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FACULTY OF APPLIED SCIENCES

DEPARTEMENT OF SCIENCES AND TECHNOLOGY

Handout of physics course 2

(Electricity and Magnetism)

Prepared by : ***Dr. Mohammed*** **BENDJEMAI**

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Expertised by :

Pr. BAGHDAD Rachid

Dr. AMMARI Abdelkader

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Foreword

This handout is addressed to the students of the first year in the domain of science and technology, conformally to the Algerian official pedagogical program. It is a course summary providing some knowledge about electrostatics and it's dual the magnetostatics which are the basics of electromagnetism. This summary course is followed by some exercises for more understanding the concepts introduced in the course.

The handout is divided in four concise chapters. The first chapter is a mathematical reminder dealing with systems of coordinates and some tools of vector analysis. Chapter two is devoted to the electrostatics and the concept of electrical charge, the properties of the conductors in equilibrium. The third chapter deals with electrokinetics that is the conductors are out of equilibrium state (electric current and the Kirchhoff laws of linear electrical circuits). Finally, the fourth chapter deals with the magnetostatics and the laws governing the magnetic field.

I hope that this work will help the students understanding the fondamental principles of electrostatics and magnetostatics.

Chapter I

Mathematical Reminder

I.1 Coordinates systems and operators of vector analysis.

There are several orthogonal coordinates systems mainly the cartesian, cylindrical, spherical, parabolic, elliptic, hyperbolic, bipolar, jacobi, …

In this course we are interested uniquely by the classical ones namely, the orthogonal system of cartesian, cylindrical and spherical coordinates.

I.1.2 Cartesian coordinates system (x, y, z)

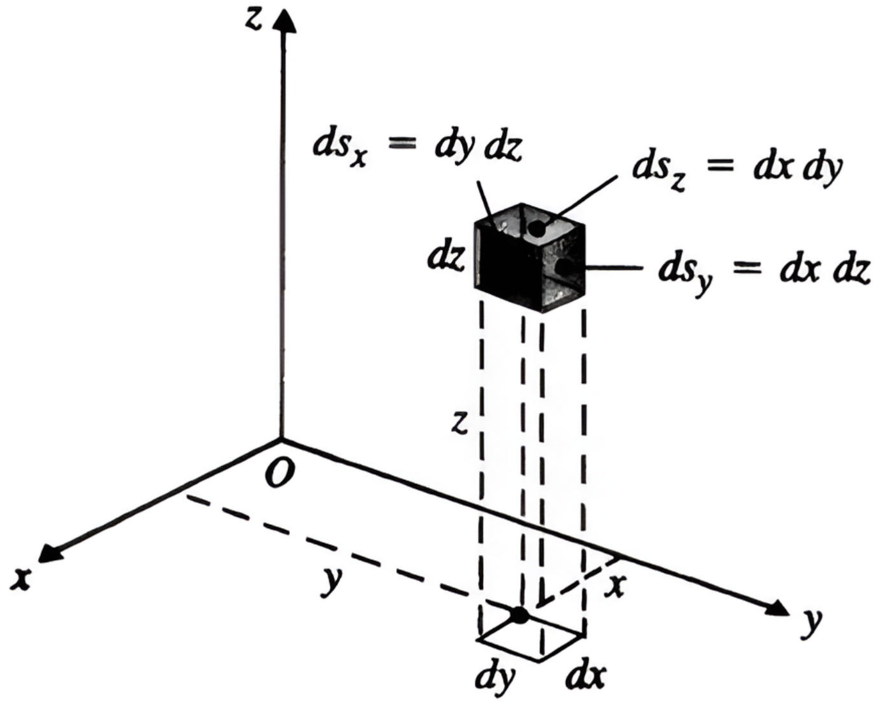


Figure 1 : cartesian coordinates and elementary variables

Lets the orthonormal frame R = (O,$\overrightarrow{e_x}$,$\overrightarrow{e_y}$,$\overrightarrow{e_z}$), in the 3-D affine space. The frame R is said cartesian and is associated to the cartesian basis (e_x, e_y, e_y). it is a canonical basis, then it is independant of the space location. The position of the point M with respect to the origin of the frame is defined as follows :

$$\overrightarrow{OM} = x\overrightarrow{e_x} + y\overrightarrow{e_y} + z\overrightarrow{e_z}$$

The infinitesimal variations of the coordinates x, y, z are dx, dy, and dz so the point M gets the elementary displacement dM defined by the following expression :

$\overrightarrow{dM} = dx\overrightarrow{e_x} + dy\overrightarrow{e_y} + dz\overrightarrow{e_z}$

The figure 1 allows us to calculate the elementary surface and volume in the cartesian basis. In fact, the elementary surfaces are given by :

$\overrightarrow{dS_x} = dydz\overrightarrow{e_x}$

$\overrightarrow{dS_y} = dxdz\overrightarrow{e_y}$

$\overrightarrow{dS_z} = dxdy\overrightarrow{e_z}$

The elementary volume is therefore : $dV = dxdydz$

I.1.3 Cylindrical coordinate system (r, θ, z)

$\overrightarrow{OM} = r\overrightarrow{e_r} + z\overrightarrow{e_z}$

$$\overrightarrow{dM} = dr\overrightarrow{e_r} + rd\overrightarrow{e_r} + dz\overrightarrow{e_z} = dr\overrightarrow{e_r} + r\left(\frac{\partial\overrightarrow{e_r}}{\partial r}dr + \frac{\partial\overrightarrow{e_r}}{\partial\theta}d\theta\right) + dz\overrightarrow{e_z}$$

$\overrightarrow{dM} = dr\overrightarrow{e_r} + rd\theta\overrightarrow{e_\theta} + dz\overrightarrow{e_z}$

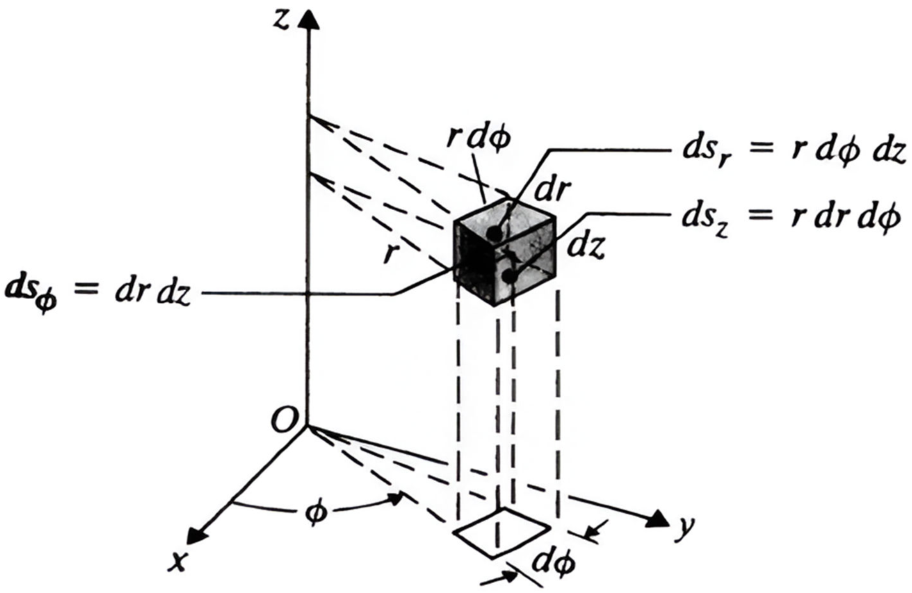


Figure 2 : elements of length, surface and volume in cylindrical coordinates.

With help of the figure 2, we have : $\overrightarrow{e_r} = \cos\theta\overrightarrow{e_x} + \sin\theta\overrightarrow{e_y}$;

$$\overrightarrow{e_\theta} = -\sin\theta\,\overrightarrow{e_x} + \cos\theta\overrightarrow{e_y} = \frac{\partial\overrightarrow{e_r}}{\partial\theta}$$

$$\overrightarrow{dS_r} = rd\theta dz\overrightarrow{e_r}$$

$$\overrightarrow{dS_\theta} = drdz\overrightarrow{e_\theta}$$

$$\overrightarrow{dS_z} = rdrd\theta\overrightarrow{e_z}$$

$$dV = rdrd\theta dz$$

I.1.4 Spherical coordinates system $(\boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{\varphi})$

The basis is : $(e_r, e_\theta, e_\varphi)$ then, $\overrightarrow{OM} = r\overrightarrow{e_r}$

$$\overrightarrow{dM} = dr\overrightarrow{e_r} + \left(r\frac{\partial\overrightarrow{e_r}}{\partial r}dr + r\frac{\partial\overrightarrow{e_r}}{\partial\theta}d\theta\right) + r\frac{\partial\overrightarrow{e_r}}{\partial\varphi}d\varphi$$

$$\overrightarrow{e_r} = sin\theta\cos\varphi\,\overrightarrow{e_x} + \sin\theta\sin\varphi\overrightarrow{e_y} + \cos\theta\,\overrightarrow{e_z}$$

$$\overrightarrow{e_\theta} = \cos\theta\cos\varphi\,\overrightarrow{e_x} + \cos\theta\sin\varphi\overrightarrow{e_y} - \sin\theta\overrightarrow{e_z}$$

$$\overrightarrow{e_\varphi} = -\sin\varphi\overrightarrow{e_x} + \cos\varphi\,\overrightarrow{e_y}$$

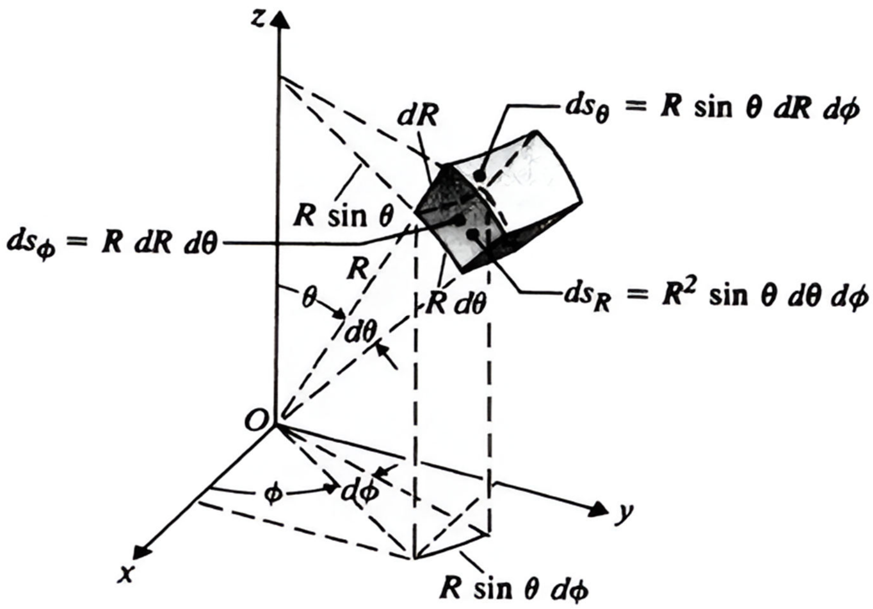


Figure 3 : spherical coordinates system

Hence we have :

$$\overrightarrow{dM} = dr\overrightarrow{e_r} + rd\theta\overrightarrow{e_\theta} + r\sin\theta d\varphi\,\overrightarrow{e_\varphi}$$

$$\overrightarrow{dS_r} = r^2\sin\theta d\theta d\varphi\overrightarrow{e_r}$$

$$\overrightarrow{dS_\theta} = r\sin\theta drd\varphi\overrightarrow{e_\theta}$$

$$\overrightarrow{dS_\varphi} = rdrd\theta\overrightarrow{e_\varphi}$$

$$dV = r^2\sin\theta drd\theta d\varphi$$

I.2 differential operators :

I.2.1 The Gradient : $\overrightarrow{\boldsymbol{grad}} = \vec{\nabla}$

The gradient operator associate a scalar function $f(x,y,z)$ a vector of components $\left(\frac{\partial f}{\partial x},\frac{\partial f}{\partial y},\frac{\partial f}{\partial z}\right)$, such as $\vec{\nabla} f = \frac{\partial f}{\partial x}\overrightarrow{e_x} + \frac{\partial f}{\partial y}\overrightarrow{e_y} + \frac{\partial f}{\partial z}\overrightarrow{e_z}$

Recall that $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$ and $\vec{\nabla} = \frac{\partial}{\partial x}\overrightarrow{e_x} + \frac{\partial}{\partial y}\overrightarrow{e_y} + \frac{\partial}{\partial z}\overrightarrow{e_z}$

Hence we obtain the gradient in any coordinates system as follows :

$df = \overrightarrow{(\nabla f)}.d\vec{r}$ where ; $d\vec{r} = dx\overrightarrow{e_x} + dy\overrightarrow{e_y} + dz\overrightarrow{e_z}$

- **Cartesian coordinates** : $f(x,y,z)$

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\overrightarrow{e_x} + \frac{\partial f}{\partial y}\overrightarrow{e_y} + \frac{\partial f}{\partial z}\overrightarrow{e_z}$$

- **Cylindrical coordinates** : $f(r,\theta,z)$

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\overrightarrow{e_r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\overrightarrow{e_\theta} + \frac{\partial f}{\partial z}\overrightarrow{e_z}$$

- **Spherical coordinates** : $f(r,\theta,\varphi)$

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\overrightarrow{e_r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\overrightarrow{e_\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\overrightarrow{e_\varphi}$$

I.2.2 Divergence of a vector :

The divergence is defined as the scalar product between the displacement element and and the gradient of the vector.

- **Cartesian coordinates** : $\vec{V}(x, y, z)$

$$\vec{\nabla}.\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_x}{\partial y} + \frac{\partial V_z}{\partial z} \qquad ; \qquad \vec{V} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

- **Cylindrical coordinates** : $\vec{V}(r, \theta, z)$

$$\vec{\nabla}.\vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z} \qquad ; \qquad \vec{V} = \begin{pmatrix} V_r \\ V_\theta \\ V_z \end{pmatrix}$$

- **Spherical coordinates** : $\vec{V}(r, \theta, \varphi)$

$$\vec{\nabla}.\vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2V_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, V_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}V_\varphi \qquad ; \qquad \vec{V} = \begin{pmatrix} V_r \\ V_\theta \\ V_\varphi \end{pmatrix}$$

I.2.3 Rotational of a vector :

The operator rotational or curl $(\vec{\nabla}\wedge)$ associates to a vector $\vec{V}$, the cross product of $\vec{\nabla}$ by this vector : $\overrightarrow{Rot}\vec{V} = \vec{\nabla}\wedge\vec{V}$

- **Cartesian coordinates** :

$$\overrightarrow{Rot}\vec{V} = \begin{pmatrix} \overrightarrow{e_x} & \overrightarrow{e_y} & \overrightarrow{e_z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ V_x & V_y & V_z \end{pmatrix} = \begin{pmatrix} \left(\dfrac{\partial V_z}{\partial y} - \dfrac{\partial V_y}{\partial z}\right) \\ \left(\dfrac{\partial V_x}{\partial z} - \dfrac{\partial V_z}{\partial x}\right) \\ \left(\dfrac{\partial V_y}{\partial x} - \dfrac{\partial V_x}{\partial y}\right) \end{pmatrix} \begin{pmatrix} \overrightarrow{e_x} \\ \overrightarrow{e_y} \\ \overrightarrow{e_z} \end{pmatrix}$$

- **Cylindrical coordinates** :

$$\left(\overrightarrow{Rot}\vec{V}\right)_r = \frac{1}{r}\frac{\partial V_z}{\partial \theta} - \frac{\partial V_\theta}{\partial z}$$

$$\left(\overrightarrow{Rot}\vec{V}\right)_\theta = \frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r}$$

$$\left(\overrightarrow{Rot}\vec{V}\right)_z = \frac{1}{r}\left[\frac{\partial}{\partial r}(rV_\theta) - \frac{\partial V_r}{\partial \theta}\right]$$

- **Spherical coordinates** :

$$\left(\overrightarrow{Rot}\vec{V}\right)_r = \frac{1}{r\sin\theta}\left[\frac{\partial\,(\sin\theta V_\varphi)}{\partial\theta} - \frac{\partial V_\theta}{\partial\varphi}\right]$$

$$\left(\overrightarrow{Rot}\vec{V}\right)_\theta = \frac{1}{r\sin\theta}\frac{\partial V_r}{\partial\varphi} - \frac{1}{r}\frac{\partial(rV_\varphi)}{\partial r}$$

$$\left(\overrightarrow{Rot}\vec{V}\right)_\varphi = \frac{1}{r}\left[\frac{\partial(rV_\theta)}{\partial r} - \frac{\partial V_r}{\partial\theta}\right]$$

I.3 Laplacian operator :

The Laplacian operator is defined by :

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

It can be applied to a scalar function to yield :

$\mathbf{\Delta f} = \frac{\partial^2 \mathbf{f}}{\partial \mathbf{x}^2} + \frac{\partial^2 \mathbf{f}}{\partial \mathbf{y}^2} + \frac{\partial^2 \mathbf{f}}{\partial \mathbf{z}^2}$ Or to a vector yielding to $\Delta\vec{\mathbf{V}} = \frac{\partial^2 \vec{\mathbf{V}}}{\partial \mathbf{x}^2} + \frac{\partial^2 \vec{\mathbf{V}}}{\partial \mathbf{y}^2} + \frac{\partial^2 \vec{\mathbf{V}}}{\partial \mathbf{z}^2}$

I.4 Vector relationships :

Scalar triple product :

$$\vec{A}\cdot(\vec{B}\wedge\vec{C}) = \vec{C}\cdot(\vec{A}\wedge\vec{B}) = \vec{B}\cdot(\vec{C}\wedge\vec{A})$$

Vector triple product :

$\vec{A} \wedge \vec{B} \wedge \vec{C} = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

For the scalar functions $f and\ p$, we have :

$\vec{\nabla}(fp) = f\vec{\nabla}p + p\vec{\nabla}f$

$\vec{\nabla} \cdot (f\vec{A}) = (\vec{\nabla}f) \cdot \vec{A} + f\vec{\nabla} \cdot \vec{A}$

$\vec{\nabla}(\vec{A} \wedge \vec{B}) = \vec{B} \cdot \vec{\nabla} \wedge \vec{A} - \vec{A} \cdot \vec{\nabla} \wedge \vec{B}$

$\vec{\nabla} \wedge (f\vec{A}) = (\vec{\nabla}\, f) \wedge \vec{A} + f\vec{\nabla} \wedge \vec{A}$

$\vec{\nabla}(\vec{\nabla}f) = \Delta f$

$\vec{\nabla}(\vec{\nabla} \wedge \vec{A}) = 0$

$\vec{\nabla} \wedge \vec{\nabla}f = \vec{0}$

$\vec{\nabla} \wedge (\vec{\nabla} \wedge \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \Delta\vec{A}$

Chapter II

ELECTROSTATICS

Previously, we have studied, in mechanics the gravitational interaction between bodies that are characterised by their masses. In the following, we have to consider another interaction, the electrostatic interaction, which involves the concept of electric charge. Electrostatics is about fixed charges with the Coulomb law.

II.1 Introduction :

From experience, we learn that two bodies rubbed against each other aquire charges one with positive charge and the other the negative one. The magnitude of transferred charge on every body depend on the number of transferred charge. Positive and negative charges are purely conventional.

II.2 Properties of charges :

Charges have peculiar properties. The charges with the same nature repel each other in contrast with the different nature charge they attract. A charged body attract a neutral body by the electrostatic force. The electric charge is an intrinsic property of any body. A neutral body has equal amount of positive and negative charges so that the charge on a neutral body is always zero.

The principle of charge conservation states that it can neither be created nor destroyed but it may simply be transferred from one body to another body. Actually, electric charge is quantized any physically existing charge is an integral multiple of the elementary charge (e).it isn’t the case for magnetism.

II.3 Coulomb's law in vacuum :

The electric force between like charges is repelent and that between unlike charges is attractive.

This law is called Coulomb's inverse square law.

$$F = k\frac{q_1 q_2}{r^2}$$

Where, k is a constant, whose value is given by : $\frac{1}{4\pi\varepsilon_0}$, if the charges are placed in vacuum. If the charges, distance and the force are measured in Coulomb (C), meter (m) and Newton respectively, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm/C^2$. The constant ε_0 is the permittivity of free space. It's value is

$8.85 \times 10^{-12} C^2/Nm^2$

If $q_1 = q_2 = 1$ Coulomb and $r = 1$ meter, then we get $9 \times 10^9 \times \frac{1\times 1}{1^2} = 9 \times 10^9$ Newton.

Conditions of validity of Coulomb's law :

We have seen that Coulomb's law between two point charges is an inverse square distance law. However, it may be applied to extended objects provided the distance between the mis much larger than their dimensions. The separation between the charges must be greater than nuclear distance (10^{-15} m) because for distances less than 10^{-15} m, the nuclear attractive forces become dominant over all other forces.

II.4 Electric field and potential :

The environment of an electric charge in wich another charge experiences a force (attractive or repulsive), is called the electric field of the electric charge.

If a charge q_0 experiences a force in the space surrounding the charge q, then charge q is called the « source charge » and the charge q_0 is called a « test charge ». Further, the test charge must be vanishingly small so that it does not modify the electric field of the source charge.

The strenght of the electric field at a point in an electric field is the ratio of the force acting on the test charge placed at that point to the magnitude of the test charge. It is a vector quantity and its direction is along the direction of the force.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Here, we have assumed that test charge q_0 is infinitesimal, therefore the definition of strenght of electric field may be expressed as :

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

If the strenght of the electric field $\vec{E}$ at a point is known, then we can calculate the force $\vec{F}$ acting on a charge q placed at that point by the following equation

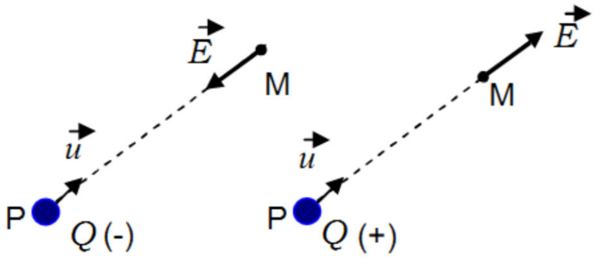
$$\boldsymbol{\vec{F} = q\vec{E}}$$

II.5 Electric field of an isolated point charge :

In the case of one source charge Q, the force exerted on the test charge q is given by the Coulomb's law : $\vec{F} = K\frac{Qq}{r^2}\vec{u}$

Hence, the electric fiel dis expressed as :

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\vec{u}$$



II.6 Electric field created by a set of point charges (superposition principle) :

A set of point charges placed at points P_i. We propose to determine the electric field created by this set of point charges at a point M distant of r_i from P_i.

This field is obtained by the superposition of the fields created by every charge Q_i. Each of the fields is calculated as if the source charge was alone.

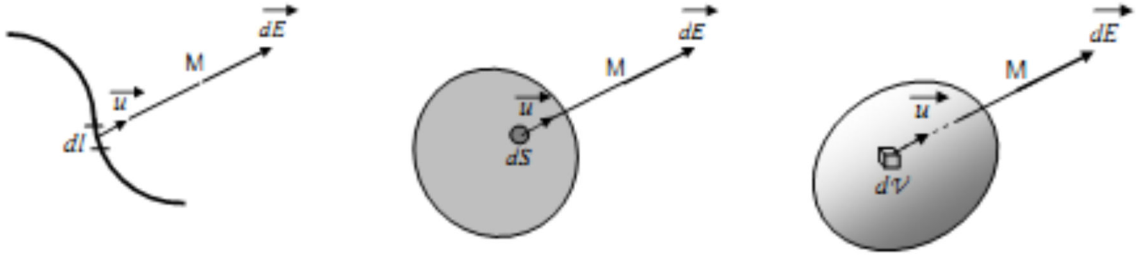
The principle of superposition results from the additive property of vectors of forces and electric fields.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{Q_i}{r_i^2}\vec{u}_i$$

II.7 Electric field due to a continuous charge distribution :

When the charge Q_i is distributed on a wire with a linear density λ, each element dl carrying a charge $dQ = \lambda dl$, and create an elementary field :

$\overrightarrow{dE} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{r^2}\vec{u}$ the fiel dis then : $\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda dl}{r^2}\vec{u}$



(a) (b) (c)

In the case of surface density σ so that $dQ = \sigma dS$, we will find that :

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\iint \frac{\sigma dS}{r^2}\vec{u}$$

In the case of volume charge density $dQ = \rho dV$, we obtain :

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho dV}{r^2}\vec{u}$$

Where dl, dS and dV are lenght, surface and volume elements.

II.8 Electric potential :

II.8.1 Circulation of a vector :

In mechanics we defined the elementary work dW of a force along an infinitesimal path $\overrightarrow{MM'} = \overrightarrow{dl}$ by the scalar product :

$$dW = \vec{F}\cdot\overrightarrow{dl} = Fdl\cos\theta$$

When the path AB is not infinitesimal, the work W of the force $\vec{F}$ between the points A and B, is equal to the sum of the elementary works dW.

$$W = \int_A^B \vec{F}\cdot\overrightarrow{dl}$$

The circulation of a vector along a path AB for an infinitesimal displacement dl is defined by the scalar product :

$$dC = \vec{A}\cdot\overrightarrow{dl} \quad \text{with } \vec{A} = A_x\vec{\imath} + A_y\vec{\jmath} + A_z\vec{k}$$

II.8.2 Calculation of the electric potential :

We calculate the circulation of the electric vector $\vec{E}$, created by a fixed charge Q, when traveling an elementary displacement : $\overrightarrow{MM'} = \overrightarrow{dl}$

$$dC = \vec{E} \cdot \overrightarrow{dl}$$

In polar coordinates, we have the radial component of the electric field, consequently :

$$dC = Edr = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dr = -d\left[\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}\right]$$

The vector $\vec{E}$, derive from a scalar function : $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} + C$; named electric potential if we let the potential vanishing at infinity, then the constant C will vanish, and we get :

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

So, we can write : $\vec{E} \cdot \overrightarrow{dl} = -dV$, then $E_x dx + E_y dy + E_z dz = dV$

Expressed in vector notation : $\vec{E} = -\overrightarrow{grad}V$

$$\int_A^B \vec{E} \cdot \overrightarrow{dl} = -\int_A^B dV = V_A - V_B$$

Consequently :

$$\oint \vec{E} \cdot \overrightarrow{dl} = 0$$

The circulation of the electric field along a closed path is null.

II.8.3 Electric potential of a charges distribution :

As for the electric field, we use the principle of superposition. The potential created by n fixed charges,Q_1,$Q_2, Q_3, \cdots Q_n$ is :

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{Q_i}{r_i}$$

When the charge Q, is sprayed on a wire with a linear density ,$\lambda = dQ/dl$ and in the case of surface charge distribution $\sigma = dQ/dS$ or a volume charge distribution $\rho = dQ/dV$

The potential for the three type of distribution is the following :

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda}{r}dl \ , \ V = \frac{1}{4\pi\varepsilon_0}\int\frac{\sigma}{r}dS \ , \ \ V = \frac{1}{4\pi\varepsilon_0}\int\frac{\varrho}{r}dV$$

Example :

Calculation of the electric field and electric potential of a charge Q evenly distributed on a surface of a disc, at a point M along the z-axis :

$Q = \sigma S$ and $S = \pi R^2$

$$dQ = \sigma dS \qquad , \qquad d\vec{E} = \frac{KdQ}{PM^2}\vec{u} \quad , \qquad dS = 2\pi r dr$$

$$dE = \frac{K\sigma dS}{PM^2}\cos\alpha \ , \quad \cos\alpha = \frac{z}{PM} \quad \text{and} \quad PM^2 = r^2 + z^2$$

The total electric field will be :

$$E = 2\pi K\sigma z\int_0^R \frac{rdr}{(r^2+z^2)^{3/2}} = \frac{\sigma}{2\varepsilon_0}z\left[\frac{1}{\sqrt{(r^2+z^2)}}\right]_0^R$$

Lets changing the variables : $u = r^2 + z^2$ so $du = 2rdr$ then we obtain :

$$E(z) = \frac{\sigma}{2\varepsilon_0}\left[\frac{z}{|z|} - \frac{z}{\sqrt{(r^2+z^2)}}\right]$$

- ✓ $z > 0, |z| = +z$ then we get : $E(z) = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{(r^2+z^2)}}\right]$
- ✓ $z < 0, |z| = -z$ then we get : $E(z) = -\frac{\sigma}{2\varepsilon_0}\left[1 + \frac{z}{\sqrt{(r^2+z^2)}}\right]$

Calculation of the electric potential :

Using the circulation of the electric field along the oz axis, we have :

$$dV = -\vec{E}\cdot\vec{dl} = -Edz \quad \Longrightarrow \quad V = -\int E(z)dz$$

For $z > 0$ and using the expression of the electric field with the change of variable : $u = r^2 + z^2$ then $du = 2rdr$, we obtain the electric potential :

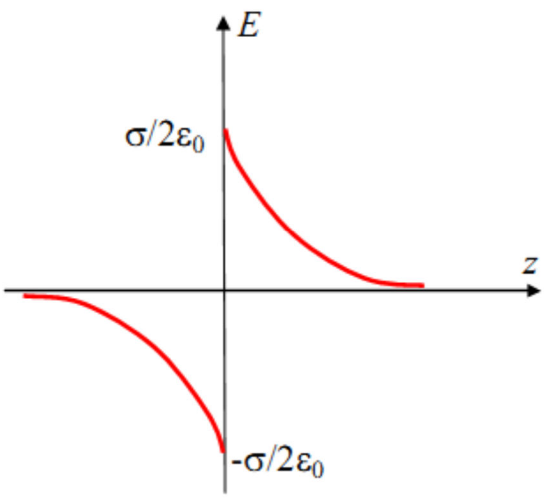
$$V(z) = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{z^2 + R^2} - z\right] + C^{te}$$

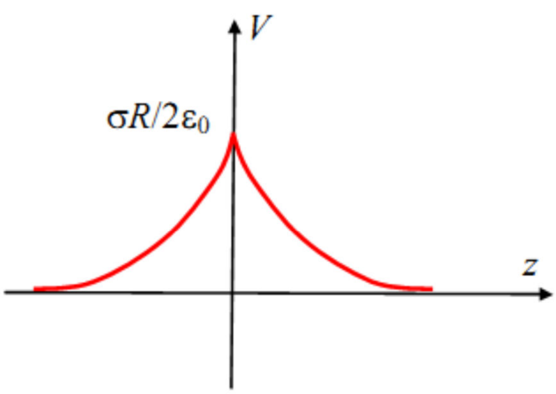
For $z < 0$ we find :

$$V(z) = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{z^2 + R^2} + z\right] + C^{te}$$

As $z \to 0$, we have $V = 0$, and the $C^{te} = 0$

At $z = 0$, the electric potential is continuous and the electric field is discontinuous as plotted bellow :





II.8.4 Topography of the electric field :

The presence of an electric charge in a region of space, does modify the electric properties of that region by creating at any point of the region an electric field.

Field line :

A field line is an oriented curve in which the fiel dis tangent at each point of curve. The field lines of positive and negative charges are drawn bellow :

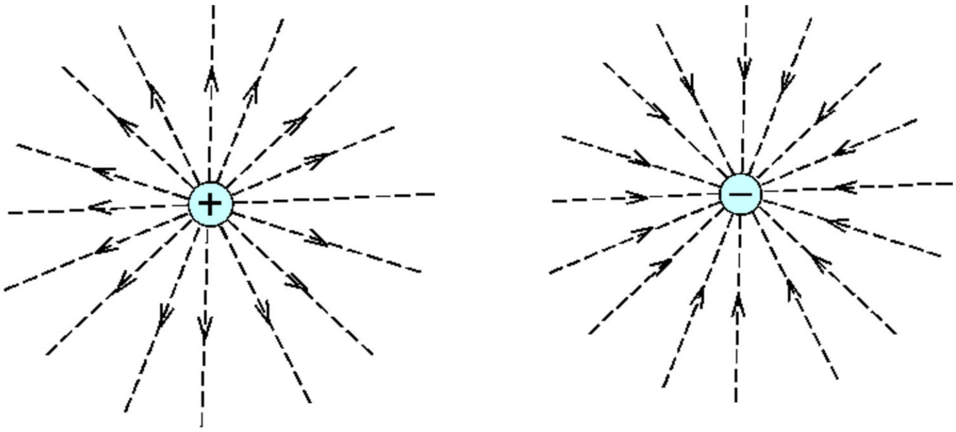


Figure II.1

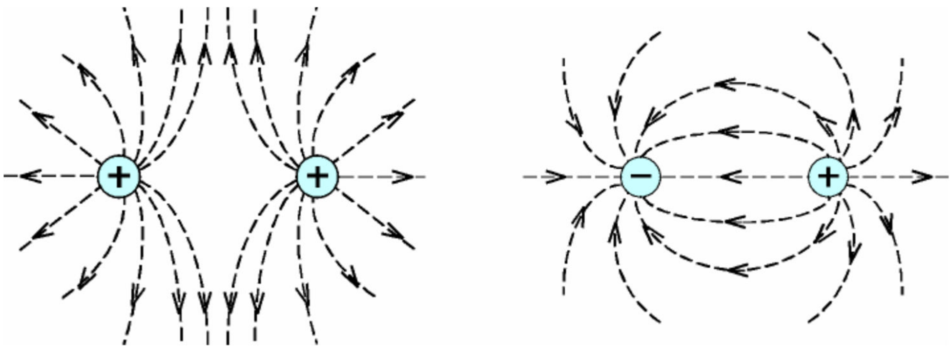


Figure II.2

As we can see, two opposite charges allow a new topography.

Field tube :

A field tube is a virtual surface formed by the field lines pushed on a closed curve.

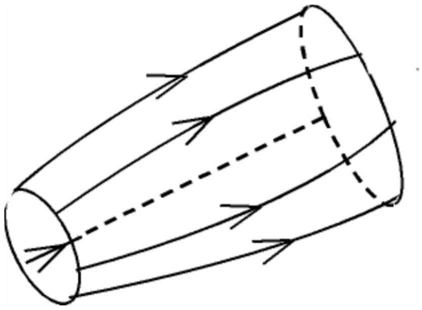


Figure II.3

Surface equipotential :

A surface equipotential is a surface where all the points are in the same potential V.

1. The field lines are perpendicular to the surface potentials.

   If we consider a displacement, $\overrightarrow{MM'} = \overrightarrow{dl}$, on a surface equipotential (figure.), we find that :

$$dV = -\vec{E} \cdot \overrightarrow{dl} = -\vec{E} \cdot \overrightarrow{MM'} = 0$$

   Hence, $\vec{E}$ is perpendicular to $\overrightarrow{MM'}$.

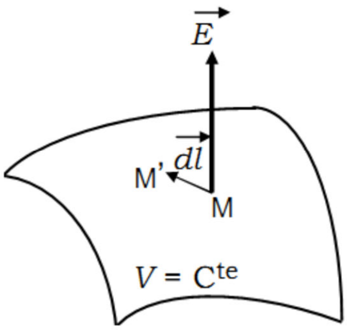


Figure II.4

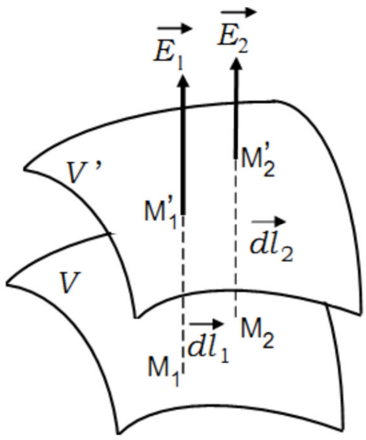


Figure II.5

2. The potential decreases along the field line.
3. Indeed, an infinitesimal displacement, $\overrightarrow{MM'} = \overrightarrow{dl}$, in the direction of $\vec{E}$ on the line field lead to :

   $dV = -\vec{E} \cdot \overrightarrow{dl} = -\vec{E} \cdot \overrightarrow{MM'} = -|\vec{E}| \cdot |\overrightarrow{MM'}|$

$$dV < 0 \Longrightarrow \; V_{M'} < V_M$$

4. So, the field line is oriented from the upper potential to the lower potential.
5. The electric field is stronger where the potentials are close to each other. Indeed, if we consider two infinitesimal displacements (Figure II.5) :

$\overrightarrow{M_1M_1{}'} = \overrightarrow{dl_1}$ and $\overrightarrow{M_2M_2{}'} = \overrightarrow{dl_2}$

We have :

$$dV = \overrightarrow{E_1} \cdot \overrightarrow{M_1M'_1} = -\overrightarrow{E_1} \cdot \overrightarrow{dl_1} = -\left|\overrightarrow{E_1}\right| \cdot \left|\overrightarrow{dl_1}\right|$$

And :

$$dV = \overrightarrow{E_2} \cdot \overrightarrow{M_2M'_2} = -\overrightarrow{E_2} \cdot \overrightarrow{dl_2} = -\left|\overrightarrow{E_2}\right| \cdot \left|\overrightarrow{dl_2}\right|$$

But : $\left|\overrightarrow{dl_1}\right| < \left|\overrightarrow{dl_2}\right| \;\; \Longrightarrow \left|\overrightarrow{E_1}\right| > \left|\overrightarrow{E_2}\right|$

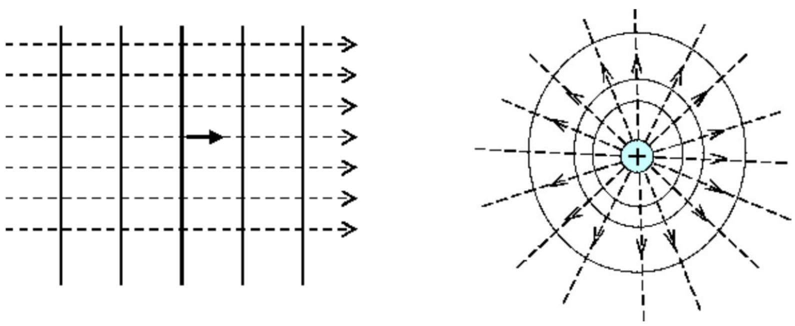


Figure II.6

II.9 Work of an electric force :

We put an electric charge q at a point of the space where exist an electric field $\vec{E}$, the charge experience the action of electric force :

$\vec{F} = q\vec{E}$, the work of the force is then :

$$dW = \vec{F} \cdot \vec{dl}$$

Along the path AB, we have :

$$W = \int_A^B \vec{F} \cdot \vec{dl} = q \int_A^B \vec{E} \cdot \vec{dl} = q(V_A - V_B)$$

II.9.1 Potential energy :

The potential energy of a point charge placed in an external electric field is defined as the work of the electric force acting on the charge, moved from a point M of potential V, to a reference point R, where the charge doesn't experience the external field. In that point, the potential is null.

$$E_p(M) = \int_M^R \vec{F} \cdot \overrightarrow{dl} = q \int_M^R \vec{E} \cdot \overrightarrow{dl} = q(V_M - V_R)$$

Hence :

$$E_p(M) = qV_M$$

The Coulomb force is then conservative and its work between any two points is independant of the path experienced.

$$-dE_p(M) = \vec{F} \cdot \overrightarrow{dl}$$

II.9.2 Internal energy of electric charge distribution :

The internal energy of two point charges is defined as the work provided by an operator for assembling the charges initially non-interacting. It is the potential energy of the second charge in the field of the first charge or vice- versa. Let two charges $q_1$and q_2, placed respectively at points $M_1$and M_2, distant from each other by $M_1M_2 = r_{12}$

The work provided by an operator to bring the charge q_2 to the point M_2 without varying the kinetic energy (adiabatically) is given by :

$$W = \int_\infty^{M_2} \vec{F} \cdot \overrightarrow{dl} = -\int_\infty^{M_2} \overrightarrow{F_{1\to 2}} \cdot \overrightarrow{dl} = q_2 \int_\infty^{M_2} \overrightarrow{E_1} \cdot \overrightarrow{dl}$$

Where $\overrightarrow{F_{1\to 2}}$, is the electric force exerted by the $q_1$charge on the charge q_2 then we have :

$$U = W = q_2 V_1 = \frac{Kq_1q_2}{r_{12}}$$

This is the internal energy of the system of the two charges $q_1$and q_2.

II.9.3 The electric dipole :

The electric dipole is constitued by two equal and opposite charges $+q\ and\ -q$, spaced by a distance d which is little compared to the distance of observation r. $d \ll r$

We can define the electric dipole moment as :

$\vec{p} = q\vec{d}$

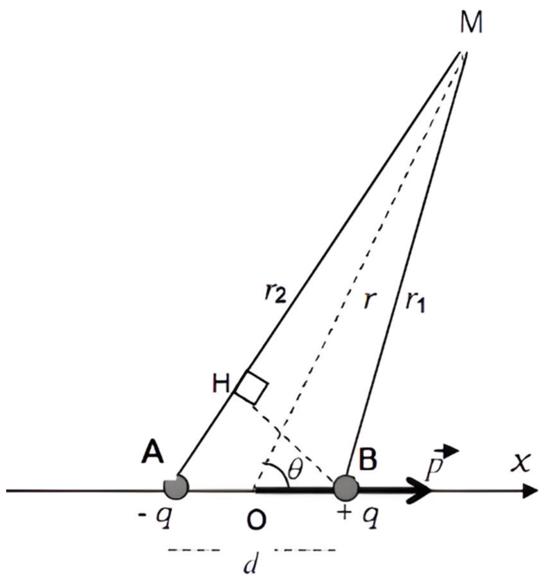


Figure II.7

Electric potential created by a dipole :

We calculate the potential produced by the dipole at the point M situated at a distance r from the middle O of the dipole :

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{q}{4\pi\varepsilon_0}\frac{r_2 - r_1}{r_1 r_2}$$

The distance of observation r, is larger than the distance d between the two charges of the dipole. Let H be the projection of B over AM :

$AH = d\cos\theta = r_2 - r_1$, we can use the approximation : $r_2 = r_1 = r$

Hence the potential will be after using the relation : $(1+\varepsilon)^n = 1 + n\varepsilon$

Whence $\varepsilon \ll 1$, so we get the potential created at M, by the dipole as :

$$V = \frac{1}{4\pi\varepsilon_0}\frac{P\cos\theta}{r^2}$$

Calculation of the electric field created by a dipole :

We determine the electric field by using the relations :

$\vec{E} = -\overrightarrow{grad}(V)$; $dV = -\vec{E}\cdot\vec{dl}$

In polar coordinates (figure below), we can get the components of the electric field in the following :

$$dV = -(E_r dr + E_\theta r d\theta) = \left(\frac{\partial V}{\partial r}\right) dr + \left(\frac{\partial V}{\partial \theta}\right) d\theta$$

$$E_r = -\left(\frac{\partial V}{\partial r}\right)_\theta = \frac{1}{4\pi\varepsilon_0}\frac{P\cos\theta}{r^3} \quad \text{and} \quad E_\theta = -\left(\frac{1}{r}\frac{\partial V}{\partial \theta}\right)_r = \frac{1}{4\pi\varepsilon_0}\frac{P\sin\theta}{r^3}$$

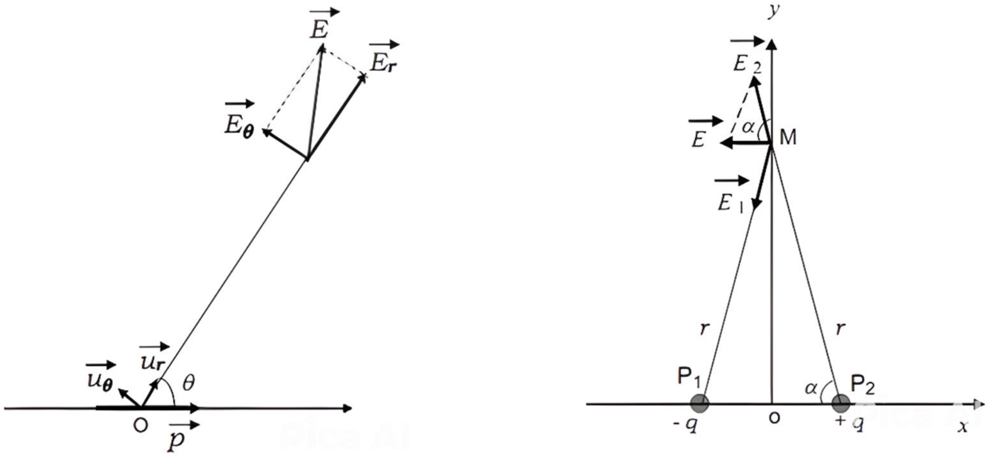
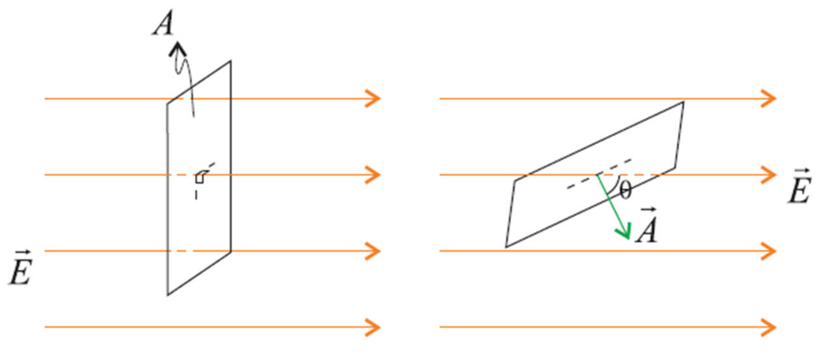


Figure II.8

II.9.4 Gauss's theorem :

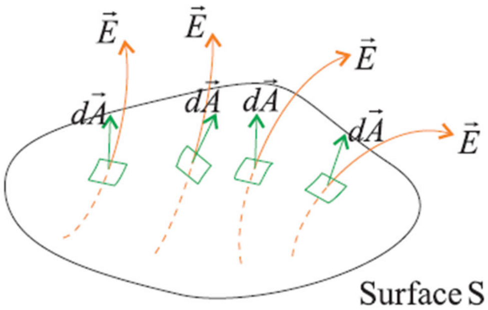
Electric flux :

Graphically : Electric flux Φ_E, represents the number of E-field lines crossing a surface. Mathematically : it is an inner product of the electric field with the surface crossed by the latter.



$\Phi_E = EA$ $\qquad$ $\Phi_E = \vec{E}\cdot\vec{A}$

For non-uniform E-field and surface, direction of the area vector is not $\vec{A}$, is not uniform.



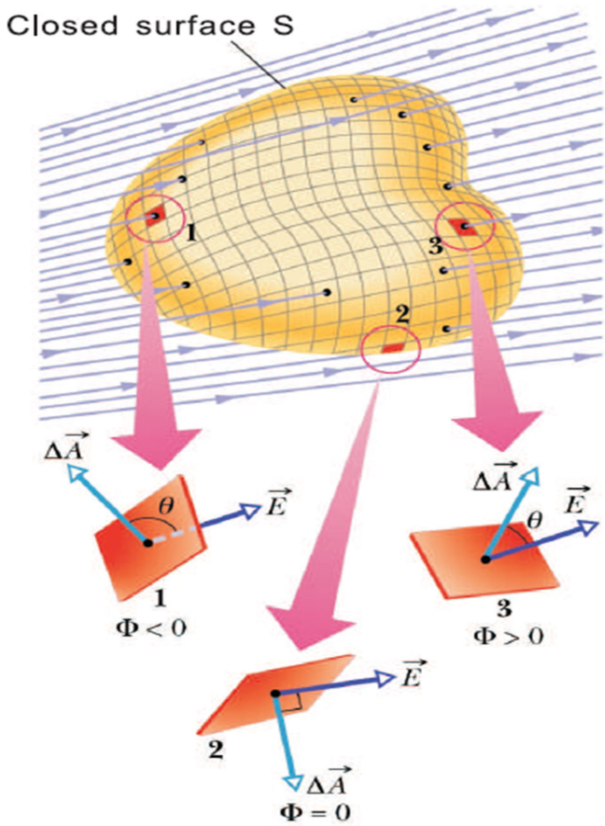
$d\vec{A}$ = Area vector for small area element dA

Electric flux $d\Phi_E = \vec{E} \cdot \overrightarrow{dA}$

The electric flux of $\vec{E}$ through surface S : $\Phi_E = \int_S \vec{E} \cdot d\vec{A}$

Example :

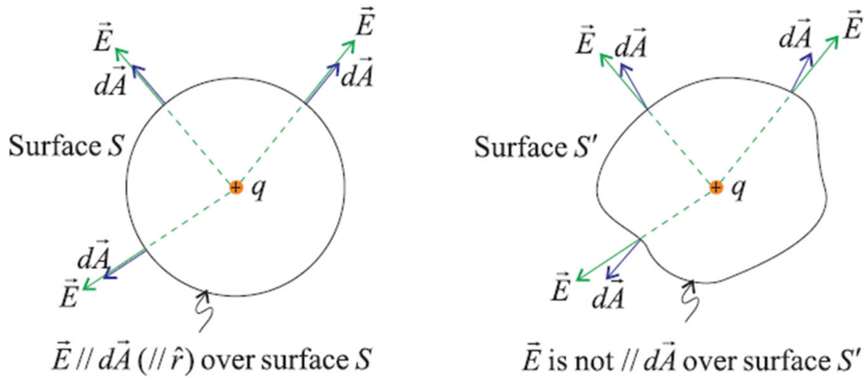
For a closed surface :



Recall that the direction of area vector $d\vec{A}$ goes from inside to outside of closed surface S.

Electric flux over a closed surface S is : $\Phi_E = \oint_S \vec{E} \cdot \overrightarrow{dA}$

We notice that if we remove the spherical symmetry of closed surface, the total number of E-field lines crossing the surface remains the same.



$$\Phi_E = \oint_S \vec{E} \cdot \overrightarrow{dA} = \oint_{S'} \vec{E} \cdot \overrightarrow{dA} = \frac{q}{\varepsilon_0}$$

For any closed surface S, and q is the net electric charge enclosed in closed surface S.

Gauss law :

$$\Phi_E = \oint_S \vec{E} \cdot \overrightarrow{dA} = \frac{q}{\varepsilon_0}$$

II.9.5 Electric field calculation with Gauss law :

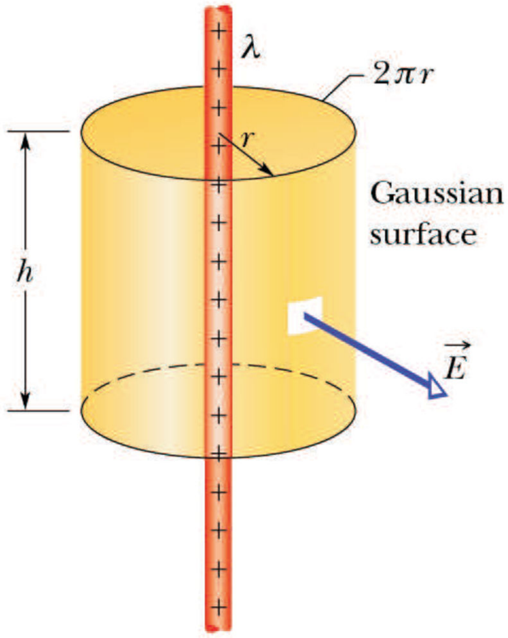
1. <u>Infinite line of charges</u> :

Linear charge density λ, cylindrical symmetry :

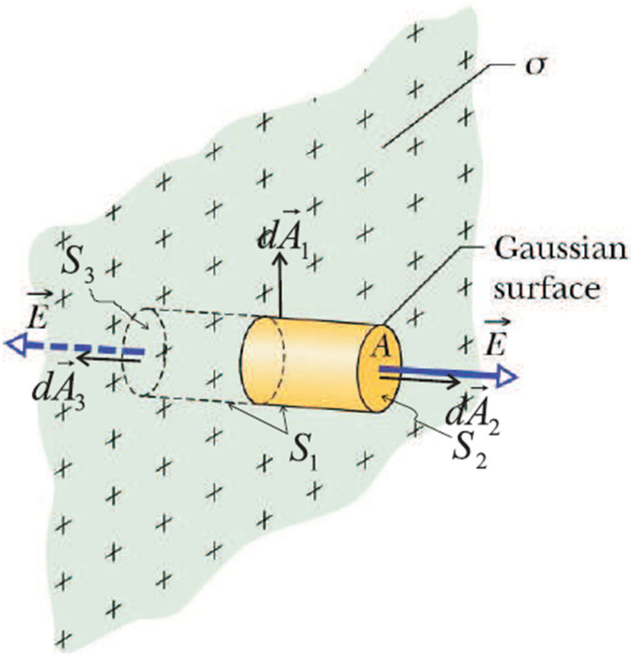
The electric field directs radially outward from the rod.

Construct a Gaussian surface S in the shape of a cylinder, making up of a curved surface S_1, and the top and bottom circles S_2, S_3. Gauss' Law :

$$\oint_S \vec{E} \cdot \overrightarrow{dA} = \frac{total\ charge}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$



2. Infinite sheet of charge :



Uniform surface charge density σ :

The electric field directs perpendicular to the sheet of charge. Construct Gaussian surface S in the shape of a cylinder of cross-sectional area A.

Gauss'Law :

$$\oint_S \vec{E} \cdot \overrightarrow{dA} = \frac{A\sigma}{\varepsilon_0}$$

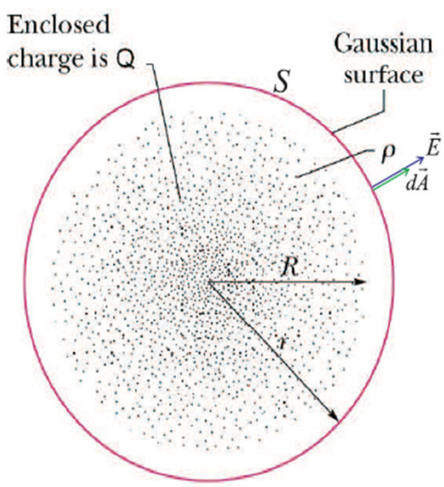
$\int_{S_1} \vec{E} \cdot \overrightarrow{dA} = 0$ $\qquad$ $\vec{E} \perp d\vec{A}$ over the whole surface S_1

$\int_{S_2} \vec{E} \cdot \overrightarrow{dA} + \int_{S_3} \vec{E} \cdot \overrightarrow{dA} = 2EA \quad (\vec{E} \parallel \overrightarrow{dA_2}, \vec{E} \parallel \overrightarrow{dA_3})$

$$2EA = \frac{A\sigma}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{\sigma}{2\varepsilon_0}$$

3. <u>Uniformly charged sphere</u> : (Total charge = Q)

(a) For $r > R$:



Consider a spherical Gaussian surface S of radius r:

$$\vec{E} \parallel d\vec{A} \parallel \hat{r}$$

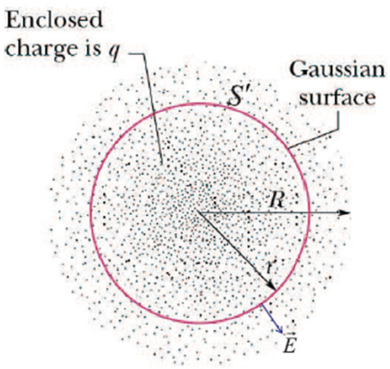
Gauss' Law: $\oint_S \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$

$$\oint_S E \cdot dA = \frac{Q}{\epsilon_0}$$

$$E \underbrace{\oint_S dA}_{\text{surface area of } S \,=\, 4\pi r^2} = \frac{Q}{\epsilon_0}$$

$$\therefore \quad \vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{r}\,; \qquad \text{for } r > R$$

(b) For $r < R$:



Consider a spherical Gaussian surface S' of radius $r < R$, then total charge included q is *proportional to the volume included by* S'

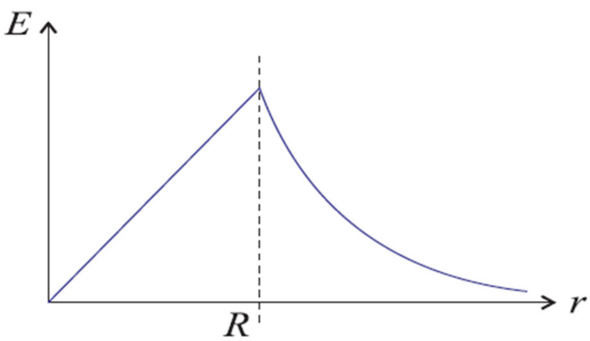
$$\therefore \quad \frac{q}{Q} = \frac{\text{Volume enclosed by } S'}{\text{Total volume of sphere}}$$

$$\frac{q}{Q} = \frac{4/3\,\pi r^3}{4/3\,\pi R^3} \quad \Rightarrow \quad q = \frac{r^3}{R^3}\,Q$$

Gauss' Law: $\displaystyle \oint_{S'} \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$

$$E \underbrace{\oint_{S'} dA}_{\text{surface area of } S' = 4\pi r^2} = \frac{r^3}{R^3} \frac{1}{\epsilon_0} \cdot Q$$

$$\therefore \quad \vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R^3}\, r\,\hat{r}\,; \qquad \text{for } r \leq R$$

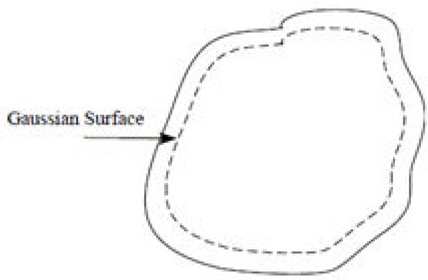


II.10 Conductors in electrostatic equilibrium :

A good electrical conductor has electrons that aren't bound to any atom and therefore are free to move about within the material. When no net motion of charge occurs within a conductor, the conductor is said to be in electrostatic equilibrium.

A conductor in electrostatic equilibrium has the following properties :

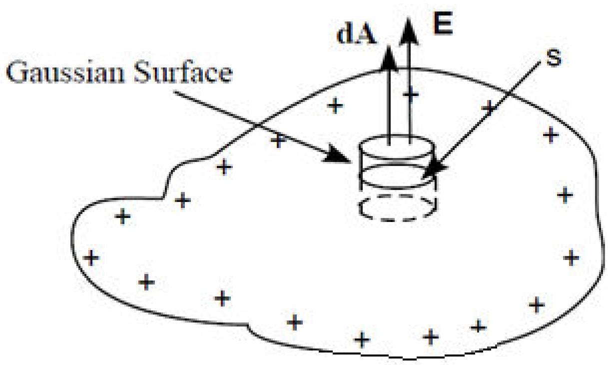
a) The electric field is zero everywhere inside the conductor.



$$\oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_o} \quad \vec{E} = 0 \text{ and } q_{enc} = 0$$

b) Any net charge on an isolated conductor must reside entirely on its surface.

c) The E-field just outside a charged conductor is perpendicular to the conductor's surface and has a magnitude $\frac{\sigma}{\varepsilon_0}$, where σ is the surface charge density at that point.

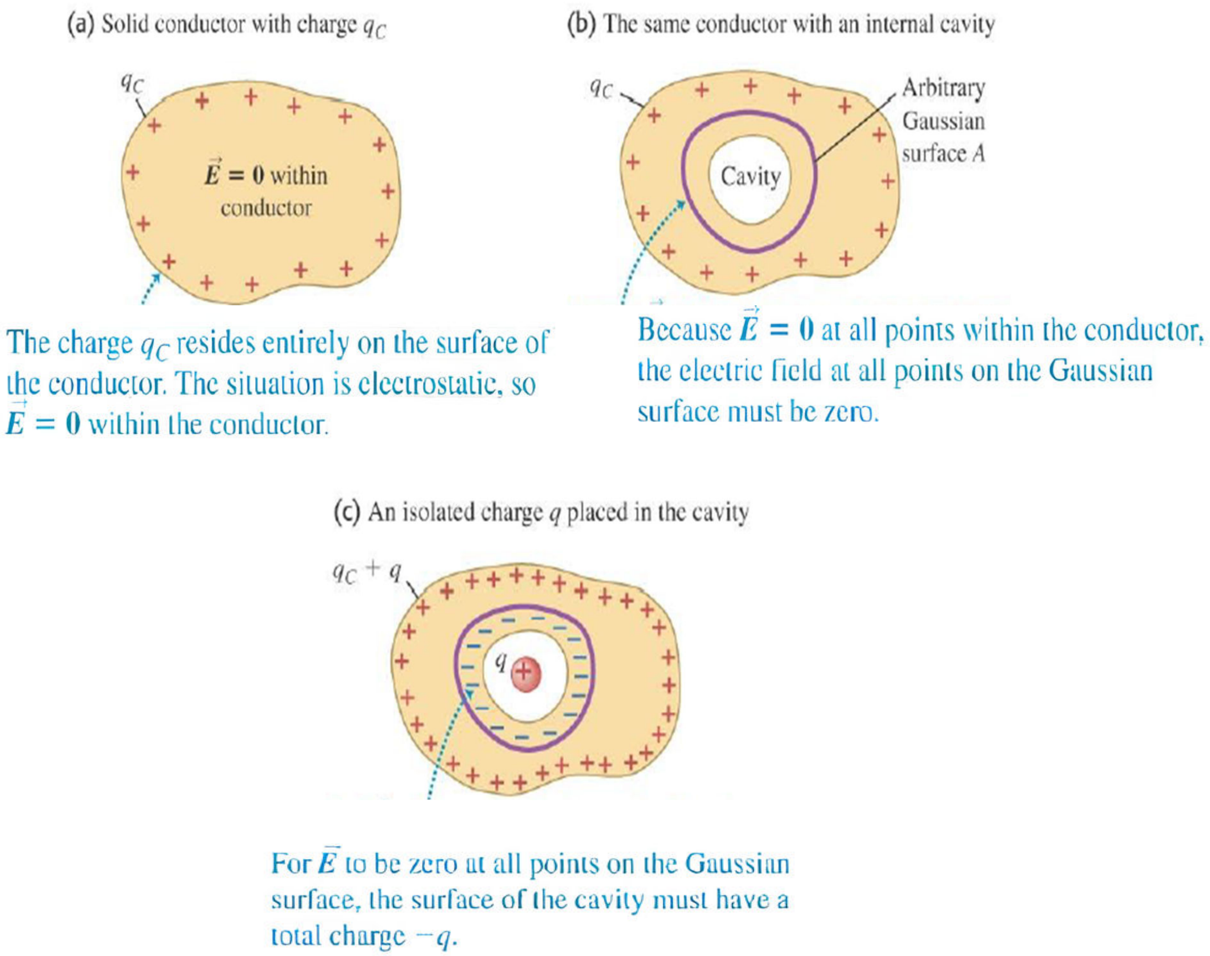


$$\oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_o}$$

$$EA = \frac{q_{enc}}{\varepsilon_o}$$

$$EA = \frac{\sigma A}{\varepsilon_o}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_o}}$$ E-field at conductor surface



d) A conductor in electrostatic equilibrium constitute an equipotential volume.

$dV = -\vec{E} \cdot \vec{dl}$ where $\vec{dl} = \overrightarrow{MM'}$ then V is constant.

The electric field is zero inside the conductor, so the potential is uniform over the conductor volume.

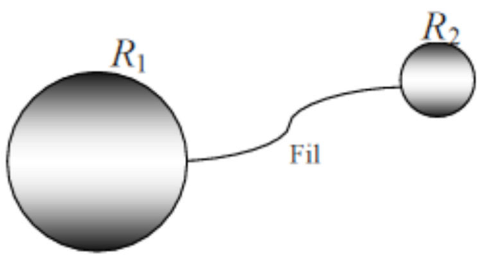
II.10.1 Electrostatic pressure :

The charges on the surface of the conductor are submitted to the repulsive forces of the bulk charges. The force exerted by unit surface, or electrostatic pressure, can be calculated by multiplying the electric mean-field on the conductor surface by the charge of the unit surface. The electric mean-field previously calculated is :

$E = \frac{\sigma}{2\varepsilon_0}$ hence, the electrostatic pressure : $p = \sigma E = \frac{\sigma^2}{2\varepsilon_0}$

II.10.2 Power of spikes :

The experiment shows that the distribution of charges on the conductor surface doesn't correspond to a constant charge density. The surface density is particularly great at the spike. So, it is the same for the electric field near the spike.



This phenomenon can be explained by considering two spheres of different rays R_1 *and* R_2 $(R_2 < R_1)$, linked by a tiny wire. For this reason, the spheres are kept in the same potential V. As they are very far from each other, we can write :

$$V_1 = V_2 \implies \frac{K}{R_1}\iint \sigma_1\, ds = \frac{K}{R_2}\iint \sigma_2 ds$$

By symmetry, the charges are evenly distributed on the surface of each sphere (σ_1 *and* σ_2 *are constants*). It follows that :

$\frac{\sigma_1}{R_1} = \frac{\sigma_2}{R_2}$ so, the little sphere carry the bigest density of charge.

II.10.3 Capacitance of a conductor :

Whenever an electric charge is deposited on a conductor, its potential increases. The deposited charge spreads over its surface. For any conductor, the electric potential (V) is directly proportional to the charge store (Q). Hence, $Q \propto V$ and $Q = CV$ where C is a constant known as Capacitance of the conductor.

The capacitance of a conductor (C) is defined as the amount of charge required to make the potential 1 unit (1 volt).

S.I Unit of capacitance is Farad (F) and we have : $C = \frac{Q}{V}$

The Farad is a very large unit hence smaller practical units are used. Smaller units used are :

1 microfarad (1 μF) = 10^{-6} F

1 nanofarad (1 nF) = 10^{-9} F

1 picofarad (1 pF) = 10^{-12} F4.

Example :

Calculation of the capacitance of a spherical conductor.

On every point situated at a distance r from the center of the sphere of radius R, the potential is given by : $V = K\frac{Q}{r}$

On a point of the surface of the sphere : $V = K\frac{Q}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

So we get the following capacitance : $C = \frac{Q}{V} = 4\pi\varepsilon_0 R$

For example the capacity of the earth is about $(R = 6400\ km)$: $C = 710\mu F$.

II.10.4 Electrostatic energy of a conductor :

Let dE_p the variation of the potential energy undergone by an elementary charge, brought back from infinity (chosen as reference potential) to the conductor :

$$dE_p = Vdq$$

Where q and V are the charge and potential respectively in an intermediate state. During charge transfer on the conductor, his total charge and so the absolute value of his potential increase. The internal energy of the conductor when the charge is completed is given by :

$$E_p = \int_0^Q -Vdq = \int_0^Q \frac{q}{C}dq$$

$E_p = \frac{1}{2}\frac{Q^2}{C} = CV^2$ or $E_p = \frac{1}{2}QV$

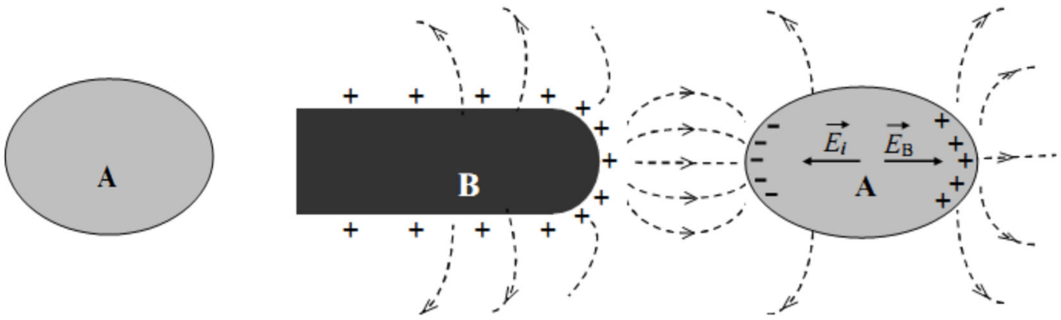
The electrostatic energy for a set conductors in equilibrium is the generalization of the relation above :

$$E_p = \sum_{i=1}^{N} Q_i V_i$$

II.11 Influence phenomenon :

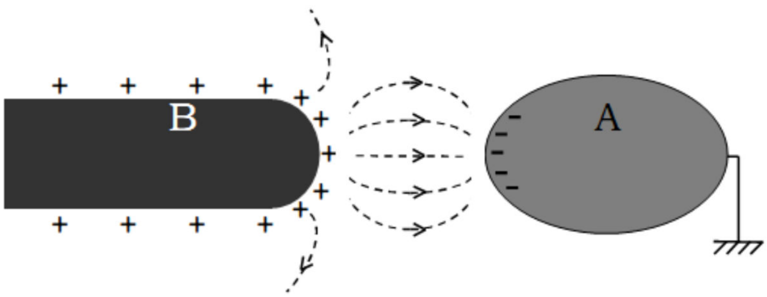
II.11.1 Partial influence :

Considering an electrically neutral conductor A at the figure below, let's approach the latter by the conductor B positively charged as in the figure. The conductor B create in the space and in particular in the conductor A an electric field $\vec{E}_B$.

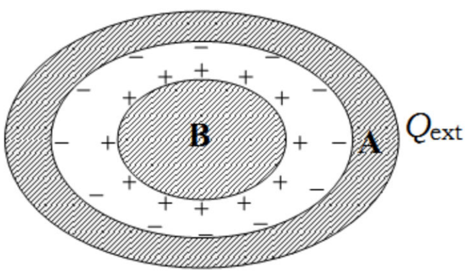


(a) (b)

Explanation : the free electrons on conductor A go, under the action of this field, in the inverse direction of $\vec{E}_B$. These electrons accumulate progressively in front of B and at the equilibrium they get negative charges so that the result is $-Q$. These charges, resulting from influence electrisation, bring their contribution to the electric field inside and outside of the conductor. They create an induced electric field $\vec{E}_i$ opposite to the field $\vec{E}_B$reducing the total electric field. Hence the system attain the equilibrium state.

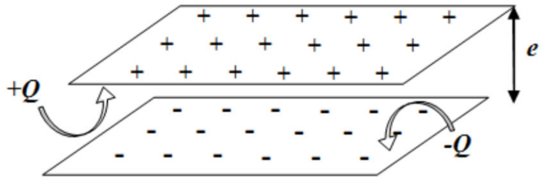


II.11.2 Total influence :



We get a total influence when the field lines going from B attain A by enclosing B by A. $Q_B = -Q_{Aint}$

II.11.3 Capacitors :



A capacitor is system formed by two conductors in total influence. When we apply a potential difference between the plates by connecting them to an external source (a generator), the capacitor gets a charge.

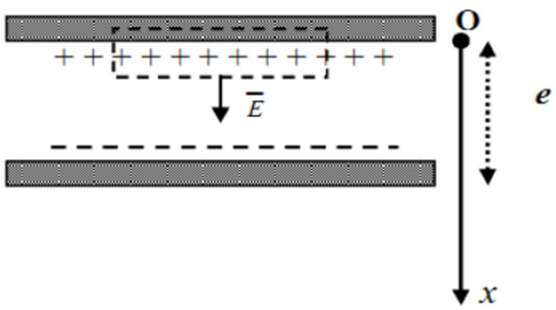
Capacitance of a capacitor :

We define the capacity of a capacitor by :

$$C = \frac{Q}{\Delta V} = \frac{Q}{V_1 - V_2}$$

Q is the charge carried by each of the plates (+Q for one and –Q for the other) and $\Delta V = V_1 - V_2$ is the difference of potential between the plates. The capacitance is an intrinsec property of every capacitor. Its value depend on the geometry of the conductors and the distance between them.

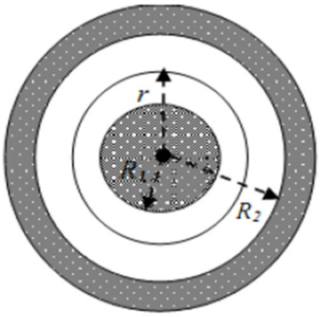
Capacitance of a parallel plate capacitor :



The electric field between the plates is uniform, it is given by : $E = \frac{\sigma}{\varepsilon_0}$; $\sigma = \frac{Q}{S}$

Q is the capacitor charge. $-dV = \vec{E} \cdot \vec{dl} = Edx \implies V = \frac{\sigma}{\varepsilon_0} e = \frac{Qe}{\varepsilon_0 S} \implies C = \frac{\varepsilon_0 S}{e}$

Capacitance of a spherical capacitor :



As shown in the figure, the spherical capacitor is made of two concentric spheres. If we apply Gauss's theorem we obtain the electric field between the two spheres :

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Knowing that : $E = -\frac{dV}{dr}$ then : $-dV = E.dr$

$V = \int_{V_1}^{V_2} -dV = \frac{Q}{4\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{dr}{r^2}$ so that, $\Delta V = V_1 - V_2 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Hence : $C = 4\pi\varepsilon_0\frac{R_1R_2}{R_1-R_2}$

II.11.4 Capacitors connection :

a. Serial connection :

We consider a set of capacitors connected in series on figure a. when a difference of potential is applied between the two ends of the connection, the left plate of the first capacitor will carry a charge Q. the total voltage of the set will be written simply by :

$\Delta V = (V_0 - V_1) + (V_1 - V_2) + (V_2 - V_3) + \cdots + (V_{N-1} - V_N)$

So, we get : $\Delta V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \cdots + \frac{Q}{C_N} = Q\sum_{i=1}^{N}\frac{1}{C_i}$

The voltage corresponds to the unique capacitor of equivalent capacitance :

$$\frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i}$$

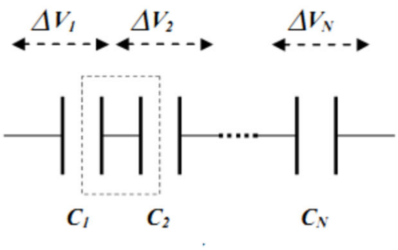
b. Parallel connection :

Lets N capacitors, parallel connected, with the same voltage V as in the figure below. Q_i *and* C_i are the charge and capacitance for the ith capacitor, we have then : $Q_i = C_i V$ the total charge carried by all the capacitors is then :

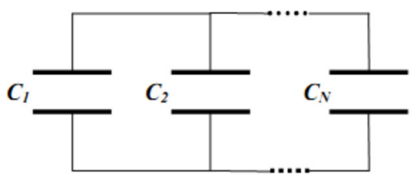
$$Q = \sum_{i=1}^{N} Q_i = \sum_{i=1}^{N} C_i V = V \sum_{i=1}^{N} C_i$$

The equivalent capacitance is the sum of the individual capacitances

$$C_{eq} = \sum_{i=1}^{N} C_i$$



(a)



(b)

CHAPTER III

Electrokinetics

In this chapter we shall discuss the dynamics of charges.

Introduction :

When the charges are stationary, an electrostatic field and static potential are developed in their vicinity. If the charges are placed in a region of non-uniform potential, they start to move and a current is set up. In conductors the electrons in the outermost orbits are relatively loosely bound to their respective atoms. When the conductors are placed in an electric field, a force starts to act on these free electrons. The direction of the force on positive charges is along the direction of the field and on negative charges is opposite to the field. The free charges start to move under the action of this force. The flow of free charges in a conductor constitutes the electric current.

III.1 Electric current :

The moving electrons are the source of the electric current.It can also be carried by ions in electrolyte. The current is a physical quantity that can be measured and expressed numerically. The current in a circuit at any instant can be measured by determining the quantity of charge passing per second through the cross-section of the wire at that instant. If the rate of flow of charge is independent of time (steady state) and q charges flows through the circuit in time t then current is given by :

$$i = \frac{q}{t}$$

If the rate of the flow of charges varies with the time than the instantaneous current is given by :

$$i = \frac{dq}{dt}$$

If the charge is measured in Coulomb and time in seconds then the unit of current is Ampere. Thus a current of 1 ampere means that there is 1 Coulomb of charge passing through the cross-section of wire every 1 second.

1 ampere = 1 Coulomb/1 second

Electric current is a scalar quantity as it doesn't follow the law of vector addition. The arrows used in the electric circuits represent the direction of flow of positive charges.

III.2 Current density vector :

If a current is flowing in the conductor then the current per unit area of it, when the area is taken along a direction normal to the current, is known as current density. Let us consider a current flowing through a conductor of lenght l and uniform cross-sectional area A. Suppose this current is due to the motion the electrons only. These electrons will possess the average drift velocity v_d in a direction opposite to that of applied field. The value of v_d for one second, in fact, gives the distance travelled by the electrons in one second. Therefore, the volume of the cylindar around the path traversed by electrons in one second is given by :

$dV = \overrightarrow{v_d} \cdot \overrightarrow{ds}$

If N is the number of charge carriers (electrons of charge e) per unit volume then the charge passing through the area $\overrightarrow{ds}$ in one second is

$$dq = Ne\left(\overrightarrow{v_d} \cdot \overrightarrow{ds}\right)$$

But charge passing per second is nothing but the current, hence

$$dI = Ne\left(\overrightarrow{v_d} \cdot \overrightarrow{ds}\right)$$

Here, the quantity $Ne\overrightarrow{v_d}$ is a vector, called the current density. The current density is represented by $\vec{J}$ and has the same direction as that of drift velocity.

$$\vec{J} = Ne\vec{v}_d$$

Thus, we can write :

$$dI = \vec{J} \cdot \overrightarrow{ds}$$

If we take a small element dI through a small area ds around a point and if d sis normal to dI then current density at that point is given by :

$$\vec{J} = \frac{dI}{ds}\vec{n}$$

When the unit vector $\vec{n}$ represents the direction of current.

If $ds_{\perp}$is the area element perpendicular to the current at a point then, current density may also be defined as :

$$\vec{J} = \frac{dI}{ds_{\perp}}$$

From the equations above, we can write an expression for the total current through a total surface S, using surface integral, as :

$$I = \iint dI = \iint \vec{J} \cdot \overrightarrow{dS} = \iint Ne\left(\overrightarrow{v_d} \cdot \overrightarrow{ds}\right)$$

Here double integral sign represent the integration over the entire closed surface taken into consideration.

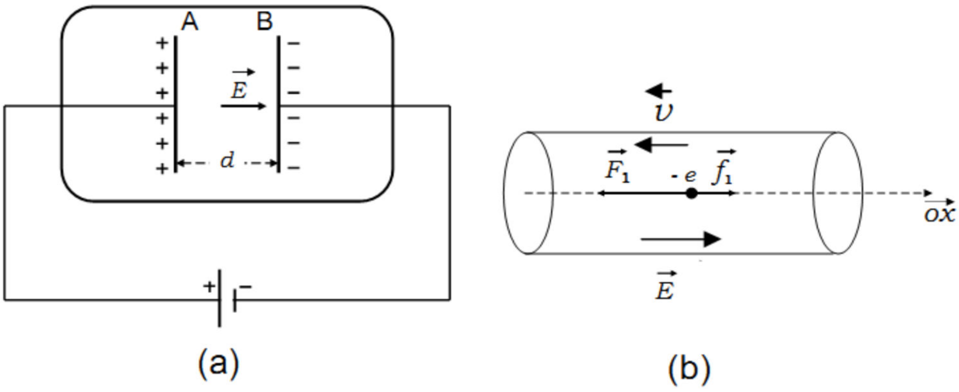
III.3 Movement of electrons in the vacuum :

Two parallel plates A and B are placed in the vacuum and separated by a distance d, submitted to a difference of potential $V=V_A- V_B$ (figure a). an electric field $\vec{E}$ is created so that :

$$E = \frac{V_A - V_B}{d} = \frac{V}{d}$$

If an electron is emited by the plate B, he will be submitted to an electrical force : $\vec{F} = -e\vec{E} = m\vec{a}$ $\quad\quad$ $\vec{a} = \frac{d\vec{v}}{dt} = -\frac{e}{m}\vec{E}$

The acceleration being constant, the movement of the electrons, is then uniformly accelerated, which is not the case in the metals.



III.4 Movement of the electrons in a conductor :

In a metal, in absence of an external electric field, the free electrons moves randomly. Their average velocity is zero, however, in presence of an electric field the drive movement lead to an electric current. If we consider the effect of the crystal lattice on the moving electrons by a frictional force of the form :

$\vec{f} = -k\vec{v}$

Writng the fondamental relation of dynamics for the electron, one has :

$$\vec{F} + \vec{f} = m\vec{a}$$

Where $\vec{a}$, is the acceleration of the electron. Projecting this relation on the ox axis, we get :

$$-e\,E - kv_x = ma_x \qquad \Rightarrow \qquad m\,\frac{dv_x}{dt} + kv_x = -eE$$

and $\dfrac{dv_x}{dt} + \dfrac{k}{m}v_x = -\dfrac{e}{m}E$

$$-eE - kv_x = ma_x \qquad \Longrightarrow \qquad m\frac{dv_x}{dt} + kv_x = -eE$$

Or the following relation :

$$\frac{dv_x}{dt} + \frac{k}{m} v_x = -\frac{e}{m} E$$

It is a first order differential equation with constant coefficients and second member wich has a general solution consisting of the sum of two solutions :

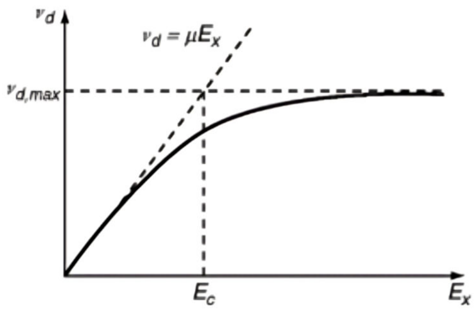
$$v_h(t) = -\frac{e}{k} E + A\, exp\left(-\frac{k}{m} t\right)$$

With the initial condition v(0)=0, we obtain the constant $A = \frac{e}{k} E$

Which gives :

$$v_h(t) = -\frac{e}{k} E\left(1 - A\, exp\left(-\frac{k}{m} t\right)\right) = -v_l\left(1 - A\, exp\left(-\frac{k}{m} t\right)\right)$$

Where : $v_l = \frac{e}{k} E$ is the limiting velocity attained by the electrons and $\tau = \frac{m}{k}$ the relaxation time. The drift velocity is plotted in the figure below.



III.5 Macroscopic Ohm's Law :

The experience shows that : the ratio of the voltage and the current in between two points of a metallic conductor is constant,

$$V = RI$$

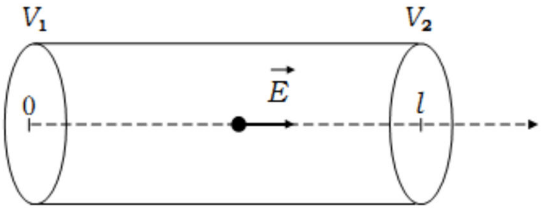
That is the Ohm's Law. The constant R is the electric resistance of the conductor, it is expressed in Ohms (Ω).

Local formulation of Ohm's Law :

A cylindrical conductor, of lenght l and cross-section S, submitted to a differential potential V then it results on any point of the conductor, an electric field $\vec{E}$ so that : $dV = -\vec{E} \cdot \overrightarrow{dl}$

$\vec{E}$ and $\overrightarrow{dl}$ are parallel, one has :

$\int_{V_1}^{V_2} dV = -E \int_0^l dl \quad \Longrightarrow \quad V = V_1 - V_2 = El$



$El = RJS$, so we have :

$J = \frac{l}{RS} E = \sigma E$ or $\sigma = \frac{l}{RS}$

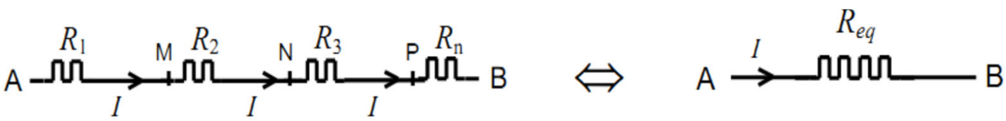
σ is the conductivity which has units $\Omega^{-1} m^{-1}$, the electrical resistance will be : $R = \frac{l}{\sigma S}$

At the microscopic scale, we can write : $J = -nev = \frac{ne^2}{m} E$ so that we obtain the conductivity as : $\sigma = \frac{ne^2}{k}$ rewritten with the relaxation time as :

$\sigma = \frac{ne^2}{k} \tau$ in vector form $\vec{J} = \sigma \vec{E}$ which is the local form of Ohm's Law.

III.6 Resistors in combination :

a) **Serial combination** :



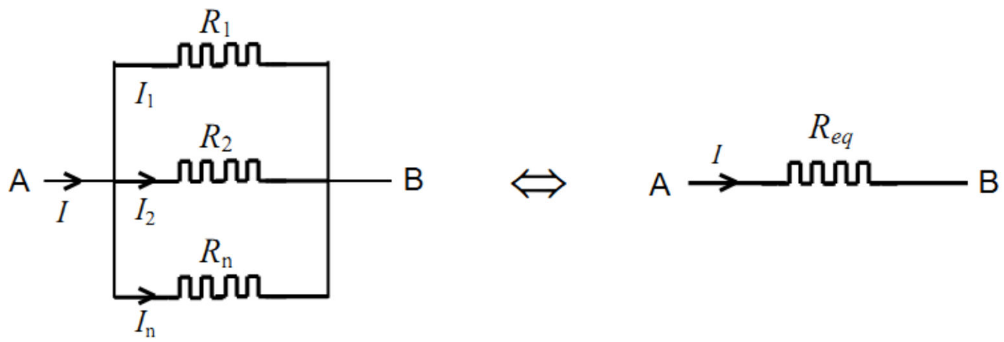
The same current flowing through all the resistors connected from A to B.

$V_A - V_B = (V_A - V_M) + (V_M - V_N) + (V_N - V_P) + \cdots$

So that : $V_A - V_B = R_1 I + R_2 I + \cdots R_N I = R_{eq} I$

$\Longrightarrow \ R_{eq} = \sum_{i=1}^{N} R_i$: the equivalent resistance is the sum of all the resistances.

b) **Parallel combination** :



Now, the same potential difference is applied on the terminals of the resistors, hence :

$V_A - V_B = R_1 I_1 = R_2 I_2 = \cdots = R_N I_N$

And : $I = I_1 + I_2 + \cdots + I_N$ then we will have :

$$\frac{(V_A - V_B)}{R_{eq}} = \frac{(V_A - V_B)}{R_1} + \frac{(V_A - V_B)}{R_2} + \cdots + \frac{(V_A - V_B)}{R_N}$$

So the equivalent resistance is :

$$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

III.7 Joule effect :

The circulation of the current in a conductor yields a loss of energy by heating. We can evaluate the dissipated energy during the passage of the current. If dq is charge that passes from point A to point B of the conductor, then the work of the electric forces is :

$$dW = (V_A - V_B)dq$$

This quantity of charge is related to the current by : $dq = Idt$ so ;

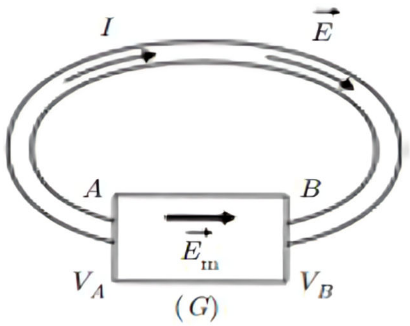
$dW = (V_A - V_B)\, Idt \quad V = (V_A - V_B) = RI$

The work will be written as : $dW = VIdt = RI^2$dt

This dissipated energy is the Joule's heating. It coresponds to an electric power : $P = \frac{dW}{dt} = RI^2$

As V and I are constant so the electric power will be constant in the time.

III.8 Generators :



Lets a generator (G), apply a potential difference $V_A - V_B > 0$ in the terminals of a conductor AB.

In the steady state, we have $div\vec{j} = 0$ in all the points of the circuit, including the generator, and the field lines are closed curves. If the conductor was closed on himself, we will have :

$\oint \vec{E} \cdot \overrightarrow{dl} = 0$ because of $\vec{E} = -\overrightarrow{grad}\,(V)$

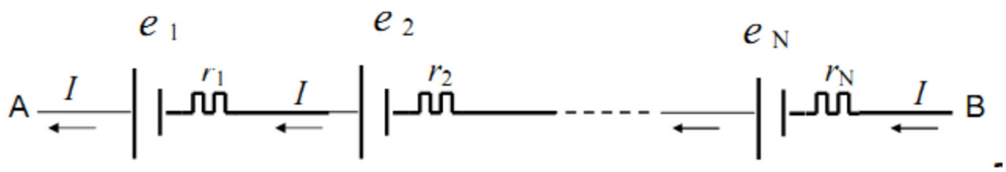
$\oint \frac{\vec{j}}{\sigma} \cdot \overrightarrow{dl} = 0$ which leads to : $\vec{j} = 0$

It is the circulation of the field $\overrightarrow{E_m}$ in the generator that ensure the potential difference, $V_A - V_B$. This circulation is called the electromotive force e of the generator, although it has the dimensions of a potential. We have :

$$e = \int \overrightarrow{E_m} \cdot \overrightarrow{dl} = V_A - V_B$$

III.9 Generators in combination :

- **Serial combination** :



Lets N generators (e_i, r_i), in serial connection as represented in the figure the same current I traversing each of them. The voltage in the terminals of the i[th] generator is written as :

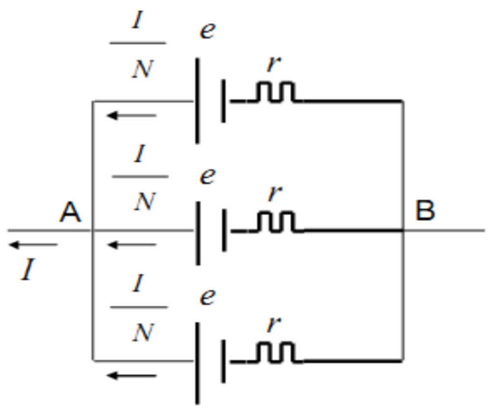
$$V_{i+} - V_{i-} = e_i - r_i I$$

The voltage in the terminals of the whole generators is :

$$V_A - V_B = (e_1 + e_2 + \cdots + e_N) = (r_1 I + r_2 I + \cdots + r_N I) = e - rI$$

- **Parallel combination** :

The figure shows that the equivalent generator delivers a current I equal to the sum of all the currents delivered by each of the generators.



$I = \sum_{i=1}^{N} I_i$

The voltage between A and B is then :

$$V_A - V_B = e - r\frac{I}{N} = e - \left(\frac{r}{N}\right) I$$

III.10 Network analysis :

III.10.1 Kirchhoff's first law :

Circuit network analysis can be carried out using Kirchhoff's laws. Kirchhoff's first law applies to currents at a junction in a circuit. It states that :

The sum of currents flowing into the node or equivalently a junction is equal to the sum of currents flowing out of the node.

It is merely a consequence of charge conservation.

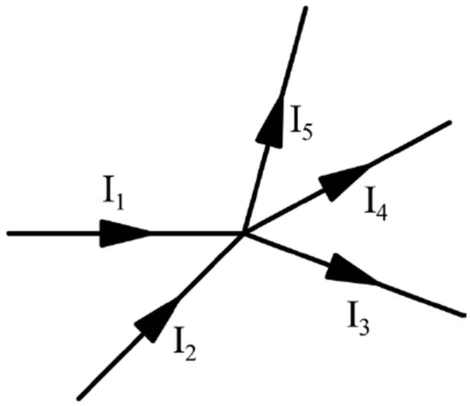


Figure III.1 currents of known direction of flow into and out of a node.

$$I_1 + I_2 = I_3 + I_4 + I_5$$

In some cases, the direction of current flow is not known and, in this situation, we can arbitrarily assign a direction, as shown in Figure III.1 assuming that all the currents are non-zero, at least one of the currents must have a negative value, indicating that the arrow has been drawn in the wrong direction. Current can not flow into the node from all three directions without current flowing out.

III.10.2 Kirchhoff's second law :

Kirchhoff's second law applies to voltage drops across components in a circuit. It states that :

Kirchhoff's second law concerns the potential differences across components in a circuit. It states that :

The directed sum of potential differences across the components of the closed loop circuit is zero.

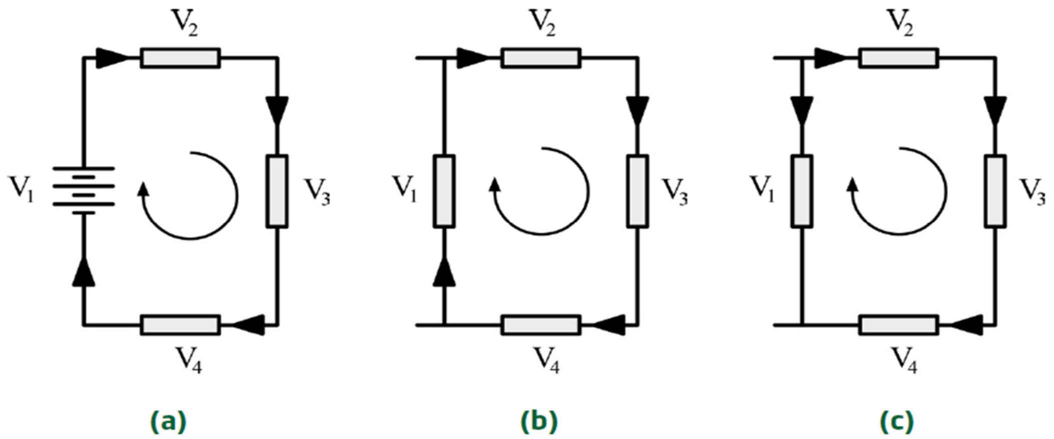


Figure III.2 (a-c) : Two closed loops with a power supply (a), without a power supply assumed current direction (b) and without a power supply with actual current direction (c).

Remember that :

- For loops, the assumed positive is where the current flows out of the cell.
- For components, the assumed positive is where it flows into the component.

  In figure (a), Kirchhoff's second law states that:

  $V_1 - V_2 - V_3 - V_4 = 0$

The same principle applies in Figure III.2(b), where an assumed current flow has been added. In this case, the diagram has added wires to indicate that it is joined to a larger circuit. This helps clarify that in this case, as there is no direct current supply, current must enter and exit the loop.

$$V_1 + V_2 + V_3 + V_4 = 0$$

Validity of Kirchhoff's current Law

KCL means Kirchhoff's current law :

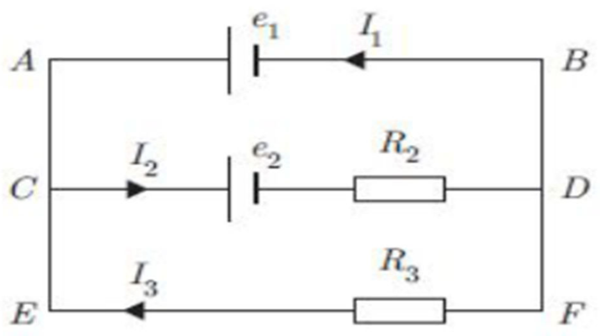
- KCL is temperature independent variation in the circuit.
- KCL is valid for linear,non-linear, passive and active elements.
- KCL is valid for lumped electrical networks only, not for distributed electrical networks. KCL is invalid at high frequencies.
- Kirchhoff's law is not valid for time-varying magnetic fields.

Validity of Kirchhoff's second law :

- KSL is independent of the variation in temperature in the circuit.
- KSL is valid for linear, non-linear, passive and active elements.
- KSL is only valid for lumped electrical networks, not for distributed electrical circuit networks.

Example :

Calculation of the currents in a network :



Calculate the currents I_1, I_2, I_3, respectively, in branches AB, CD, EF. The orientation of the currents is arbitrary as in the figure.

In the node C we have :

$$I_2 = I_1 + I_3$$

From the loop CDFEC, we have :

$$e_2 + R_2 I_2 + R_3 I_3 = 0$$

From the loop CDBAC, we have :

$$-e_2 - R_2 I_2 + e_1 = 0$$

The resolution of the system of 3 equations and 3 unknowns I_1, I_2, I_3

$$I_1 = \frac{(R_2 + R_3)e_1 - R_3 e_2}{R_2 R_3}$$

$$I_2 = \frac{e_1 - e_2}{R_2} \qquad I_3 = -\frac{e_1}{R_3}$$

From these expressions, knowing the numerical values of e_1 and e_2 of the resistances, we can determine the veritable orientation of the currents.

CHAPTER IV

ELECTROMAGNETISM

In the second chapter, we have studied, the interaction of two electrical bodies mainly, electrostatics. We will consider presently, another interaction, the magnetic interaction, in fact they are dual to each other.

IV.1. definition of the Magnetic field :

Consider a particle of charge q and moving at a velocity $\vec{v}$. From the experience we have the following observations :

(1) The magnitude and direction of $\overrightarrow{F_B}$ depends on $\vec{v}$ and $\vec{B}$.

(2) The magnetic force $\overrightarrow{F_B}$ vanishes when $\vec{v}$ is parallel to $\vec{B}$. However, when $\vec{v}$ makes an angle θ with $\vec{B}$, the direction of $\overrightarrow{F_B}$ is perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$, and the magnitude of $\overrightarrow{F_B}$ is proportional to $\sin\theta$.

(3) When the sign of the charge of the particle is switched from positive to negative (or vice versa), the direction of the magnetic force also reverses.

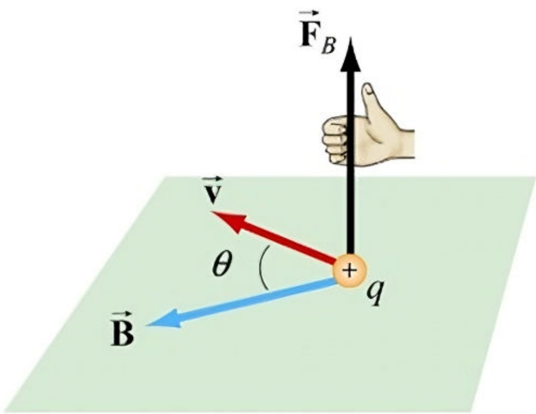


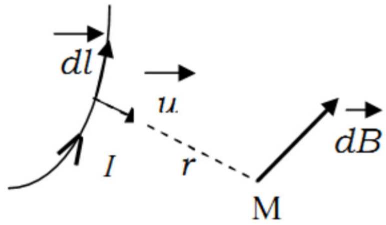
Figure IV.1 : the direction of the magnetic force.

The above observations can be summarized by the following equation :

$$\overrightarrow{F_B} = q\vec{v} \times \vec{B}$$

The Lorentz force is : $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ which is the combination of the elctric force and the magnetic force applied on the charge q.

IV.2 Biot-Savart law :



The physists Biot and Savart found the expression of the magnetic field obtained by Oersted in his experiment in 1820.

A wire conductor of infinite lenght, traversed by a current I, create at a point M in of the space situated at a distance r from the wire, a magnetic field where :

- The direction is that the field lines are circles enclosing the wire.
- The magnitude is :

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

Where, μ_0, is the magnetic permeability of the vacum. In the MKSA system units, $\mu_0 = 4\pi 10^{-7}$ Henry per meter : H/m

Every infinitesimal current of an oriented lenght $\overrightarrow{dl}$ traversed by a current of magnitude I , produce an elemental magnetic field in the point M :

$$\overrightarrow{dB}(M) = \frac{\mu_0}{4\pi}\frac{I\overrightarrow{dl} \wedge \vec{u}}{r^2}$$

This the Biot-Savart law in its differential form, we can integrate it over a loop to find out that :

$$\vec{B} = \oint \overrightarrow{dB} = \frac{\mu_0}{4\pi}\oint \frac{I\overrightarrow{dl} \wedge \vec{u}}{r^2}$$

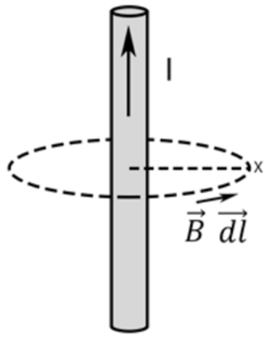
Where, the integration is taken over a closed wire.

IV.2 Biot-Savart law :

IV.3 Derivation of Ampere's law : (Ampere's theorem)

From Biot-Savart's law, the magnetic fiel dis due to a long straight wire is,

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$



Since $\vec{B}$ and $\vec{dl}$ are in the same direction,

$$\vec{B}\cdot\vec{dl} = Bdl\cos 0 = Bdl$$

$$\oint \vec{B}\cdot\vec{dl} = \frac{\mu_0}{2\pi}\frac{I}{r}\oint dl = \frac{\mu_0}{2\pi}\frac{I}{r}(2\pi r) \quad \Longrightarrow \quad \oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

IV.4 Laplace force :

A current element, placed in an external magnetic field $\vec{B}$, is submitted to an electromagnetic force (Laplace force) :

$$d\vec{f} = \vec{\imath}\, d\tau \wedge \vec{B} \text{ or } d\vec{f} = \vec{J_s} dS \wedge \vec{B} \text{ where } d\vec{f} = I\, dl \wedge \vec{B}$$

For respectively, volumic current of density $\vec{\imath}$, surface current of density $\vec{J_s}$ and a linear current of intensity I.

IV.5 Electromagnetic Induction phenomena. (Faraday law) :

These phenomenon will be studied in the quasi-steady approximation, that is :

- Considering the intensity of the current $i(t)$ the same along the wire.

- Using the same formalism as for the steady regime.

The phenomenon of electromagnetic induction is the appearance of an electric field, so an electromotive induction force, in a circuit under the influence of a magnetic field, when the circuit is in a relative displacement with respect to the induction lines created by the magnetic field.

There are two key laws that describe electromagnetic induction:

1. **Faraday's law**, this relates the rate of change of magnetic flux through a loop to the magnetude of the electromotive force ϵ induced in the loop. The relationship is : $\boldsymbol{\epsilon = -\frac{d\Phi}{dt}}$

   The electromotive force refers to the potential difference across the *unloaded* loop (i.e. when the resistance in the circuit is high). In practice it is often sufficient to think of the electromotive force as voltage since both voltage and the electromotive force are measured using the same unit, the volt. This is the analog of the third Newton law in mechanics (back-reaction)

2. **Lenz law**, is a consequenceof energy conservation applied to electromagnetic induction. It was formulated by Heinrich Emil Lenz in 1833. Lenz's law tells us the direction that current will flow. It states that the direction is always such that it will oppose the change in flux which produced it. This means that any magnetic field produced by an induced current will be in the opposite direction to the change in the original field.

   Lenz's law is incorporated into Faraday's law with a minus sign, the inclusion of which allows the same coordinate system to be used for both the flux and the electromotive force. The result is sometimes called the Faraday-Lenz law, $\boldsymbol{\epsilon = -\frac{d\Phi}{dt}}$

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