choose as a Gaussian surface a sphere of radius r , concentric with the charged sphere, due to spherical symmetry.

The electric field is radial and depends only on the distance r



Total Flux:

$$\Phi = \oiint \vec{E} \cdot d\vec{S} = \oiint_S E \cdot dS \quad \left| \quad \Phi = E \times S_G \quad \right| \quad \Phi = E \times 4\pi r^2$$

Region 1: $r < R_1$

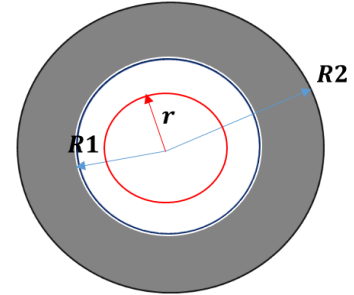
$$Q_{enc} = 0$$

Application of Gauss's Law :

$$\oiint \vec{E} \cdot d\vec{S} = \frac{\Sigma Q_{enc}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{0}{\epsilon_0}$$

$$\boxed{E(r) = 0}$$



Region2: $R_1 < r < R_2$

$$Q_{enc} = \rho (V_{sph}(r) - V_{sph}(R_1))$$

$$Q_{enc} = \rho \left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi R_1^3 \right)$$

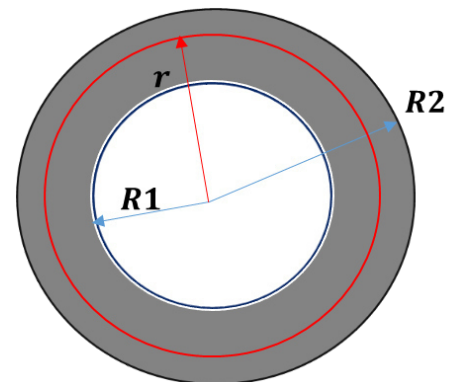
$$Q_{enc} = \rho \frac{4}{3}\pi (r^3 - R_1^3)$$

Application of Gauss's Law :

$$\oiint \vec{E} \cdot d\vec{S} = \frac{\Sigma Q_{enc}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{\rho \frac{4}{3}\pi (r^3 - R_1^3)}{\epsilon_0}$$

$$\boxed{E(r) = \frac{\rho (r^3 - R_1^3)}{3\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0} \left(r - \frac{R_1^3}{r^2} \right)}$$



Region3: $r > R_2$

$$Q_{enc} = \rho (V_{sph}(R_2) - V_{sph}(R_1))$$

$$Q_{enc} = \rho \left(\frac{4}{3} \pi R_2^3 - \frac{4}{3} \pi R_1^3 \right)$$

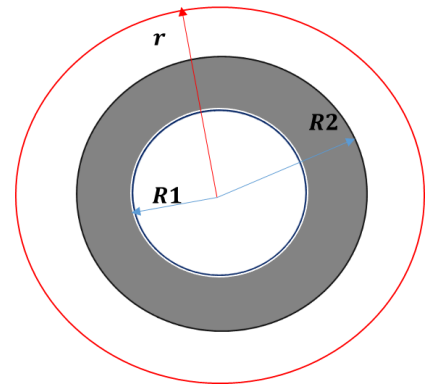
$$Q_{enc} = \rho \frac{4}{3} \pi (R_2^3 - R_1^3)$$

Application of Gauss's Law :

$$\oiint \vec{E} \cdot d\vec{S} = \frac{\Sigma Q_{enc}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{\rho \frac{4}{3} \pi (R_2^3 - R_1^3)}{\epsilon_0}$$

$$E(r) = \frac{\rho (R_2^3 - R_1^3)}{3\epsilon_0 r^2}$$



$$\begin{cases} \vec{E}(r) = 0 \vec{u}_r & r < R_1 \\ \vec{E}(r) = \frac{\rho}{3\epsilon_0} \left(r - \frac{R_1^3}{r^2} \right) \vec{u}_r & R_1 < r < R_2 \\ \vec{E}(r) = \frac{\rho (R_2^3 - R_1^3)}{3\epsilon_0 r^2} \vec{u}_r & r > R_2 \end{cases}$$

2. Electric potential V(r):

We use:

$$\vec{E} = -\overrightarrow{\text{grad}} V$$

Since the field is radial:

$$E(r) = -\frac{dV}{dr}$$

So:

$$dV = -E(r) dr$$

Region 1: $r < R_1$

$$E_1(r) = 0$$

$$dV_1 = -\frac{\rho r}{2\epsilon_0} dr$$

$$V_1(r) = -\int 0 dr$$

$$V_1(r) = C_1$$

Region 2: $R_1 < r < R_2$

$$E_2(r) = \frac{\rho}{3\epsilon_0} \left(r - \frac{R_1^3}{r^2} \right)$$

$$V_2(r) = -\frac{\rho}{3\epsilon_0} \int \left(r - \frac{R_1^3}{r^2} \right) dr$$

$$dV_2 = -\frac{\rho}{3\epsilon_0} \left(r - \frac{R_1^3}{r^2} \right) dr$$

$$V_2(r) = -\frac{\rho}{2\epsilon_0} \left(\frac{1}{2} r^2 + \frac{R_1^3}{r} \right) + C_2$$

$$V_2(r) = -\frac{\rho}{6\epsilon_0} r^2 - \frac{\rho R_1^3}{2\epsilon_0 r} + C_2$$

Region 3: $r > R_2$

$$E_3(r) = \frac{\rho (R_2^3 - R_1^3)}{3\epsilon_0 r^2}$$

$$V_3(r) = -\int \left(\frac{\rho (R_2^3 - R_1^3)}{3\epsilon_0 r^2} \right) dr$$

$$dV_3 = -\left(\frac{\rho (R_2^3 - R_1^3)}{3\epsilon_0 r^2} \right) dr$$

$$V_3(r) = -\rho \left(\frac{R_2^3 - R_1^3}{3\epsilon_0} \right) \left(-\frac{1}{r} \right) + C_3$$

$$V_3(r) = -\frac{\rho r^2}{6\epsilon_0} - \frac{\rho R_1^3}{2\epsilon_0 r} + C_2$$

$$V(r) = \begin{cases} C_1 & r < R_1 \\ -\frac{\rho r^2}{6\epsilon_0} - \frac{\rho R_1^3}{2\epsilon_0 r} + C_2 & R_1 < r < R_2 \\ \frac{\rho (R_2^3 - R_1^3)}{3\epsilon_0 r} + C_3 & r > R_2 \end{cases}$$

5. Conductors in Electrostatic Equilibrium

5.1. Introduction

A conductor is a material that contains free charges, i.e., charges that can move easily under the influence of an electric field.

Nature of free charges:

- ✓ In **metallic materials**: free charges are called conduction electrons (*copper Cu, aluminum Al, silver Ag*)
- ✓ In **ionic solutions (electrolytes)**: free charges are called ions (Na^+ , Cl^- ; H^+ , SO_4^{2-})

Under the influence of an electric field, free charges move within a conductor and continue to do so until an internal field is generated that cancels out the external field, reaching a state of stability known as electrostatic equilibrium, where the net charge movement ceases.

5.2. Conductor in electrostatic equilibrium :

A conductor is in electrostatic equilibrium when its free charges are at rest and no electric force causes further motion; therefore, the electric field inside the conductor is zero.

5.3. Properties of a conductor in equilibrium:

a) Electric field inside a conductor in electrostatic equilibrium :

$$\begin{aligned}\sum \vec{F} &= \vec{F}_{inside} = q \vec{E}_{inside} = \vec{0} \\ &\rightarrow \vec{E}_{in} = \vec{0}\end{aligned}$$

b) Electric potential of a conductor in electrostatic equilibrium:

$$\vec{E} = -\overrightarrow{grad} V_{in} = \vec{0} \rightarrow V_{in} = Constant$$

The electric potential is uniform throughout the conductor; it has the same value inside and on the surface:

$$V_{in} = V_S = Constant$$

c) Volume charge density of a conductor in electrostatic equilibrium:

$$\oint (\vec{E}) \cdot d\vec{S} = \oint \vec{E}_{inside} \cdot d\vec{S} = \frac{\sum Q_{enc}}{\epsilon_0} = 0 \rightarrow Q_{enc} = 0$$

$$\rightarrow Q_{enc} = \iiint \rho dV = 0 \rightarrow \rho = 0$$

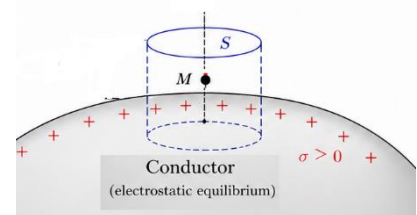
5.4. Electric Field near the Surface of a Charged Conductor in Electrostatic Equilibrium:

Let a conductor be in electrostatic equilibrium with a positive surface charge density $\sigma > 0$. Let **M** be a point located very close to the surface, just outside the conductor.

Electric field at point M:

By applying Gauss's Law and choosing a cylindrical Gaussian surface containing the point M, as shown in the figure, we obtain:

$$\begin{aligned}d\Phi &= d\Phi_{lateral} + d\Phi_{top} + d\Phi_{bottom} \\ \Phi &= \iint_{S_1} \vec{E} \cdot d\vec{S}_1 + \iint_{S_2} \vec{E} \cdot d\vec{S}_2 + \iint_{S_3} \vec{E} \cdot d\vec{S}_3\end{aligned}$$



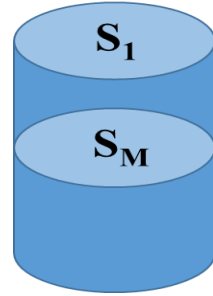
$$\Phi = \underbrace{\iint_{S_1(\text{lateral})} \vec{E} \cdot d\vec{S}_1}_{=0} + \underbrace{\iint_{S_2(\text{top})} \vec{E} \cdot d\vec{S}_2}_{(\vec{E} \parallel d\vec{S}_2)} + \underbrace{\iint_{S_3(\text{bottom})} \vec{E} \cdot d\vec{S}_3}_{=0} \quad (\vec{E} = \vec{E}_{in} = \vec{0})$$

$$\Phi = \iint_{S_2(\text{top})} \vec{E} \cdot d\vec{S}_1 = \iint_{S_1(\text{top})} E \cdot dS_1 \cos(0) = E \times S_{1(\text{top})}$$

$$S_{1(\text{top})} = S_M = \pi r^2$$

$$\boxed{\Phi = E \cdot S_{1(\text{top})} = E \cdot S_M}$$

$$E S_M = \frac{\sigma S_M}{\epsilon_0} \rightarrow E(M) = \frac{\sigma}{\epsilon_0}$$



Interpretation:

Charges redistribute on the surface such that:

- The internal field cancels ($E = 0$)
- The field exists only outside, giving the full value $\frac{\sigma}{\epsilon_0}$

5.5. Electrostatic Pressure

When a conductor is charged, the charges on its surface repel each other. This repulsion creates a force that tends to push the surface outward. This effect is describe as electrostatic pressure.

$$P = \frac{d\|\vec{F}\|}{dS}$$

$$d\|\vec{F}\| = dq\|\vec{E}\| \quad \left| \quad dq = \sigma dS \quad \right| \quad E = \frac{\sigma}{2\epsilon_0}$$

$$P = \frac{d\|\vec{F}\|}{dS} = \frac{dq\|\vec{E}\|}{dS} = \frac{\sigma dS E}{dS} = \frac{\sigma dS (\sigma/2\epsilon_0)}{dS} = \frac{\sigma^2}{2\epsilon_0}$$

$$P = \frac{\sigma^2}{2\epsilon_0}$$

5.6. Electrical energy of a conductor in electrostatic equilibrium

The electrical energy of a charged conductor is the work required to charge it from zero to a charge Q and potential V.

$$U = \frac{1}{2} QV$$

Equivalent forms:

$Q = CV$	$W = \frac{1}{2} CV^2$	$U = \frac{Q^2}{2C}$
----------	------------------------	----------------------

Exercise:

Consider two conducting metallic spheres of radii R_1 and R_2 , carrying charges Q_1 and Q_2 , respectively.

They are placed far enough apart so that their mutual influence can be neglect.

They are then connect by a conducting wire.

1. Discuss the potentials of the two spheres before and after the (connection).
2. Determine the final charges Q_1' and Q_2' on each sphere at electrostatic equilibrium.
3. Calculate the electrostatic energy of the system:
 - before the connection
 - after the connection

Given:

$$R_1 = 0.25 \text{ m} , \quad R_2 = 0.05 \text{ m} , \quad Q_1 = 1.5 \times 10^{-4} \text{ C} , \quad Q_2 = 0 \text{ C}$$

Solution :

1. Potentials before connection:

For an isolated conducting sphere of radius R carrying charge Q, the electric potential is:

<p>For sphere 1:</p> $V_1 = \frac{k Q_1}{R_1}$ $V_1 = \frac{(9 \times 10^9)(1.5 \times 10^{-4})}{0.25}$ $V_1 = 5.4 \times 10^6 \text{ V}$	<p>For sphere 2:</p> $V_2 = \frac{k Q_2}{R_2} = 0$
--	---

So, when the two spheres are connected by a conducting wire, electric charge flows from sphere 1 to sphere 2 until both spheres reach the same potential.

2. Final potential after connection

At equilibrium, the two connected conductors must have the same potential:

$V_1' = V_2' = V'$	$V' = \frac{kQ_1'}{R_1} \rightarrow$ $V'R_1 = kQ_1' \dots \dots \dots (1)$	$V' = \frac{kQ_2'}{R_2} \rightarrow$ $V'R_2 = kQ_2' \dots \dots \dots (2)$
--------------------	--	--

The total charge is conserved during the connection:

$$Q_1' + Q_2' = Q_1 + Q_2$$

$$V'(R_1 + R_2) = k(Q_1' + Q_2') \rightarrow V'(R_1 + R_2) = k(Q_1 + Q_2)$$

Thus:

$$V' = \frac{k(Q_1 + Q_2)}{R_1 + R_2} = \frac{9 \times 10^9(1.5 \times 10^{-4} + 0)}{0.25 + 0.05} = 4500 \text{ KV}$$

3. Final charges at electrostatic equilibrium:

Therefore:

$$V' = \frac{kQ_1'}{R_1} \rightarrow Q_1' = \frac{V'R_1}{k}$$

$$Q_1' = \frac{4.5 \times 10^6 \times 0.25}{9 \times 10^9}$$

$$Q_1' = 125 \times 10^{-6} \text{ C}$$

$$V' = \frac{kQ_2'}{R_2} \rightarrow Q_2' = \frac{V'R_2}{k}$$

$$Q_2' = \frac{4.5 \times 10^6 \times 0.05}{9 \times 10^9}$$

$$Q_2' = 25 \times 10^{-6} \text{ C}$$

4. Electrostatic energy of the system

- **Energy before connection :**

The energy of a charged conducting sphere is:

$$U_i = \frac{1}{2}(Q_1V_1 + Q_2V_2) = \frac{1}{2}(1.5 \cdot 10^{-4} \times 5.4 \times 10^6 + 0 \times 0) = 405 \text{ J}$$

- **Energy after connection:**

$$U_f = \frac{1}{2}(Q_2'V_2' + Q_1'V_1') = \frac{1}{2} \times 5.4 \times 10^6(125 \cdot 10^{-6} + 25 \cdot 10^{-6}) = 337.5 \text{ J}$$

$$\Delta U = U_i - U_f = 405 - 337.5 = 67.5 \text{ J}$$

The lost energy is dissipated mainly as heat in the conducting wire.

6. Capacitors

6.1. Introduction

Capacitors are fundamental components in electronics and electrical engineering. They store electrical energy and are used in many circuits (filtering, timing, energy storage, etc.).

6.2. Definition

A **capacitor** is a two-terminal device consisting of two conductive plates separated by an insulating material called a dielectric.

6.3. Structure of a Capacitor

A capacitor consists of:

- Two metallic plates (conductors)
- An insulating material (air, paper, ceramic, plastic...)

6.4. Capacitance

Capacitance represents the ability of a capacitor to store electric charge.

$$C = \frac{Q}{U}$$

Where:

- ✓ C: capacitance (Farad, F)
- ✓ Q: charge (Coulomb, C)
- ✓ U: voltage (Volt, V)

Factors affecting capacitance:

- ✓ Plate surface area
- ✓ Distance between plates
- ✓ Nature of the dielectric

6.5. Special Capacitors

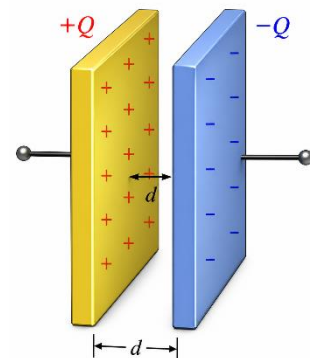
a) Parallel-Plate Capacitor :

Consists of two parallel plates separated by a distance d .

$$\text{Capacitance: } C = \frac{\epsilon \times S}{d}$$

Where:

- ✓ ϵ : permittivity of the dielectric
- ✓ S: plate surface area
- ✓ d : distance between plates



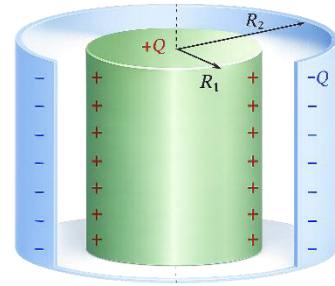
b) Cylindrical Capacitor

Formed by two coaxial cylinders with radii R_1 and R_2 .

Capacitance $C = \frac{2\pi \epsilon L}{\ln(R_2 / R_1)}$

Where:

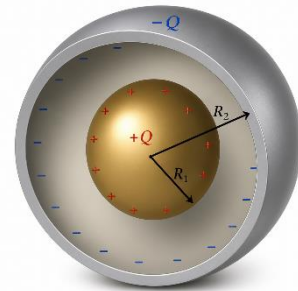
- L: length of the cylinder
- ϵ : permittivity



c) Spherical Capacitor

Formed by two concentric spheres with radii R_1 and R_2 .

Capacitance : $C = \frac{4\pi \epsilon R_1 R_2}{R_2 - R_1}$



6.6. Stored Energy:

A capacitor stores electrical energy.

$$E = \frac{1}{2} C V^2$$

6.7. Charging and Discharging :

In an RC circuit:

Charging:

The voltage gradually increases until it reaches the supply voltage.

Discharging:

The voltage gradually decreases.

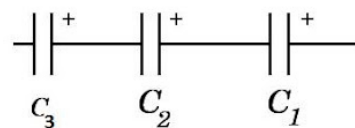
Time constant: $\tau = R \times C$

✓ τ represents the speed of charging/discharging

6.8. Capacitor Combinations:

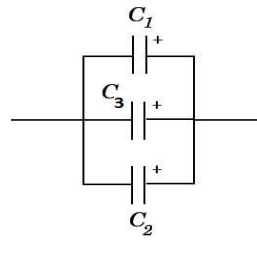
1) Series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$



2) Parallel

$$C_{eq} = C_1 + C_2 + \dots$$



6.9. Types of Capacitors:

- ✓ Ceramic capacitors
- ✓ Electrolytic capacitors
- ✓ Film capacitors
- ✓ Tantalum capacitors



6.10. Applications:

- 1) Signal filtering
- 2) Voltage stabilization
- 3) Timing circuits (RC circuits)
- 4) Energy storage
- 5) Oscillating circuits

Exercise1: (Parallel-Plate Capacitor)

A parallel-plate capacitor is made of two metallic plates of area $S = 0.02 \text{ m}^2$, separated by a distance $d = 2 \text{ mm}$. The space between the plates is filled with air of permittivity $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$.

- Calculate the capacitance C of the capacitor.

Solution:

$$S = 0.02 \text{ m}^2, \quad d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}, \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$C = \frac{\epsilon_0 \times S}{d}$$

Substitution:

$$C = \frac{(8.85 \times 10^{-12} \times 0.02)}{2 \times 10^{-3}}$$

$$C = \frac{0.02}{0.002} \times 8.85 \times 10^{-12}$$

$$C = 8.85 \times 10^{-12} \times 10 = 8.85 \times 10^{-11} \text{ F}$$

$$C = 88.5 \text{ pF}$$

Exercise2: (Cylindrical Capacitor)

A cylindrical capacitor consists of two coaxial conducting cylinders. The inner cylinder has a radius $R_1=1$ cm, while the outer cylinder has a radius $R_2=3$ cm. The length of the capacitor is $L=20$ cm, and the space between the cylinders is filled with air.

- Calculate the capacitance C of the capacitor

Solution:

$$R_1 = 0.01 \text{ m}, R_2 = 0.03 \text{ m}, L = 0.2 \text{ m}$$

$$C = \frac{2\pi \epsilon_0 L}{\ln(R_2 / R_1)}$$

$$\ln\left(\frac{R_2}{R_1}\right) = \ln\left(\frac{0.03}{0.01}\right) = \ln 3 = 1.0986$$

Substitution:

$$C = \frac{(2\pi \times 8.85 \times 10^{-12} \times 0.2)}{\ln\left(\frac{0.03}{0.01}\right)} = \frac{1.2566 \times 8.85 \times 10^{-12}}{1.0986} = 1.01 \times 10^{-11} \text{ F}$$

$$C = 10.1 \text{ pF}$$

Exercise3: (Spherical Capacitor)

Two concentric conducting spheres form a spherical capacitor. The inner sphere has a radius $R_1=5$ cm, and the outer sphere has a radius $R_2=10$ cm. The region between them filled with air.

- Find the capacitance C of the capacitor

Solution:

$$R_1 = 0.05 \text{ m}, R_2 = 0.1 \text{ m}$$

$$C = \frac{4\pi \epsilon_0 R_1 R_2}{R_2 - R_1}$$

Substitution:

$$C = \frac{(4\pi \times 8.85 \times 10^{-12} \times 0.05 \times 0.1)}{0.1 - 0.05} = \frac{0.005}{0.05} \times 4\pi \times 8.85 \times 10^{-12}$$

$$C \approx 1.11 \times 10^{-11} \text{ F} = 11.1 \text{ pF}$$

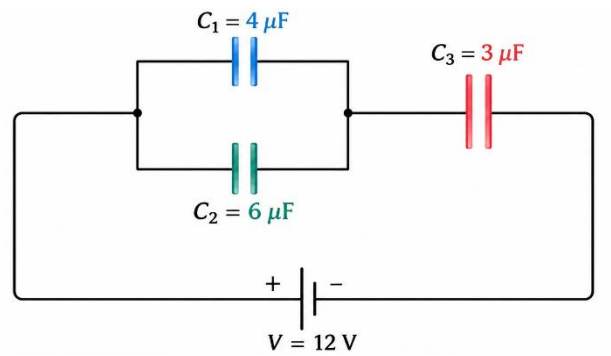
Exercise4: (Capacitor Combination)

Three capacitors are connected as follows:

$C_1 = 4 \mu F$ and $C_2 = 6 \mu F$ are connected in parallel

This parallel combination is connected in series with a third capacitor $C_3 = 3 \mu F$

1. Calculate the equivalent capacitance of the parallel part
2. Deduce the total equivalent capacitance C_{eq}
3. If a voltage $V = 12 V$ is applied across the whole circuit, determine the total charge Q .



Glossary Tables for Chapter II: Electrostatics

مصطلحات المحور الثاني: الكهرباء الساكنة

English	العربية
Capacitance	السعة
Capacitor	مكثفة
Capacitor combinations	توصيل المكثفات
Charge density	كثافة الشحنة
Charged conductor	ناقل مشحون
Charging	الشحن
Conductor	ناقل كهربائي
Continuous charge distribution	توزيع شحني مستمر
Coulomb's law	قانون كولوم
Cylindrical surface charge	شحنة سطحية أسطوانية
Cylindrical volume charge	شحنة حجمية أسطوانية
Dielectric	عازل كهربائي
Dipole moment	عزم ثنائي القطب
Discharging	التفريغ
Discrete charge distribution	توزيع شحني منفصل
Electric charge	الشحنة الكهربائية
Electric dipole	ثنائي القطب الكهربائي
Electric field	الحقل الكهربائي
Electric field line	خط الحقل الكهربائي
Electric field near surface	الحقل الكهربائي قرب سطح ناقل
Electric flux	التدفق الكهربائي
Electric susceptibility	القابلية الكهربائية
Electrical energy	الطاقة الكهربائية
Electrostatic equilibrium	التوازن الكهروستاتيكي

Electrostatic field	الحقل الكهروستاتيكي
Electrostatic force	القوة الكهروستاتيكية
Electrostatic potential	الكمون الكهروستاتيكي
Electrostatic pressure	الضغط الكهروستاتيكي
Electrostatics	الكهروستاتيكا (الكهرباء الساكنة)
Equipotential surface	سطح تساوي الكمون
Farad	فاراد
Gauss's theorem	مبرهنة غوص
Gaussian surface	سطح غوص
Induced charge	شحنة مستحثة
Infinite line charge	شحنة خطية لا نهائية
Infinite plane	مستوى لا نهائي
Insulator	عازل
Linear charge density	كثافة شحنة خطية
Negative charge	شحنة سالبة
Plates of capacitor	لبوسا المكثف
Point charge	شحنة نقطية
Polarization	الاستقطاب
Positive charge	شحنة موجبة
Potential difference	فرق الكمون
Potential energy	طاقة الوضع الكهربائية
Relation between E and V	العلاقة بين الحقل والكمون
Spherical surface charge	شحنة سطحية كروية
Stored energy	الطاقة المخزنة
Surface charge density	كثافة شحنة سطحية
Volume charge density	كثافة شحنة حجمية
Work of electrostatic force	شغل القوة الكهروستاتيكية

• **CHAPTER** •

III

ELECTROKINETICS

ELECTRICAL CONDUCTOR

OHM'S LAW

JOULE'S LAW

ELECTRICAL CIRCUITS

APPLICATION OF OHM'S LAW TO NETWORKS

KIRCHHOFF'S LAWS

Chapter III : Electrokinetics

1. Introduction :

Electrokinetics is the branch of physics that studies the motion of electric charges in conductors and the operation of electrical circuits. It is based on fundamental laws such as Ohm's law, Joule's law, and Kirchhoff's laws, which rely on the conservation of electric charge and energy.

2. Electrical Conductor :

An electrical conductor is a material in which electric charges can move under the action of an electric field. In metallic conductors, the mobile charges are mainly free electrons.

A conductor is therefore characterized by the existence of charge carriers that are able to move through the material when a potential difference is applied.

Examples

Common examples of conductors include copper, aluminum, iron, and other metals.

3. Origin of Electric Current :

The electric current originates from the motion of electric charges under the effect of a potential difference, also called voltage.

When a voltage is applied between two points of a conductor, an electric field appears inside the conductor. This field exerts an electric force on the charges:

$$\vec{F} = q\vec{E}$$

where:

- \vec{F} : is the electric force, measured in newtons (N)
- q : is the electric charge, measured in coulombs (C)
- \vec{E} : is the electric field, measured in newtons per coulomb (N/C) or volts per meter (V/m)

Inside a metallic conductor, electrons already have random thermal motion. Under the action of the electric field, they acquire a small average motion called drift velocity. This organized motion of charges creates the electric current.

4. Conventional Direction of Current :

The conventional direction of current is defined as the direction of motion of positive charges.

Therefore:

- **Conventional current direction:** from the positive terminal to the negative terminal.
- **Real electron motion in metals:** from the negative terminal to the positive terminal.

This convention is used in circuit analysis even though electrons move in the opposite direction.

5. Electric Current Intensity :

The electric current intensity is the amount of electric charge passing through a cross-section of a conductor per unit time.

It measures the rate of flow of electric charge.

$$I = \frac{dq}{dt}$$

where:

- **I:** is the current intensity, measured in amperes (A), ($1 \text{ A} = 1 \text{ C/s}$)
- **dq:** is the infinitesimal amount of charge, measured in coulombs (C),
- **dt:** is the infinitesimal time interval, measured in seconds (s).

This means that a current of one ampere corresponds to one coulomb of charge crossing a section of the conductor every second.

6. Current Density :

The current density is a vector quantity that describes the distribution of electric current inside a conductor. It represents the current flowing per unit area through a surface perpendicular to the direction of current.

$$\vec{J} = \frac{I}{S} \vec{n}$$

where:

- \vec{J} : is the current density, measured in (A/m^2),
- **I:** is the current intensity, measured in amperes (A)
- **S:** is the cross-sectional area, measured in square meters (m^2)
- \vec{n} : is a unit vector normal to the surface.

For a uniform conductor, the magnitude of current density is:

$$J = \frac{I}{S}$$

Example:

If $I = 4 \text{ A}$ flows through a section $S = 2 \times 10^{-4} \text{ m}^2$, then:

$$J = \frac{I}{S} = \frac{4}{2 \cdot 10^{-4}} = 2 \times 10^4 \text{ A/m}^2$$

7. Ohm's Law :

Ohm's law is a fundamental law of electrokinetics. It states that, for an ohmic conductor, the voltage across its terminals is proportional to the current passing through it.

$$\boxed{V = R I}$$

where:

- **V**: is the voltage, measured in volts (V),
- **I**: is the current intensity, measured in amperes (A),
- **R**: is the electrical resistance, measured in ohms (Ω).

The resistance expresses the opposition of a conductor to the flow of electric current. A larger resistance means a smaller current for the same applied voltage.

Resistance of a Uniform Conductor:

For a conductor of length **L**, cross-sectional area **S**, and resistivity ρ :

$$\boxed{R = \rho \frac{L}{S}}$$

where:

- ρ : resistivity of the material, measured in (Ωm)
- **L**: length of the conductor, measured in meters (*m*)
- **S**: cross-sectional area, measured in (m^2)

Example:

If $R = 10 \Omega$ and $I = 2 \text{ A}$, then:

$$V = R I = 10 \times 2 = 20 \text{ V}$$

8. Joule's Law :

Joule's law describes the transformation of electrical energy into thermal energy when an electric current flows through a resistor.

$$\boxed{P_J = R I^2}$$

where:

- P_J : Joule power dissipated as heat, measured in watts (W)
- **R**: resistance, measured in ohms (Ω)
- **I**: current intensity, measured in amperes (A)

Energy Dissipated:

If the current flows during a time **t**, the thermal energy dissipated is:

$$\boxed{E_J = R I^2 t}$$

where:

- E_J : dissipated energy, measured in joules (J)
- t : time, measured in seconds (s)

Note: Joule's law is mainly applied to resistors, where electrical energy is dissipated as heat.

Example:

If $R = 5 \Omega$, $I = 3 \text{ A}$, and $t = 10 \text{ s}$, then:

$$P_J = RI^2 = 5 \times 3^2 = 45 \text{ W}$$

$$E_J = RI^2t = 5 \times 3^2 \times 10 = 450 \text{ V}$$

9. Electrical Power :

The electrical power is the amount of electrical energy exchanged or converted by a dipole per unit time.

$$P = VI$$

where:

- P : electrical power, measured in watts (W)
- U : voltage across the dipole, measured in volts (V)
- I : current through the dipole, measured in amperes (A)
- E : electrical energy, measured in joules (J)
- t : time, measured in seconds (s)

✓ Power in a Resistor:

For a resistor, using Ohm's law $V = RI$, we obtain:

$$P = RI^2$$

✓ Power of a Generator:

For a real generator with electromotive force E and internal resistance r , the total power produced is:

$$P = EI$$

10. Electrical Circuits :

An electrical circuit is a set of connected electrical components forming one or more closed paths through which electric current can flow.

✓ Generator:

A generator is an active dipole that supplies electrical energy to a circuit by maintaining a potential difference between its terminals.

It converts another form of energy, such as chemical, mechanical, or solar energy, into electrical energy.

✓ Electromotive Force:

The electromotive force, denoted by E , is the voltage provided by a generator when no current flows through it. It represents the ability of the generator to set electric charges in motion.

For a real generator with internal resistance r , the terminal voltage is:

$$V = E - rI$$

where:

- **V**: terminal voltage (V)
- **E**: electromotive force (V)
- **r**: internal resistance (Ω)
- **I**: current intensity (A)

Example

If $E = 9\text{ V}$, $r = 0.5\ \Omega$, and $I = 2\text{ A}$, then: $V = E - rI = 9 - 0.5 \times 2 = 8\text{ V}$

✓ Receiver

A receiver is a dipole that absorbs electrical energy and transforms it into another form of energy, such as thermal, mechanical, chemical, or light energy.

Examples:

Electric motor, lamp, resistor, battery during charging.

✓ Counter-Electromotive Force

Some receivers, such as electric motors, develop a counter-electromotive force, denoted by E' , which opposes the applied voltage and the current.

For a receiver with internal resistance r , the voltage across its terminals is:

$$V = E' + rI$$

where:

- **V**: voltage across the receiver (V)
- **E'**: counter-electromotive force (V)
- **r**: internal resistance (Ω)
- **I**: current intensity (A)

Here, E' corresponds to the useful energy conversion, for example mechanical energy in a motor.

Example:

If $E' = 6\text{ V}$, $r = 2\ \Omega$, and $I = 3\text{ A}$, then: $V = E' + rI = 6 + 2 \times 3 = 12\text{ V}$

11. Association of Resistors

The association of resistors consists of replacing a group of resistors by a single equivalent resistance that has the same electrical effect on the circuit.

a) Resistors in Series:

Resistors are connected in series when they are traversed by the same current.

For n resistors in series:

$$R_{eq} = R_1 + R_2 + R_3 + \dots + R_n$$

where:

- R_{eq} : equivalent resistance (Ω)
- R_1, R_2, \dots, R_n : individual resistances (Ω)

The equivalent resistance is greater than each individual resistance.

Example:

If $R_1=2\ \Omega$, $R_2=3\Omega$, and $R_3=5\Omega$ in series, then: $R_{eq} = 2 + 3 + 5 = 10\ \Omega$

b) Resistors in Parallel:

Resistors are connect in parallel when they have the same voltage across their terminals.

For n resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

The equivalent resistance is smaller than the smallest individual resistance.

Example:

If $R_1 = 6\Omega$ and $R_2 = 3\Omega$ in parallel, then:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{6} + \frac{1}{3} = \frac{3}{6} \rightarrow R_{eq} = 2\Omega$$

12. Kirchhoff's Laws

Kirchhoff's laws are fundamental laws used to analyze electrical circuits. They are based on two physical principles:

- conservation of electric charge
- conservation of energy

12.1. Basic Circuit Terms :

- **Node:** a point where three or more branches meet.
- **Branch:** a part of a circuit between two nodes, containing one or more dipoles.
- **Loop:** a closed path in an electrical circuit.
- **Circuit:** a complete network of connected dipoles allowing current flow.

12.2. Kirchhoff's Current Law: Node Law:

The node law, also called Kirchhoff's current law (KCL), states that at any node of a circuit, the sum of currents entering the node is equal to the sum of currents leaving it.

$$\sum I_{in} = \sum I_{out}$$

12.3. Kirchhoff's Voltage Law: Loop Law:

The loop law, also called Kirchhoff's voltage law (KVL), states that in any closed loop of a circuit, the algebraic sum of voltages is equal to zero.

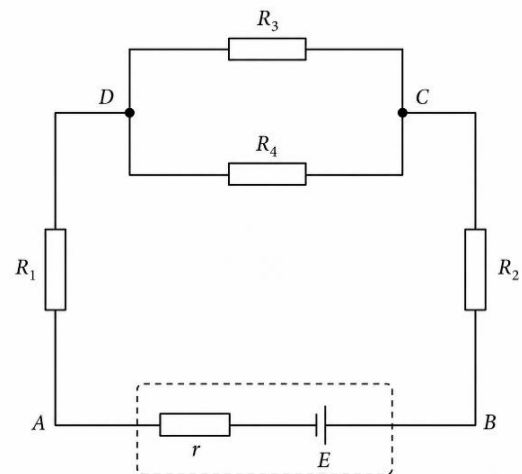
$$\sum v = 0$$

This law is a consequence of the conservation of energy. When a charge completes a closed loop, the total energy gained is equal to the total energy lost.

Exercise1:

A generator with an electromotive force $E=24\text{ V}$ and an internal resistance $r=1\ \Omega$ is connected to an electrical circuit composed of several resistors, as shown in the figure. The circuit contains two resistors, $R_1=4\ \Omega$ and $R_2=6\ \Omega$, connected in series. This series association is followed by a parallel combination formed by the resistors $R_3=5\ \Omega$ and $R_4=10\ \Omega$, connected between the junction points C and D.

The current supplied by the generator passes through the series resistors before splitting between the two branches of the parallel network and recombining afterward.



1. Determine the equivalent resistance of the parallel combination formed by R_3 and R_4 .
2. Deduce the total equivalent resistance of the external circuit.
3. Calculate the total current supplied by the generator, taking into account the internal resistance r .
4. Determine the voltage drop across each resistor.
5. Calculate the current flowing through each branch of the parallel circuit.
6. Determine the electrical power dissipated in each resistor.
7. Calculate the power supplied by the generator and compare it with the total Joule power dissipated in the circuit.

Solution:

1. Equivalent resistance of the parallel combination (R_3, R_4):

$$\frac{1}{R_{34}} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$$

$$\rightarrow R_{34} = 3.33\Omega$$

2. Total equivalent resistance of the external circuit:

$$R_{eq} = R_1 + R_2 + R_{34} = 4 + 6 + 3.33 = 13.33\Omega$$

3. Total current supplied by the generator:

The total resistance including the internal resistance is:

$$R_{total} = R_{eq} + r = 13.33 + 1 = 14.33\Omega$$

$$I = \frac{E}{R_{total}} = \frac{24}{14.33} \approx 1.67A$$

4. Voltage drop across each resistor:

For R_1 :

$$V_1 = R_1 I = 1.67 \times 4 = 6.68V$$

For R_2 :

$$V_2 = R_2 I = 1.67 \times 6 = 10.02V$$

For the parallel branch:

$$V_{CD} = R_{34} I = 1.67 \times 3.33 = 5.56V$$

Since R_3 and R_4 are in parallel:

$$V_3 = V_4 = V_{CD} = 5.56V$$

5. Current flowing through each branch of the parallel circuit:

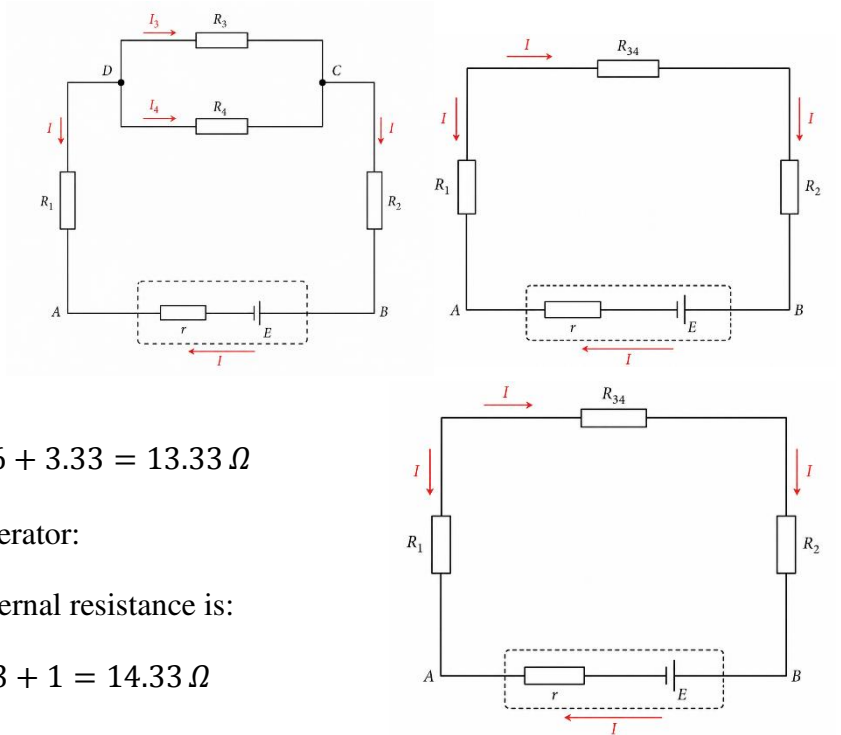
For R_3 :

$$I_3 = \frac{V_3}{R_3} = \frac{5.56}{5} = 1.11A$$

For R_4 :

$$I_4 = \frac{V_4}{R_4} = \frac{5.56}{10} = 0.56A$$

6. Electrical power dissipated in each resistor:



For R_1 :

$$P_1 = R_1 \times I_1^2 = 4 \times (1.67)^2 = 11.16W$$

For R_2 :

$$P_2 = R_2 \times I_2^2 = 6 \times (1.67)^2 = 16.74W$$

For R_3 :

$$P_3 = R_3 \times I_3^2 = 5 \times (1.11)^2 = 6.16W$$

For R_4 :

$$P_4 = R_4 \times I_4^2 = 10 \times (0.56)^2 = 3.14W$$

7. The power supplied by the generator :

$$P_{generator} = E \times I$$

$$P_{generator} = 24 \times 1.67 \approx 40.08 W$$

The power dissipated by the internal resistance is:

$$P_r = rI^2 = 1 \times (1.67)^2 \approx 2.79 W$$

The total Joule power dissipated in all resistors is:

$$P_J = P_1 + P_2 + P_3 + P_4 + P_r$$

$$P_J = 11.16 + 16.74 + 6.16 + 3.14 + 2.79 \approx 40 W$$

Therefore,

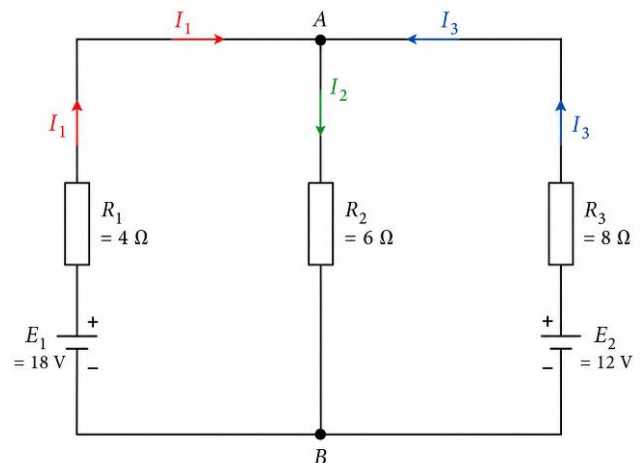
$$P_{generator} \approx P_J$$

The small difference is due to rounding.

Hence, the power supplied by the generator is approximately equal to the total Joule power dissipated in all resistors, including the internal resistance of the generator.

Exercise2:

Consider the electrical circuit shown in the figure. The circuit consists of two ideal generators and three resistors connected in three different branches between the two nodes A and B. The left branch contains a generator of electromotive force $E_1=18V$ connected in series with a resistor $R_1=4\Omega$. The middle branch contains a resistor $R_2=6\Omega$, while the right branch contains a generator of electromotive force $E_2=12V$ connected in series with a resistor $R_3=8\Omega$.



1. Determine the number of nodes in the circuit, the number of branches, and the number of independent and non-independent meshes.

2. Apply Kirchhoff's current law (KCL) at node A.
3. Apply Kirchhoff's voltage law (KVL) to the left mesh and then to the right mesh.
4. Establish the system of equations governing the currents I_1 , I_2 , and I_3 .
5. Determine the values of the currents I_1 , I_2 , and I_3 .

Solution

1. Nodes, branches, and meshes

The circuit has two nodes: $N = 2$

The circuit has three branches: $B = 3$

The number of independent meshes is: $M_{ind} = B - N + 1 = 3 - 2 + 1 = 2$

There are **2 independent meshes** and **1 non-independent mesh**.

2. Kirchhoff's current law at node A

Since I_1 and I_3 enter node A, while I_2 leaves node A:

$$I_1 + I_3 = I_2 \dots \dots \dots (1)$$

3. Kirchhoff's voltage law

For the left mesh:

$$E_1 - R_1 I_1 - R_2 I_2 = 0$$

$$18 - 4I_1 - 6I_2 = 0 \Rightarrow 4I_1 + 6I_2 = 18 \dots \dots \dots (2)$$

For the right mesh:

$$E_2 - R_3 I_3 - R_2 I_2 = 0$$

$$12 - 8I_3 - 6I_2 = 0 \Rightarrow 6I_2 + 8I_3 = 12 \dots \dots \dots (3)$$

$$(2) \Rightarrow 4I_1 + 6(I_1 + I_3) = 18 \Rightarrow 10I_1 + 6I_3 = 18 \dots \dots \dots (4)$$

$$(3) \Rightarrow 6(I_1 + I_3) + 8I_3 = 12 \Rightarrow 6I_1 + 14I_3 = 12 \dots \dots \dots (5)$$

$$\begin{cases} 10I_1 + 6I_3 = 18 \dots \dots \dots (4) \\ 6I_1 + 14I_3 = 12 \dots \dots \dots (5) \end{cases} \rightarrow \begin{cases} 5I_1 + 3I_3 = 9 \dots \dots \dots (4) \\ 3I_1 + 7I_3 = 6 \dots \dots \dots (5) \end{cases} \rightarrow \begin{cases} 5I_1 + 3I_3 = 9 \dots \dots \dots (4) \\ 3I_1 + 7I_3 = 6 \dots \dots \dots (5) \end{cases}$$

$$\begin{cases} -15I_1 - 9I_3 = -27 \dots \dots \dots (6) \\ 15I_1 + 35I_3 = 30 \dots \dots \dots (7) \end{cases}$$

$$(6)+(7) \Rightarrow -15I_1 - 9I_3 + 15I_1 + 35I_3 = -27 + 30$$

$$20I_3 = 3 \Rightarrow I_3 = \frac{3}{20} = 0.15A$$

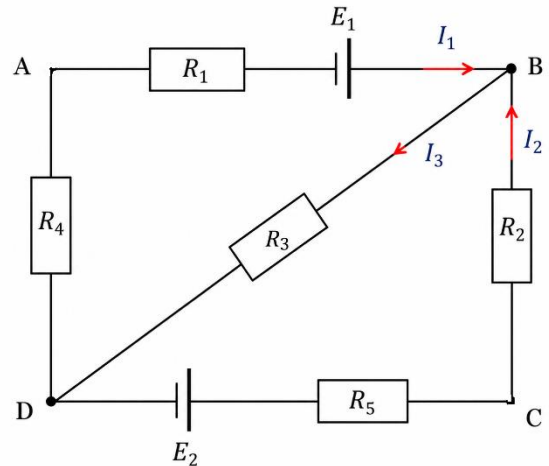
$$(4) \Rightarrow I_1 = \frac{9 - 3I_3}{5} = \frac{9 - 3 \times 0.115}{5} = 1.731A$$

$$(1) \Rightarrow I_2 = I_1 + I_3 = 1.71 + 0.15 = 1.846A$$

$$I_1 = 1.71A, \quad I_2 = 1.86A, \quad I_3 = 0.15A$$

Exercise 3 :

Consider the electrical circuit shown in the figure. The circuit contains five resistors and two ideal generators. The resistors R_1 and R_4 have the same resistance $R_1 = R_4 = 5 \Omega$, while the resistors R_2 and R_5 have the same resistance $R_2 = R_5 = 10 \Omega$. The resistor R_3 has a resistance $R_3 = 20 \Omega$. The circuit also contains two generators of electromotive forces $E_1 = 10 \text{ V}$ and $E_2 = 15 \text{ V}$. The currents flowing through the different branches are denoting by I_1 , I_2 , and I_3 .



1. Determine the number of nodes, branches, and meshes in the circuit.
2. Calculate the currents flowing through the different branches (I_1 , I_2 , I_3).
3. Determine the potential difference: $V_B - V_D$.
4. Calculate the power supplied by the two generators and the power dissipated in each resistor.
5. Determine the equivalent resistances of the resistor combinations (R_2 , R_5) and (R_1 , R_4).

Solution:

1. The number of nodes, branches, and meshes in the circuit

The circuit has two nodes: $N = 2$

The circuit has three branches: $B = 3$

The number of independent meshes is: $M_{ind} = B - N + 1 = 3 - 2 + 1 = 2$

2. The currents flowing through the different branches (I_1 , I_2 , I_3)

Law of nodes :

$$I_3 = I_1 + I_2 \dots \dots \dots (1)$$

Law of meshes:

Mesh 1 :

$$(R_4 + R_1)I_1 + R_3I_3 - E_1 = 0 \rightarrow 10I_1 + 20I_3 = 10$$

$$I_1 + 2I_3 = 1 \rightarrow I_1 + 2(I_1 + I_2) = 1$$

$$3I_1 + 2I_2 = 1 \dots \dots \dots (2)$$

Mesh 2:

$$(R_5 + R_4)I_2 + R_3I_3 - E_2 = 0 \rightarrow 20I_2 + 20I_3 = 15$$

$$4I_2 + 4I_3 = 3 \dots \dots \dots (3)$$

$$\begin{cases} -4 \times (3I_1 + 2I_2 = 1) \dots \dots \dots (2) \\ 4I_1 + 8I_2 = 3 \dots \dots \dots (3) \end{cases} \rightarrow \begin{cases} -12I_1 - 8I_2 = -4 \dots \dots \dots (4) \\ 4I_1 + 8I_2 = 3 \dots \dots \dots (3) \end{cases}$$

$$(4) + (3) \Leftrightarrow -8I_1 = -1$$

$$I_1 = \frac{-1}{-8} = 0.125 \text{ A}$$

$$(2) \Leftrightarrow I_2 = \frac{1-3I_1}{2} = 0.3125 \text{ A}$$

$$(1) \Leftrightarrow I_3 = 0.125 + 0.3125 = 0.4375 \text{ A}$$

3. Determine the potential difference: $V_B - V_D$

$$V_B - V_D = R_3I_3 = 20I_3 = 20 \times 0.4375 = 8.75 \text{ V}$$

4. The power supplied by the two generators and the power dissipated in each resistor

$$P_{fE1} = E_1 \cdot I_1 = 10 \times 0.125 = 1.25 \text{ W}$$

$$P_{fE2} = E_2 \cdot I_2 = 15 \times 0.3125 = 4.687 \text{ W}$$

$$P_{dis R1} = R_1 I_1^2 = 5 \times 0.125^2 = 0.078 \text{ W}$$

$$P_{dis R4} = R_4 I_1^2 = 5 \times 0.125^2 = 0.078 \text{ W}$$

$$P_{dis R2} = R_2 I_2^2 = 10 \times 0.3125^2 = 0.976 \text{ W}$$

$$P_{dis R5} = R_5 I_2^2 = 10 \times 0.3125^2 = 0.976 \text{ W}$$

$$P_{dis R3} = R_3 I_3^2 = 20 \times 0.4375^2 = 3.828 \text{ W}$$

5. The equivalent resistances of the resistor combinations (R_2, R_5) and (R_1, R_4)

$$R_{1/4} = R_1 + R_4 = 5 + 5 = 10 \Omega$$

$$, \quad R_{2/5} = R_2 + R_5 = 10 + 10 = 20 \Omega$$

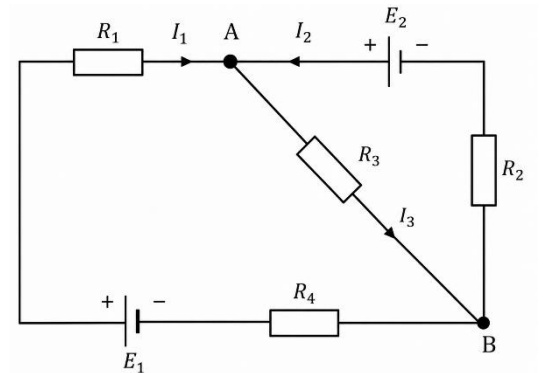
Exercise4:

Consider the electrical circuit shown in the figure. The circuit contains four resistors and two ideal generators. The resistor values are: $R_1 = 2 \Omega, R_2 = 4 \Omega, R_3 = 6 \Omega, R_4 = 2 \Omega$

The electromotive forces of the generators are:

$$E_1 = 12 V, E_2 = 18 V$$

The currents flowing through the different branches of the circuit are denoted by $I_1, I_2,$ and $I_3,$ and their directions are indicated in the figure.



1. Determine the number of nodes, branches, and independent meshes in the circuit.
2. Calculate the electric currents $I_1, I_2,$ and I_3 flowing through the different branches.
3. Determine the potential difference: $V_B - V_A$
4. Determine the total power supplied by the generators P_f and the total power dissipated by the resistors $P_d.$ Compare the two quantities and state your conclusion.

Solution:

1. The number of nodes, branches, and meshes in the circuit

The circuit has two nodes: $N = 2$

The circuit has three branches: $B = 3$

The number of independent meshes is: $M_{ind} = B - N + 1 = 3 - 2 + 1 = 2$

2. The currents flowing through the different branches (I_1, I_2, I_3)

Kirchhoff's current law at node A:

$$I_1 + I_2 = I_3 \dots\dots\dots (1)$$

Kirchhoff's voltage law :

Mesh 1 :

$$(R_4 + R_1)I_1 + R_3I_3 - E_1 = 0 \rightarrow 4I_1 + 6I_3 = 12$$

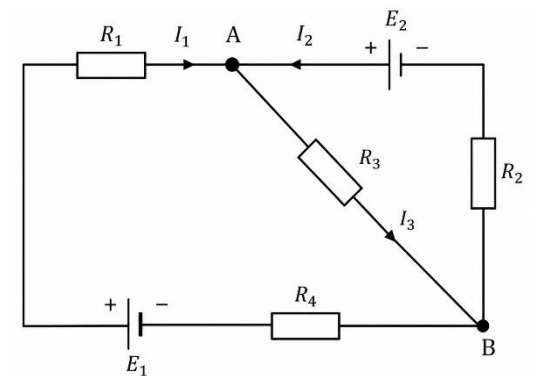
$$4I_1 + 6(I_1 + I_2) = 12$$

$$10I_1 + 6I_2 = 12 \dots\dots\dots (2)$$

Mesh 2 :

$$R_2I_2 + R_3I_3 - E_2 = 0 \rightarrow 4I_2 + 6I_3 = 18 \rightarrow 4I_2 + 6(I_1 + I_2) = 18$$

$$6I_1 + 10I_2 = 18 \dots\dots\dots (3)$$



$$\begin{cases} (10I_1 + 6I_2 = 12) \dots\dots\dots (2) \\ 6I_1 + 10I_2 = 18 \dots\dots\dots (3) \end{cases} \rightarrow \begin{cases} 5I_1 + 3I_2 = 6 \dots\dots\dots (4) \\ 3I_1 + 5I_2 = 9 \dots\dots\dots (5) \end{cases}$$

$$\begin{cases} 3 \times (5I_1 + 3I_2 = 6) \dots\dots\dots (4) \\ -5 \times (3I_1 + 5I_2 = 9) \dots\dots\dots (5) \end{cases} \rightarrow \begin{cases} 15I_1 + 9I_2 = 18 \dots\dots\dots (6) \\ -15I_1 - 25I_2 = -45 \dots\dots\dots (7) \end{cases}$$

$$(6) + (7) \Leftrightarrow -16 I_2 = -27$$

$$I_2 = \frac{-27}{-16} = \mathbf{1.69 \text{ A}}$$

$$(5) \Leftrightarrow I_1 = \frac{9-5I_2}{3} = \mathbf{0.18 \text{ A}}$$

$$(1) \Leftrightarrow I_3 = \mathbf{1.69 + 0.18 = 1.87 \text{ A}}$$

3. Determine the potential difference: $V_B - V_A$

$$V_B - V_A = -R_3 I_3 = -6 \times 1.87 = -11.22 \text{ V}$$

4. The power supplied by the two generators and the power dissipated in each resistor

$$P_{E1} = E_1 \cdot I_1 = 12 \times 0.18 = 2.16 \text{ W}$$

$$P_{E2} = E_2 \cdot I_2 = 18 \times 1.69 = 30.42 \text{ W}$$

$$P_{R1} = R_1 I_1^2 = 2 \times 0.18^2 = 0.065 \text{ W}$$

$$P_{R4} = R_4 I_1^2 = 2 \times 0.18^2 = 0.065 \text{ W}$$

$$P_{R2} = R_2 I_2^2 = 4 \times 1.69^2 = 11.42 \text{ W}$$

$$P_{R3} = R_3 I_3^2 = 6 \times 1.87^2 = 20.98 \text{ W}$$

$$P_d = P_{R1} + P_{R2} + P_{R3} + P_{R4} = 32.625 \text{ W}$$

$$P_f = P_{E1} + P_{E2} = 32.625 \text{ W}$$

$$P_f = P_d$$

The total power supplied by the generators is equal to the total power dissipated by the resistors, which confirms the conservation of energy in the electrical circuit.

Glossary Tables for Chapter III: Electrokinetics

مصطلحات المحور الثالث: الكهرباء المتحركة

English	العربية
Alternating current	تيار متناوب
Ammeter	أمبير متر
Branch	فرع
Conductance	الناقلية الكهربائية
Conductivity	الموصلية الكهربائية
Conventional current direction	الاتجاه الاصطلاحي للتيار
Current density	كثافة التيار
Current intensity	شدة التيار الكهربائي
Direct current	تيار مستمر
Electric charge	الشحنة الكهربائية
Electric current	التيار الكهربائي
Electrical circuit	دائرة كهربائية
Electrical conductor	ناقل كهربائي
Electrical energy	الطاقة الكهربائية
Electrical power	الاستطاعة الكهربائية
Electrokinetics	الكهرباء الحركية
Electromotive force	القوة المحركة الكهربائية
Equivalent resistance	المقاومة المكافئة
Fuse	مصهر كهربائي
Generator	مولد كهربائي
Internal resistance	المقاومة الداخلية
Joule effect	تأثير جول
Joule's law	قانون جول
Kirchhoff's current law	قانون كيرشوف للتيارات
Kirchhoff's laws	قوانين كيرشوف

Kirchhoff's voltage law	قانون كيرشوف للتوترات
Loop / Mesh	حلقة
Node	عقدة
Node law	قانون العقد
Ohm's law	قانون أوم
Open circuit	دائرة مفتوحة
Origin of electric current	منشأ التيار الكهربائي
Parallel association	توصيل على التوازي
Potential difference	فرق الكمون
Power dissipation	تبيد الاستطاعة
Receiver	مستقبل كهربائي
Resistance	المقاومة الكهربائية
Resistor	مقاومة كهربائية
Series association	توصيل على التسلسل
Short circuit	دائرة قصيرة
Terminal voltage	التوتر بين الطرفين
Voltage	التوتر الكهربائي
Voltmeter	فولط متر
Wattmeter	واط متر

— • **CHAPTER** • —

» **IV** «

◆

ELECTROMAGNETISM

← • →

DEFINITION OF A MAGNETIC FIELD

LORENTZ FORCE

LAPLACE LAW

FARADAY'S LAW

BIOT-SAVART LAW

MAGNETIC FIELD DUE TO A BAR MAGNET



1. Introduction:

Electromagnetism is one of the fundamental branches of physics. It studies the relationship between electricity and magnetism.

Electric charges in motion produce magnetic fields and magnetic fields can influence moving charges.

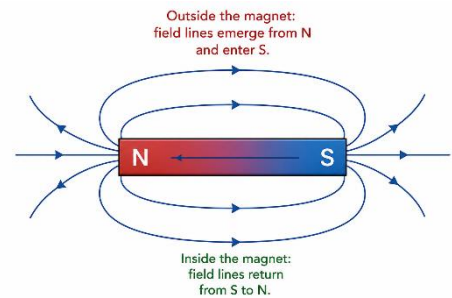
Electromagnetism has many applications in modern life:

- Electric motors
- Generators
- Transformers
- Magnetic resonance imaging (MRI)
- Loudspeakers
- Wireless communication

2. Definition of a Magnetic Field:

A magnetic field is a region of space where magnetic forces can be detected. It is produced by permanent magnets, electric currents, and moving electric charges.

The magnetic field is represented by the vector \vec{B} and its SI unit is the tesla (T).



Properties of Magnetic Field Lines:

- ✓ They emerge from the North Pole and enter the South Pole.
- ✓ They never intersect.
- ✓ Their density indicates the intensity of the field.
- ✓ Inside a magnet, the lines return from South to North.

Magnetic Field from Other Sources:

a) Current-Carrying Wire:

The magnetic field around a straight current-carrying wire forms concentric circles around the wire.

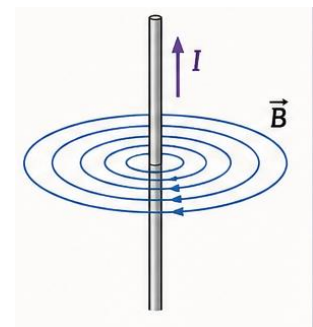
The direction of the field is determined by the right-hand rule.

Magnetic field around a straight conductor:

$$B = \frac{\mu_0 I}{2\pi r}$$

Where:

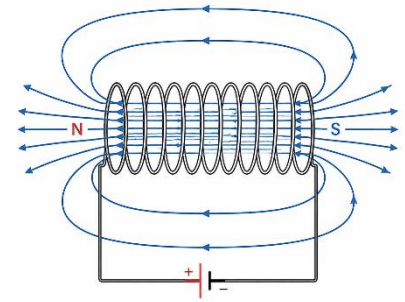
- B : magnetic field intensity (tesla, T)



- μ_0 : permeability of vacuum = $4\pi \times 10^{-7}$ T·m/A
- I : electric current intensity (ampere, A)
- r : distance from the conductor (meter, m)

b) Solenoid:

A solenoid produces a magnetic field similar to that of a bar magnet.
 Inside the solenoid, the magnetic field is strong and nearly uniform.



3. Lorentz Force:

The Lorentz force is the force experienced by a charged particle moving in a magnetic field.

It acts perpendicular to both the particle's velocity and the magnetic field direction.

The vector form of the Lorentz force is:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

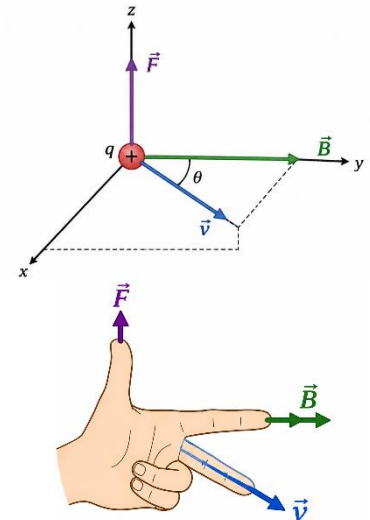
Its magnitude is:

$$F = qvB\sin\theta$$

Where:

- F: Lorentz force
- q: electric charge
- v: particle velocity
- B: magnetic field intensity
- θ : angle between \vec{v} and \vec{B}

The direction of the force is determined using the right-hand rule.



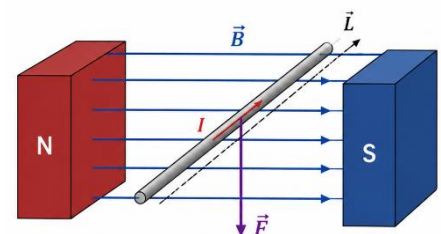
4. Laplace Law:

The Laplace force is the magnetic force exerted on a current-carrying conductor placed inside a magnetic field.

This force acts perpendicular to both the direction of the electric current and the magnetic field.

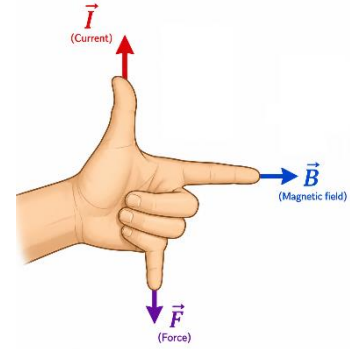
The vector form of Laplace force is:

$$\vec{F} = I(\vec{L} \times \vec{B})$$



Where:

- \vec{F} : Laplace force
- I: electric current intensity
- \vec{L} : length and direction of the conductor
- \vec{B} : magnetic field intensity



Magnitude of the Laplace Force:

$$F = BIL\sin(\theta)$$

Where:

- F: Laplace force (N)
- B: magnetic field intensity (T)
- I: electric current intensity (A)
- L: length of the conductor inside the magnetic field (m)
- θ : angle between the current direction and the magnetic field

Note:

- The force is maximum when the conductor is perpendicular to the magnetic field ($\theta = 90^\circ$).
- The force becomes zero when the conductor is parallel to the magnetic field.

Applications :

1. Electric motors
2. Loudspeakers
3. Electromagnetic relays
4. Moving-coil measuring devices
5. Magnetic cranes for lifting metals

5. Faraday's Law of Electromagnetic Induction:

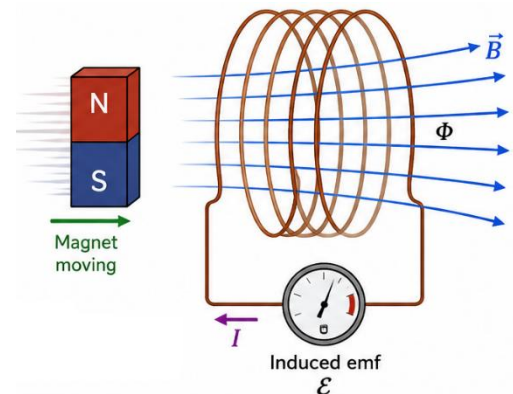
Faraday's law states that an electromotive force (emf) induced in a conductor whenever the magnetic flux through it changes with time.

The mathematical expression of Faraday's law is:

$$\varepsilon = -\frac{d\Phi}{dt}$$

Where:

- ✓ ε : induced electromotive force (emf) in volts (V)
- ✓ Φ : magnetic flux in weber (Wb)
- ✓ t: time in seconds (s)



Note:

The negative sign indicates Lenz's law, meaning that the induced current opposes the change in magnetic flux that produced it.

Applications :

1. Electric generators
2. Transformers
3. Induction cookers
4. Wireless charging systems
5. Bicycle dynamos

6. Biot–Savart Law:

The Biot–Savart law describes the magnetic field produced by a small element of a current-carrying conductor.

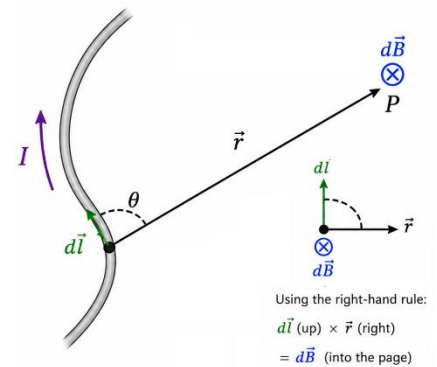
It allows the calculation of the magnetic field at any point in space generated by electric currents.

The mathematical expression of the law is:

$$d\vec{B} = \frac{\mu_0 I (d\vec{l} \times \vec{r})}{4\pi r^3}$$

Where:

- ✓ $d\vec{B}$: elementary magnetic field (T)
- ✓ μ_0 : permeability of vacuum
- ✓ I : electric current intensity (A)
- ✓ $d\vec{l}$: elementary length vector of the conductor (m)
- ✓ \vec{r} : vector from the current element to the observation point
- ✓ r : distance between the current element and the observation point



Note :

- The direction of the magnetic field is determined using the right-hand rule.
- The magnetic field is perpendicular to both $d\vec{l}$ and \vec{r} .
- This law mainly used to calculate magnetic fields generated by wires, circular loops, and complex current distributions.

Applications :

1. Magnetic field calculation around conductors
2. Electromagnets
3. Circular current loops
4. Magnetic devices and sensors

7. Magnetic Dipole Moment:

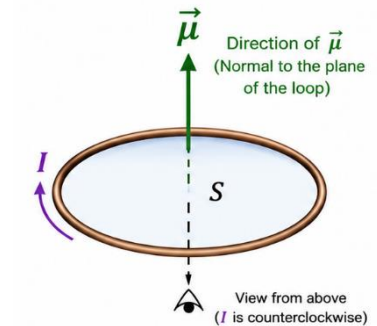
A magnetic dipole is a system that produces a magnetic field similar to that of a small magnet. A current-carrying loop behaves like a magnetic dipole and is characterized by its magnetic dipole moment.

The magnetic dipole moment is given by:

$$\mu = NIS$$

Where:

- ✓ μ : magnetic dipole moment ($A \cdot m^2$)
- ✓ N : number of turns of the coil
- ✓ I : electric current intensity (A)
- ✓ S : surface area of the loop (m^2)



Note :

- The direction of the magnetic dipole moment is determined using the right-hand rule.
- Increasing the current, the number of turns, or the loop area increases the magnetic dipole moment.

Applications :

1. Electric motors
2. Galvanometers
3. Electromagnets
4. Magnetic sensors
5. Compasses and magnetic devices

Exercise 1:

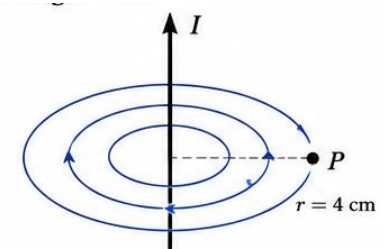
A long straight conductor carries an electric current of intensity: $I = 8 A$

We want to study the magnetic field created around the wire at a point located 4 cm from the conductor.

Calculate the magnetic field intensity at this point.

Given:

$$\mu_0 = 4\pi \times 10^{-7} T \cdot m/A$$



Solution :

$$B = \frac{\mu_0 I}{2\pi r}$$

$$r = 4 \text{ cm}$$

$$B = \frac{4\pi \times 10^{-7} \times 8}{2\pi \times 0.04}$$

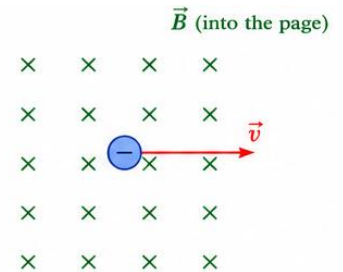
$$B = 4 \times 10^{-5} \text{ T}$$

Exercise 2 :

An electron moves inside a magnetic field with a velocity of $2 \times 10^6 \text{ m/s}$

The velocity is perpendicular to a magnetic field of intensity: 0.3 T

1. Calculate the magnitude of the Lorentz force acting on the electron.
2. Determine the direction of the force using the right-hand rule, taking into account that the electron has a negative charge.



Given: $e = 1.6 \times 10^{-19} \text{ C}$

Solution

$$F = qvB\sin\theta$$

Since the velocity is perpendicular to the magnetic field:

$$\theta = 90^\circ, \sin^\circ = 1$$

$$F = 1.6 \times 10^{-19} \times 2 \times 10^6 \times 0.3$$

$$F = 9.6 \times 10^{-14} \text{ N}$$

Because the particle is an electron, the force direction is opposite to the right-hand rule.

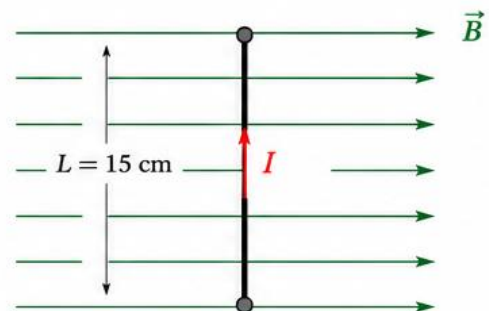
Exercise 3 :

A straight conductor of length 15 cm is placed inside a uniform magnetic field of intensity $B = 0.4 \text{ T}$

The conductor carries an electric current of intensity $I = 3 \text{ A}$

The conductor is perpendicular to the magnetic field.

1. Calculate the magnitude of the Laplace force acting on the conductor.
2. Determine the direction of the force using the right-hand rule.
3. What happens to the force if the conductor becomes parallel to the magnetic field?



Solution

$$F = BIL\sin\theta$$

$$L = 15 \text{ cm} = 0.15 \text{ m}$$

$$F = 0.4 \times 3 \times 0.15 \times 1$$

$$F = 0.18N$$

If the conductor becomes parallel to the magnetic field:

$$\theta = 0^\circ, \sin 0^\circ = 0$$

So:

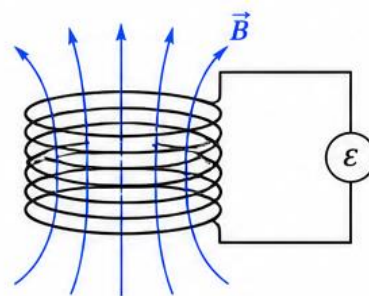
$$F = 0$$

Exercise 4:

The magnetic flux through a coil changes from 0.12 Wb to 0.03 Wb during a time interval of 0.2 s

Calculate the induced electromotive force (emf).

1. Does the magnetic flux increase or decrease?
2. According to Lenz's law, what is the direction of the induced current relative to the change in flux?



Solution:

$$\varepsilon = -\frac{\Delta\Phi}{\Delta t}$$

$$\Delta\Phi = \Phi_f - \Phi_i = 0.03 - 0.12 = -0.09 \text{ Wb}$$

$$\varepsilon = 0.45 \text{ V}$$

The magnetic flux decreases.

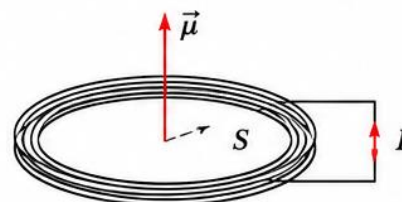
According to Lenz's law, the induced current creates a magnetic field that opposes this decrease.

Exercise 5:

A circular coil consists of 100 turns and carries an electric current of intensity $I = 0.5 \text{ A}$.

The area of each turn is 0.01 m^2 .

1. Calculate the magnetic dipole moment of the coil.
2. Explain how the magnetic dipole moment can be increased.



Solution:

$$\mu = NIS$$

$$\mu = 100 \times 0.5 \times 0.01$$

$$\mu = 0.5 \text{ A} \cdot \text{m}^2$$

The magnetic dipole moment can be increased by:

- Increasing the number of turns N

- Increasing the current I
- Increasing the surface area S

Exercise 6:

Consider a small current element of a conductor carrying an electric current. This element produces a small magnetic field at a point in space.

The magnetic field is given by:

$$d\vec{B} = \frac{\mu_0 I (d\vec{l} \times \vec{r})}{4\pi r^3}$$

1. Explain the meaning of each symbol in the equation.
2. What factors affect the magnitude of the magnetic field $d\vec{B}$?
3. How can the direction of the magnetic field be determined?
4. What happens to the magnetic field when the distance between the current element and the observation point increases?

Solution:

$$d\vec{B} = \frac{\mu_0 I (d\vec{l} \times \vec{r})}{4\pi r^3}$$

The magnetic field depends on:

- Current intensity I
- Length element $d\vec{l}$
- Distance r
- Angle between $d\vec{l}$ and \vec{r}

The direction of $d\vec{B}$ is determined by the right-hand rule for the cross product:

$$d\vec{l} \times \vec{r}$$

When the distance r increases, the magnetic field decreases.

Glossary Tables for Chapter IV: Electromagnetism

مصطلحات المحور الرابع: الكهرومغناطيسية

English	العربية
Angle	زاوية
Biot-Savart law	قانون بيو-سافار
Charged particle	جسيم مشحون
Compass	بوصلة
Conductor length	طول الناقل
Current element	عنصر تيار
Current-carrying wire	سلك حامل للتيار
Distance from conductor	البعد عن الناقل
Distance vector	شعاع المسافة
Electric generator	مولد كهربائي
Electric motor	محرك كهربائي
Electromagnetic induction	الحث الكهرومغناطيسي
Electromagnetic relay	مرحل كهرومغناطيسي
Electromagnetic wave	موجة كهرومغناطيسية
Electromagnetism	الكهرومغناطيسية
Elementary length vector	شعاع طول عنصري
Elementary magnetic field	حقل مغناطيسي عنصري
Faraday's law	قانون فاراداي
Galvanometer	غلفانومتر
Hall effect	تأثير هول
Induced current	تيار مستحث
Induced electromotive force	القوة الدافعة الكهربائية المستحثة
Inductance	التحريض الذاتي
Laplace force	قوة لابلاس
Lenz's law	قانون لنز

Loop area	مساحة الحلقة
Lorentz force	قوة لورنتز
Loudspeaker	مكبر صوت
Magnetic dipole	ثنائي قطب مغناطيسي
Magnetic dipole moment	العزم المغناطيسي لثنائي القطب
Magnetic field	المجال المغناطيسي
Magnetic field around straight wire	المجال المغناطيسي حول سلك مستقيم
Magnetic field lines	خطوط المجال المغناطيسي
Magnetic flux	التدفق المغناطيسي
Magnetic permeability	النفاذية المغناطيسية
Magnetization	المغطة
Magnitude of force	شدة القوة
Moving charges	شحنات متحركة
Mutual inductance	التحريض المتبادل
North pole	القطب الشمالي
Number of turns	عدد اللفات
Observation point	نقطة الملاحظة
Permanent magnet	مغناطيس دائم
Permeability of vacuum	نفاذية الفراغ
Right-hand rule	قاعدة اليد اليمنى
Solenoid	وشيجة
South pole	القطب الجنوبي
Tesla	تسلا
Transformer	محول كهربائي
Uniform magnetic field	حقل مغناطيسي منتظم
Vector product	الجداء الشعاعي
Velocity	السرعة
Weber	ويبر

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