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FACULTY OF APPLIED SCIENCES

DEPARTMENT OF SCIENCES AND TECHNOLOGIES

Handout

Electricity and magnetism

Course

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Contents

List of figures:

List of tables

Abbreviation:

EP – Electric Potential

EMF – Electromotive Force

AC – Alternating Current

DC – Direct Current

HE – Hall effect

Introduction:

Electricity and magnetism are among the most important foundations of modern science and engineering. From transmitting electrical power from dams to cities, to powering delicate digital electronics such as computers, to electromagnetic communication systems, the widespread use of electrical and magnetic phenomena is seen. This document provides a structured introduction to both static and dynamic electricity, as well as magnetism, which form the basis of classical electromagnetic theory.

The first chapter introduces the phenomena arising from electrostatics, which is the phenomenon resulting from static electric charges and the fields they produce, as well as the interaction between the charges. Topics such as Coulomb's law, electric fields, and electric potential (EP) will be covered. Furthermore, we focus primarily on Gauss's law due to its importance in modern technological applications, while presenting a number of its applications in practical engineering problems.

In the second chapter, we move to study the electrokinetic, examining moving charges (currents) and the resulting electric fields. This chapter includes a detailed discussion of Ohm's law and Kirchhoff's laws that govern electric circuits, as well as the behavior of circuits involving resistors, capacitors, and generators.

In the final chapter, we present a study of the magnetic phenomena arising from electric currents within the framework of classical physics. Phenomena such as the effect of electric currents on magnetized objects will be studied, in addition to phenomena such as the Hall effect and magnetic fields arising from electric currents.

This course will utilize relatively advanced mathematical tools such as vector calculus and differential and integral calculus, which we have comprehensively reviewed in this course, along with a physical explanation of electrical and magnetic phenomena. This prepares first-year engineering students in "Science and Technology" for more advanced studies in electromagnetism, electronics, and electromechanical systems. Examples and illustrations will help connect theory to engineering applications, ensuring a practical and conceptual understanding of the subject.

1. Mathematical background:

1.1. Introduction

1.2. Coordinate system:

1.2.1. Cartesian coordinates:

The Cartesian coordinate system is a coordinate system that is defined by an origin point O and three axes (Ox, Oy, Oz) perpendicular to each other (see Figure 1). The unit vectors carried by the axes are: $\vec{\imath}, \ \vec{\jmath}, and\ \vec{k}$

We can define any point M in space by the three components of the three-unit vectors as:

$$\overrightarrow{OM} = x\,\vec{\imath} + y\,\vec{\jmath} + z\,\vec{k}$$

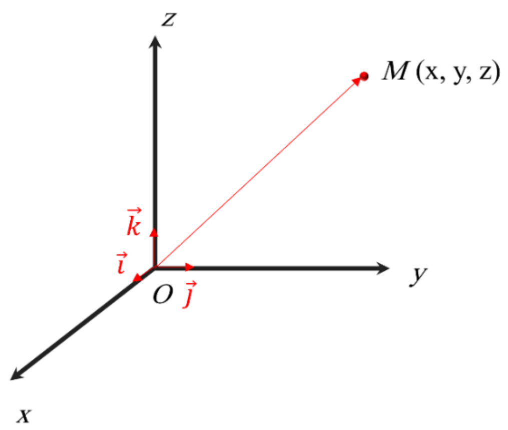


Figure 1 The position of the point M in Cartesian coordinates

1.2.1.a) Differential element of length in Cartesian coordinates:

In Cartesian coordinates, the differential element of length in three-dimensional space is given as:

$$dl = \sqrt{dx^2 + dy^2 + dz^2}$$

1.2.1.b) Displacement (differential) in Cartesian coordinates:

The displacement is given as:

$$d\overrightarrow{OM} = dx\,\vec{\imath} + dy\,\vec{\jmath} + dz\,\vec{k}$$

1.2.1.c) Element of the surface:

In Cartesian coordinates, the differential element of surface in three-dimensional space is given as:

$$ds = dx\,dy$$

1.2.1.d) Element of the volume:

In Cartesian coordinates, the differential element of volume in three-dimensional space is given as:

$$dV = dx\, dy\, dz$$

1.2.2. Cylindrical coordinates:

The cylindrical coordinate system is a three-dimensional coordinate system defined by three components:

ρ: The projection of the magnitude of the vector $\overrightarrow{OM}$ on the plan (x, y).

φ: The azimuthal angle in the xy-plane.

z: The height or vertical coordinate, representing the position along the z-axis.

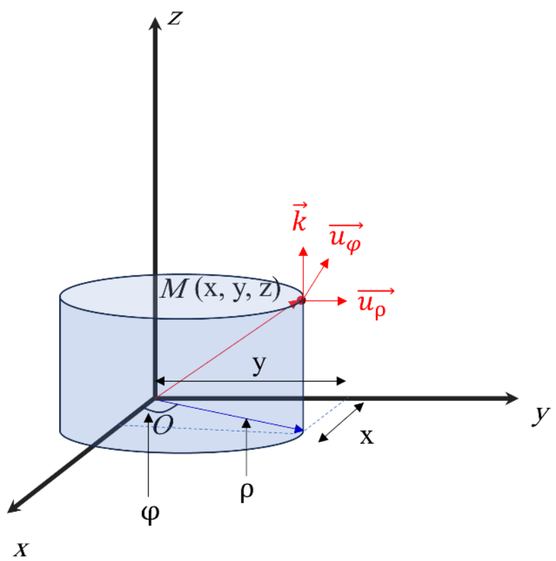


Figure 2 The position of point M in cylindric coordinates

Conversion to Cartesian Coordinates

Given a point (ρ, φ, z) in cylindrical coordinates, the corresponding Cartesian coordinates (x, y, z) are:

$x = \rho\cos(\varphi)$

$y = \rho\sin(\varphi)$

$z = z$

Conversion from Cartesian to Cylindrical

For a point (x, y, z) in Cartesian coordinates:

$$\rho = \sqrt{x^2 + y^2}$$

$$\varphi = arctan(\frac{y}{x})$$

$z = z$

1.2.2.a) Differential element of length in cylindrical coordinates:

The differential displacement in cylindrical coordinates is:

$$d\vec{l} = d\rho\, \vec{u}_\rho + \rho d\varphi\, \vec{u}_\varphi + dz\, \vec{k}$$

The magnitude of the differential length element is given by:

$$dl = \sqrt{d\rho^2 + (\rho d\varphi)^2 + z^2}$$

1.2.2.b) Element of the surface:

In cylindrical coordinates (ρ,φ,z), the differential surface element depends on the surface being considered, as it is defined by the normal vector and the area element on a specific coordinate surface. There are three primary surfaces in cylindrical coordinates: constant ρ, constant φ, and constant z.

Surface of constant ρ:

For a surface with a constant ρ, the surface element is:

Along φ: $dl_\varphi = \rho\, d\varphi$

Along z: $dl_z = dz$

Therefore,

$$ds = \rho\, d\varphi\, dz$$

Surface of Constant φ:

For a surface with a constant ρ, the surface element is:

Along ρ: $dl_\rho = d\rho$

Along z: $dl_z = dz$

Therefore,

$$ds = d\rho\, dz$$

Surface of Constant z:

For a surface with a constant z, the surface element is:

Along ρ: $dl_\rho = d\rho$

Along φ: $dl_{\varphi} = \rho\, d\varphi$

Therefore,

$$ds = d\rho\, \rho\, d\varphi$$

1.2.2.d) Element of the volume:

In Cylindrical coordinates, the differential element of volume in three-dimensional space is given as:

$$dV = d\rho\, \rho\, d\varphi\, dz$$

1.2.3. Spherical coordinates:

Spherical coordinates are a three-dimensional coordinate system that extends polar coordinates to describe points in space using a radial distance, a polar angle, and an azimuthal angle.
A point in spherical coordinates is defined by three components:

r (radial distance): The distance from the origin to the point $r \geq 0$.

θ: The angle from the positive z-axis (typically $0 \leq \theta \leq \pi$).

ϕ (azimuthal angle): The angle in the xy-plane from the positive x-axis (typically $0 \leq \phi < 2\pi$).

Conversion from Spherical to Cartesian coordinates:

Let's take a point (r, θ, ϕ) in spherical coordinates, the corresponding Cartesian coordinates are:

$x = \mathrm{r} \sin(\theta) \cos(\phi)$

$y = \mathrm{r} \sin(\theta) \sin(\phi)$

$z = \mathrm{r} \cos(\theta)$

Conversion from Cartesian to Spherical coordinates:

$$\mathrm{r} = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \arccos\left(\frac{z}{r}\right)$$

$$\phi = arctan\left(\frac{y}{x}\right)$$

1.2.3. a) Differential element of length in Spherical coordinates:

The differential displacement vector in Spherical coordinates:

$$d\vec{l} = dr\, \vec{u}_{\mathrm{r}} + \mathrm{r}d\theta\, \vec{u}_{\theta} + r\sin(\theta)\, d\phi\, \vec{u}_{\phi}$$

The magnitude of the differential length element is given by:

$$dl = \sqrt{dr^{2} + (rd\theta)^2 + (r\sin(\theta)\, d\phi)^2}$$

1.2.3. b) Differential Element of Surface in Spherical coordinates:

<u>Surface of Constant *r*:</u>

If r is constant, both coordinates θ and ϕ vary, therefore, the differential displacement becomes:

Along θ: $dl_\theta = r\, d\theta$

Along ϕ: $dl_\phi = r\sin(\theta)\, d\,\phi$

The surface element is:

$$ds = r^2\, d\theta\, \sin(\theta)\, d\,\phi$$

<u>Surface of Constant θ:</u>

If θ is constant, both coordinates r and ϕ vary, therefore, the differential displacement becomes:

Along r: $dl_r = dr$

Along ϕ: $dl_\phi = r\sin(\theta)\, d\,\phi$

The surface element is:

$$ds = r\, dr \sin(\theta)\, d\,\phi$$

<u>Surface of Constant ϕ:</u>

Along r: $dl_r = dr$

Along θ: $dl_\theta = r\, d\theta$

The surface element is:

$$ds = r\, dr\, d\theta$$

<u>Differential Element of Volume in Spherical Coordinates:</u>

$$dV = r^2 \sin(\theta)\, d\,r\, d\theta\, d\phi$$

1.3. Operators:

An operator is a mathematical object that acts on a function to produce another function. For example, differential operators act on a function by computing its derivatives.

$$\Omega\, f(x) = \frac{df(x)}{dx}$$

1.3.1 Gradient:

The gradient of a scalar field $f(x,y,z)$ is a vector field that points in the direction of the steepest increase of f and whose magnitude is the rate of that increase.

$$\nabla f = \frac{\partial f}{\partial x}\vec{\imath} + \frac{\partial f}{\partial y}\vec{\jmath} + \frac{\partial f}{\partial z}\vec{k}$$

1.3.1.a) Divergence:

The divergence of a vector field $\vec{F}$=$F_x\vec{\imath}$+$F_y\vec{\jmath}$+$F_z\vec{k}$ measures the "outward flux" per unit volume at a point.

$$\nabla \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

1.3.1.b) Curl:

The curl of a vector field of a vector field $\vec{F}$=$F_x\vec{\imath}$+$F_y\vec{\jmath}$+$F_z\vec{k}$ measure the rotation or circulation of the field $\vec{F}$ is:

$$\nabla \times \vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\vec{\imath} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\vec{\jmath} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\vec{k}$$

Chapter 2

Electrostatic

2. Electrostatic

2.1 Introduction:

Electrostatics is a branch of classical physics that focuses on the study of static electric charges and the interactions between them. This includes the forces, fields, potentials, and energies of electric charges arising from or interacting with electrical charges.

Coulomb's laws of electric force explain how electric charges interact with each other, repelling each other if they have the same sign and attracting each other if they have opposite signs. Concepts such as the electric field provide a deeper understanding of the instantaneous effect of electric forces, while concepts such as the electric potential (EP) allow us to measure the energy associated with the distribution of charges.

Despite its focus on the foundations of classical physics, electrostatics forms the basis of many technologies, including some modern electronics, energy storage systems, and sensors, as well as natural phenomena such as lightning.

2.2 Definition of the term:

The term Electrostatic is divided into two parts(Bleaney and Bleaney, 2013):

Electricity is the phenomenon resulting from electrical charges, including their behavior, interactions, and effects.

Static: means that this charge is in a state of rest.

<u>Examples of electrostatic:</u>

<u>Example 1:</u>

We deal with electrostatic phenomena almost daily. This can be observed, for example, in small electric shocks when touching a metal door handle, such as a car door, after walking on a carpet, for example. This phenomenon occurs as a result of the accumulation and discharge of electric charge.

Where does the electric charge come from?

When walking on a carpet, for example, friction occurs between the sole of the shoe and the carpet, which leads to the transfer of electrons. Electrons accumulate on the shoe, making the body negatively charged.

How do the electrical shocks happen?

When touching a metal door handle, electrons are transferred from the body to the metal door as a result of the electric potential (EP) difference between the door and the charged body.

This results in small electric shocks.

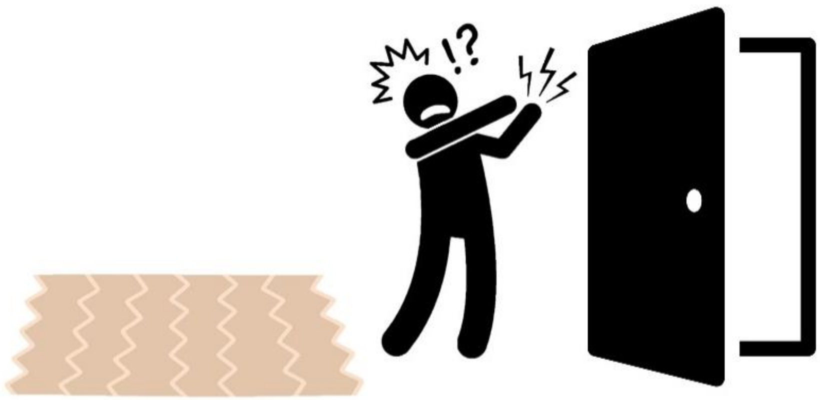


Figure 3 Electrostatic example

2.3. The electrical charge:

2.3.1. The quantification of the electrical charge:

Electricity arises from a fundamental property of elementary particles, ***the elementary electric charge***. This charge is a ***scalar quantity*** and is negative for particles such as electrons and positive for other particles such as protons. The magnitude of this elementary charge is a fixed value and represents ***the smallest unit of electric charge in nature,*** denoted *e*:(Millikan., 1913)

$$e = 1.69\ 10^{-19}\ \mathrm{C} \qquad (1)$$

All electric charges in the universe are quantized, meaning they are integer multiples of this elementary charge. It is not possible to observe a charge smaller than this fundamental value in nature.

"The charge of any particle is equal to an integer multiple of the elementary charge."

While classical physics treats electricity as a phenomenon arising from the motion or interaction of charged particles like electrons, a deeper understanding requires quantum mechanics and quantum field theory. These advanced frameworks provide a more accurate description of the behavior and interactions of fundamental particles such as electrons, explaining phenomena like wave-particle duality and the quantization of energy.

In this classical lecture, however, we will consider the electron as a fundamental particle carrying a fixed negative electric charge, focusing on its macroscopic effects (electrical force, electrical field …) rather than delving into quantum-level explanations.

2.3.2. The unit of the electrical charge:

The unit of the electrical charge is ***the coulomb*** (C); one coulomb is the amount of charge transported by a current of one ampere in one second. Both ampere and second are fundamental units in the International System of Units (SI).

$$1\ C = 1\ A\ s. \quad (2)$$

One coulomb is equivalent to $6.241\ 10^{18}$ elementary charge.

2.3.3. Charge transfer:

The electrical charge can transfer between physical objects through different processes, including friction, conduction, ion exchange, and tunneling.

Friction:

This Charge transfer occurs when two materials, such as a plastic rod and a piece of fur, are rubbed together. Electrons move from a material with a low electron affinity to a material with a high electron affinity. When the two materials are separated, one becomes positively charged due to losing electrons, and the other becomes negatively charged due to gaining electrons.

Conduction:

The phenomenon of charge transfer by electrical conduction occurs when an electrically charged conductor (such as a metal) comes into contact with another electrically neutral or oppositely charged conductor (also known as a conductor). This allows the transfer of electrical charge between the two bodies until equilibrium is reached.

Ion exchange:

Cations and anions move between substances when they come into contact with a medium that facilitates their movement, such as water or chemical solutions. These chemical compounds interact with each other, exchanging electrical charges in chemical reactions.

Tunnel effect:(Burke, 1952)

Tunneling charge transfer is a quantum phenomenon whereby elementary particles with wavelike properties can cross thin insulating barriers even if they lack the energy required to conventionally cross the barrier.

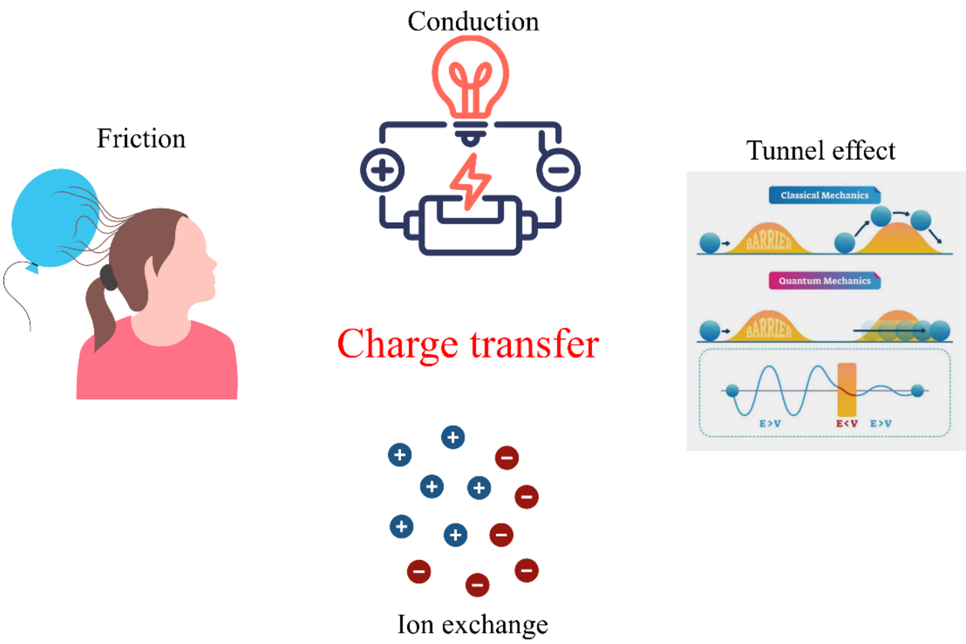


Figure 4 The types of charge transfer

2.3.4. The conservation of the electrical charge:

In addition to its quantized nature, the electrical charge has another important property, which is its conservation nature; ***the electric charge in an isolated system cannot be created or destroyed.*** This is a fundamental principle in physics that governs the behavior of the charge and is widely observed in many experiences.

Examples of electrical charge conservation:

Charging by Friction (Triboelectric Effect):

Rubbing a glass rod with silk fabric results in the transfer of electrons from the glass to the silk. The glass acquires a positive charge from the lack of electrons, while the silk attains a negative charge from the extra number of electrons. In the context of the glass rod and silk fabric system as an isolated object, the total charge remains unchanged before and after the frictional interaction.

Chemical Reactions (Batteries):

In electrical batteries, such as a zinc-copper battery, zinc atoms undergo oxidation by losing electrons, whereas copper ions experience reduction by gaining electrons. The quantity of electrons given by the zinc atoms is equivalent to the quantity of electrons acquired by the copper ions. The overall charge inside the system is unchanging.

Nuclear Reactions:

In nuclear processes, such as beta decay, a neutron transforms into a proton, an electron, and an antineutrino. A neutron (charge = 0) transforms into a proton (charge = +1) and an electron (charge = -1). The total charge before to and subsequent to the decay is null. The proton's positive charge is counterbalanced by the electron's negative charge.

The conductors and insulators:

The materials could be divided generally into kinds: conductors, and insulators.

Conductors: The conductors are materials that allow the electrical charge to move freely.

Insulators: The insulators are materials that resist the movement of the electrical charge, "the charge cannot move freely".

This classification is based on the mobility of the electrical charge inside the material. The mobility of the electrical charge inside materials depends on many parameters, particularly the atomic electronic structure. Atoms that have free electrons tend to have a conductor behavior; however, atoms that have bound electrons tend to have an insulator behavior.

Examples of conductor materials: Copper, silver, gold, nickel, lead, aluminum, and iron.

Examples of insulator materials: Plastic, glass, and ceramics.

Problem 1:

Two conducting spheres are identical: they have the same radius. The first sphere has a charge of 6.0 nC and the second has a charge of −9.0 nC. The two spheres are brought together until they touch, then they are moved apart.

What is the charge on each sphere after they are moved apart?

Solution:

We take Q_1 is the charge of the first sphere Q_1 = 6 nC, and Q_2 is the charge of the second charge Q_2 = -9 nC.

The total charge of the system is $Q_{Total} = Q_1 + Q_2 = 6 – 9$ nC = -3 nC.

Because the two spheres are identical, the charge is distributed equally between them; therefore:

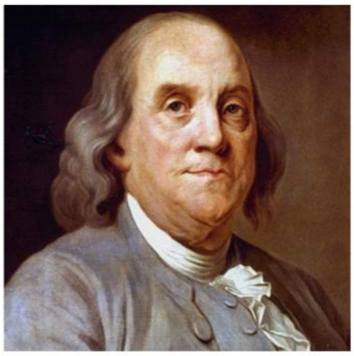
$$Q_f = Q_{total} /2 = -1.5 \text{ nC}.$$

2.4. Electrical force:

2.4.1 Coulomb law:

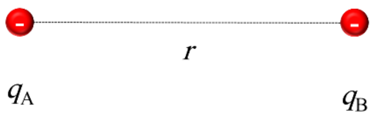
From the examples and experience that we mentioned above, it is clear that the charged objects are sub to an electrical force. These observations were noted by many scientists around the time; however, systematic experiments began in the 18th century by scientists like Benjamin Franklin (Morgan, 2002) and Joseph Priestley. In 1785, a French physicist called Charles-Augustin de Coulomb worked on groundbreaking experiments that focused on the quantification of the electrical force between charged objects.

Benjamin Franklin 1706-1790



2.4.1.a) The mathematical expression the electrical force:

We assume that q_A and q_B are two electrical charges located at points A and B.



The mathematical expression of the Coulomb force can be given as follows:

$$\vec{F}_{A/B} = K\,\frac{q_A\, q_B}{r^2}\,\overrightarrow{u_{AB}}$$

Where K is the constant of Coulomb, its value is 8.99×10^9 N m^2/C^2, q_A, q_B, are the electrical charges, r is the distance between the charges q_A, and q_B, and $\overrightarrow{u_{AB}}$ is the unit vector of the electrical force.

Suppose that we have two electrical charges q_A, and q_B fixed at points A, and B, and spaced by a distance r. The charge q_A is submitted by a force $\overrightarrow{F_{BA}}$ created by the charge q_B, and the charge q_B is submitted by a force $\overrightarrow{F_{AB}}$ created by the charge A. The two forces $\overrightarrow{F_{BA}}$, and $\overrightarrow{F_{AB}}$ have the same magnitude:

$$F_{A/B} = \;\; F_{B/A} = K\,\frac{q_A\, q_B}{r^2}$$

In the case of two electrical charges with the same sign (both charges are positive or negative). The electrical force between these two charges is repulsive. However, when the charges are not the same, the electrical force between these two charges is attractive. This is a fundamental principle of electrostatics.

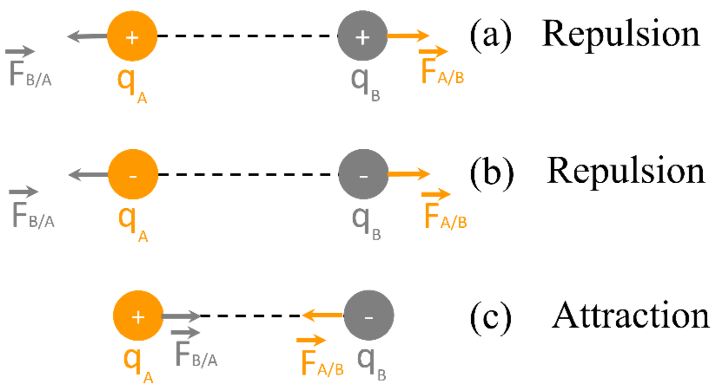


Figure 5 Repulsion and attraction phenomena

<u>Problem:</u>

An electrical charge $q_1 = 1\ \mu C$ is located at the position $\vec{r}_1 = 2\vec{\imath}\ (cm)$, and another electrical charge $q_2 = 3\ \mu C$ located at position $\vec{r}_1 = (7\vec{\imath} + 4\vec{\jmath})\ (cm)$.

Find the electrostatic force created by the charge q_1 on the charge q_2.

<u>Problem:</u>

Two small conducting spheres are suspended using non-conductive strings of length L=30.0 cm. Each sphere has a mass of 100 g. When the spheres each carry a charge q, they repel each other such that the angle between each string and the vertical is θ=1.20°, as shown in the following Figure 6. Calculate the charge q.

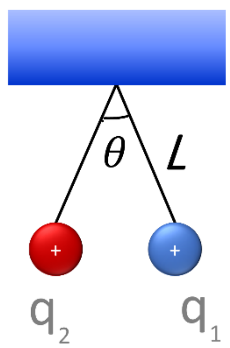


Figure 6 Problem description

Problem:

Four identical free charges each of value q are located at the corners of a square of side a.

What must be the charge Q that has to be placed at the center of the system so that the system stays in equilibrium?

2.4.2. The principle of superposition:

When several charges exert a force on a charge q_0, the total electrical force created on the charge q_0 is the algebraic sum of these forces. This principle is called the superposition principle.

$$\overrightarrow{F_T} = \sum_{i}^{n} \overrightarrow{F_i}$$

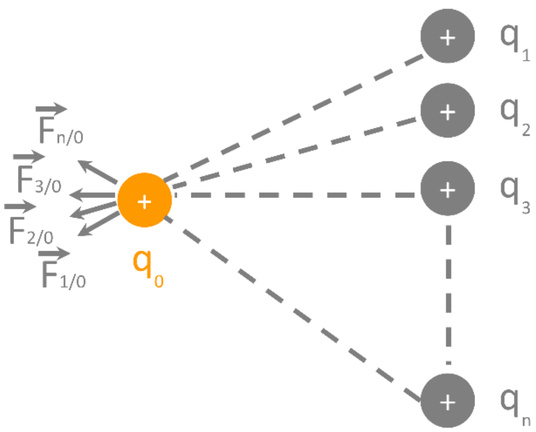


Figure 7 The principle of superposition

<u>Problem:</u>

Three points charges are located in a plane:

- $q_1 = +2\ \mu C$ at $\vec{r}_1 = 2\,\vec{\imath}$ (cm).
- $q2 = -3\ \mu C$ at $\vec{r}_2 = 7\vec{\imath} + 4\vec{\jmath}$ (cm).
- $q3 = +4\ \mu C$ at $\vec{r}_2 = 5\vec{\imath} - 3\vec{\jmath}$ (cm).

Calculate the net electrostatic force on q_3.

<u>Problem:</u>

Four points charges are located at the corners of a square of side a = 5 m:

1- $q_1 = 2\ \mu C$ at (0,0).
2- $q_2 = -2\ \mu C$ at (5 cm, 0).
3- $q_3 = 2\ \mu C$ at (5 cm, 5 cm).
4- $q_4 = 2\ \mu C$ at (0, 5 cm).

Calculate the net electrostatic force on $q1$ due to the other three charges.

<u>Problem:</u>

Four point charges are located at the corners of a square of side a = 5 m:

1- $q_1 = 5\ \mu C$ at (0 cm,0).
2- $q_2 = -6\ \mu C$ at (1 cm, -1).

3- $q_3 = 7$ µC at (5 cm, 3 cm).

4- $q_4 = 2.5$ µC at (1, 5 cm).

Calculate the net electrostatic force on $q1$ due to the other three charges.

2.5. Electrical field:

It is clear that Coulomb's law shows that the effect of electric force is an instantaneous effect, not related to time. This represents a deep problem in physics, as the existence of an instantaneous effect of a force not related to time completely contradicts physical concepts.

The English physicist, Michael Faraday(Faraday, 1991), addressed this problem by proposing another mechanism to explain the instantaneous effect of electric forces. According to Faraday, an electric charge q_0 creates lines of force around it; these lines are called the electric field. This electric field spreads throughout the universe, and the electric charge q_0 is considered the source of this electric field. The expression for this electric field is given as follows:

$$\vec{E}_0 = K\,\frac{q_0}{r^2}\,\vec{u}$$

In the case of an electric charge q at a point in space, the interaction between the lines of force, or as we call it, the electric field, leads to the instantaneous appearance of the electric force. Thus, the force is the result of the interaction of the electric field and the electric charge.

$$\vec{F}_0 = q\,\vec{E}_0\vec{u}$$

2.5.1. The lines of the electrical field:

Electric field lines represent the direction and intensity of an electric field around an electrical charge and have the following characteristics:

1. Direction: They start from positive charges (or infinity) and converge toward negative charges (or infinity).
2. Density and Magnitude: The density of field lines in a certain area is directly proportional to the magnitude of the electric field in that area. When the lines are closely spaced, the field is strong; when they are farther distanced, the field is diminished.
3. Never Cross: Electric field lines never intersect each other.

The electric field lines are a visual tool to represent the direction and strength of an electric field in space.

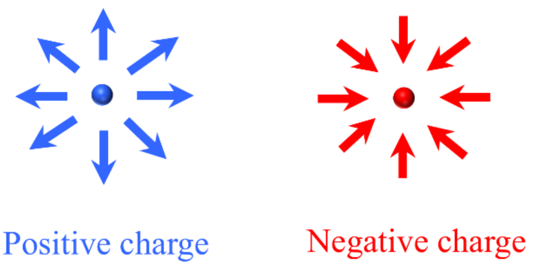


Figure 8 Line of the electric field

2.5.2. Continuous distributions of charges:

In the abovementioned discussion, we dealt with electrical forces and fields that are created from separated charges. However, in many real applications, the electrical charges are so close and numerous. For example, a metallic sphere S (see Figure 9.) has a charge of -2 nC, this charge is due to the existence of 12 10^9 electrons. If we try to study the electrical field created from this huge number of electrons in a point M (x,y,z) from the sphere, it is not possible to calculate the electrical field created from each electron in sphere S at the point M. Therefore, we suppose that the electrical charges continue. We will study below the continuous distributions of electric charge, including linear, surface, and volume charge distributions.

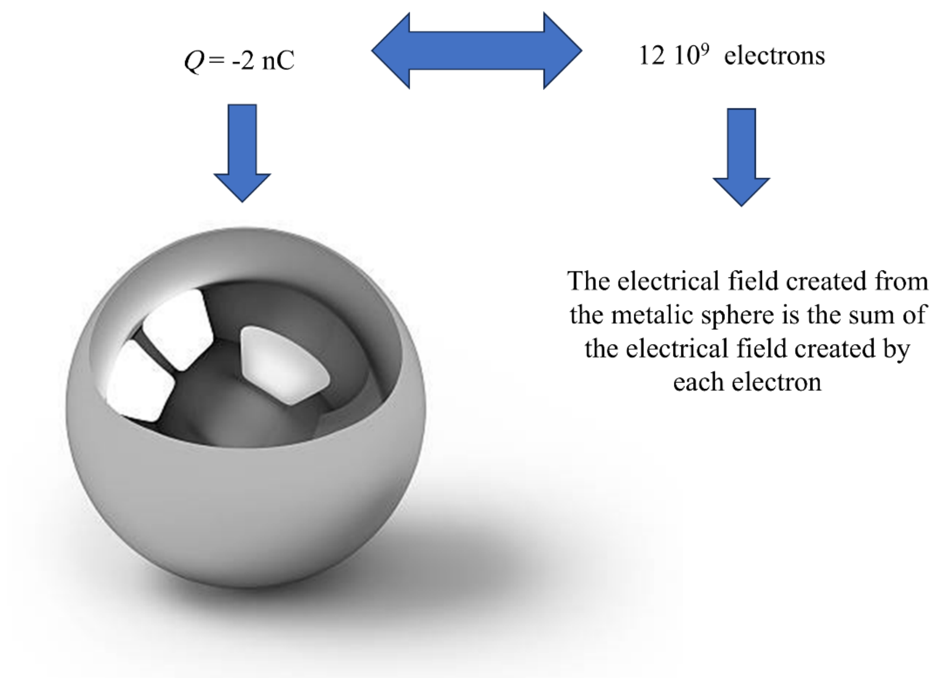


Figure 9 Continuous distributions of charges

2.5.3. The linear distribution of electrical charges:

Let us first take a piece of metal with a length of L as shown in the figure below, and neglect the width of this piece. Let us assume that this piece contains a charge Q uniformly distributed along the length L of this piece.

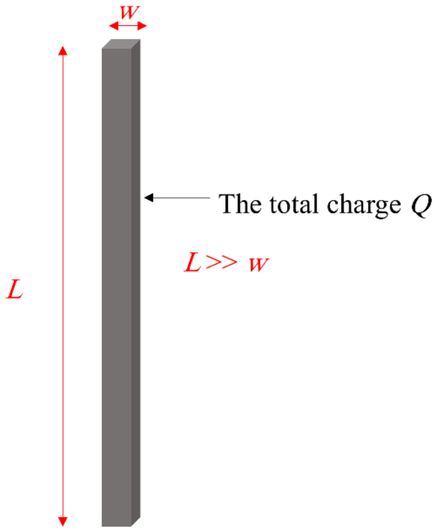


Figure 10 A piece of metal contains a charge Q

We can now define a variable λ as the density of electric charge, which is given by the following mathematical equation:

$$\lambda = \frac{dQ}{dL}$$

Since the electric charge Q is uniformly distributed along the length L of the metal piece, the value of λ remains constant.

λ can also be defined as the linear charge, and its unit in the International System of Units is (C/m). The linear charge can take negative or positive values depending on the sign of the charge distributed along the length L.

We can define an infinitesimal amount of the total charge dQ as follows:

d$Q = \lambda\, dL$

where dL is an infinitesimal quantity of the length L.

2.5.4. The surface distribution of electrical charge:

When a charge Q is distributed over an area S, let's say it's the area of a metal surface, we can define another physical quantity σ, which is the surface charge density or surface charge, as follows:

$$\sigma = \frac{dQ}{dS}$$

The unit of the surface charge density is (C/m^2).

The sign of surface charge density is also related to the sign of the electric charge distributed on the surface S. When the charge is positive, the surface charge density is positive, and when the charge is negative, the surface charge density is negative.

We can also define an infinitesimal amount of the total charge as follows:

$dQ = \sigma\, dS$

Where dS is an infinitesimal amount of the surface S.

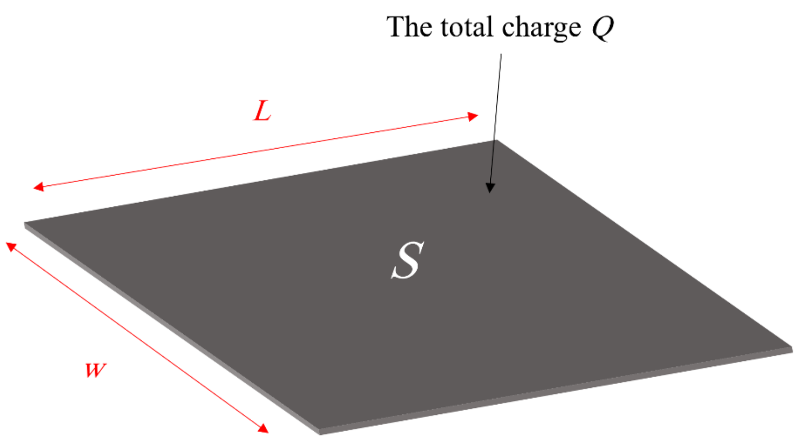


Figure 11 Charged surface

2.5.5. The volume distribution of electrical charge:

When distributing a charge Q over a volume, we can define another physical quantity, which is the volumetric charge density or volumetric charge, as follows:

$$\rho = \frac{dQ}{dV}$$

The unit of this physical quantity is C/m^3.

Similarly, the sign of this quantity is related to the sign of the charge. When the charge is positive, the volumetric charge density is positive, and when the charge is negative, the volumetric charge density is negative.

We can also formulate any infinitesimal amount of the total charge as follows:

$dQ = \rho\, dV$

The charge densities defined above will play a fundamental role in our analysis of many electrical phenomena, such as the electric fields produced by a piece of metal of negligible width, an electrically charged surface of infinite area, or a charged cylinder.

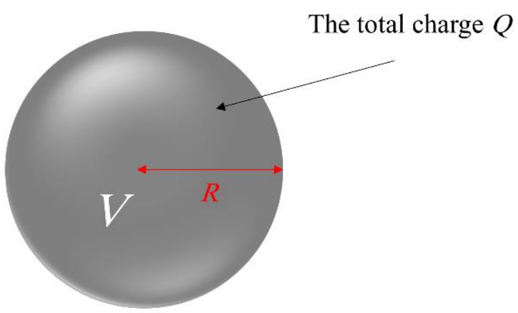


Figure 12 Charged volume

Since our study will deal with calculating the effect of a continuous electric charge, we will have to use differential and integral calculus. It would be good here to provide a review of the knowledge acquired about differential and integral calculus.

2.6. A brief overview of calculus:

The first knowledge of differential and integral calculus goes back to the pioneering work of the Greek mathematician Archimedes(Lévy, 2011) while trying to calculate the surface of a disc. Archimedes was a brilliant mathematician and geometer, and calculating the area of non-polygonal shapes, such as the disc, was one of the challenges he faced. Archimedes relied on approximating the circular shape to be measured using polygons. By increasing the number of polygons, the space between the polygon and the disc is exhausted, thus approximating the area of the circle.

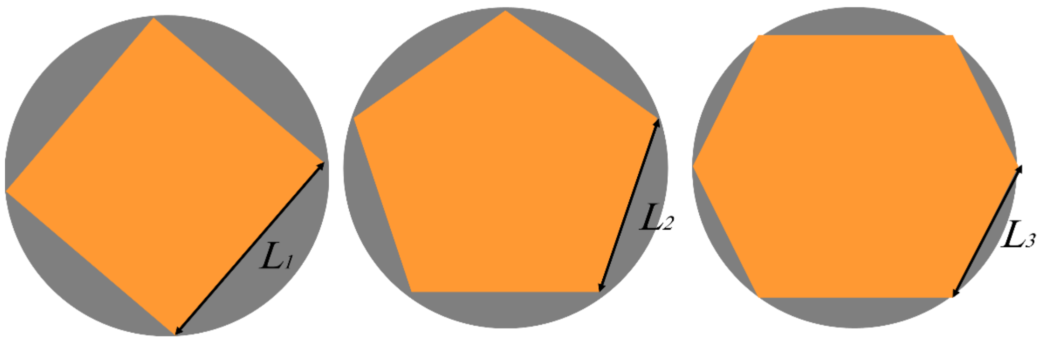


Figure 13 Approximation of the disc surface

As you can see in Figure 13, we can cover a larger area of the disc the more we increase the number of sides (or in other words, the more we decrease the length of the side 'L' of the polygon). In fact, what Archimedes arrived at was the implicit concept of the limit. ***By reducing the length of the side by enough, the area of the polygon will equal the area of the disc***.

After Archimedes, the luster of these ideas faded for centuries in Europe. In contrast, the Islamic world and India made essential contributions to this field:

In the Islamic world, the mathematician Al-Khwarizmi developed algebra and was able to formulate everyday problems in the form of algebraic equations.

In the European world, the mathematician Descartes was able to connect the two extremes of mathematics: algebra and geometry. While the scientists of that age viewed each branch as independent of the other, Descartes proved that we can draw mathematical equations geometrically and formulate curves according to algebraic equations.

In the late Middle Ages and the beginning of the European Renaissance, urban and technological developments led to the emergence of many problems related to motion, variation, calculating areas, and finding the tangents to curves. Scientists such as Nicola Orsme, the Chiral School in India, and later Cavalieri, Fermat, Descartes, Pascal, and Barrow made groundbreaking contributions:

Fermat developed a method for finding the maximum and minimum values of functions (similar to taking the derivative and setting it equal to zero) and a method for finding tangents to curves.

Barrow was Newton's teacher and realized an inverse relationship between the problem of finding areas (integration) and the problem of finding tangents (differentiation), bringing him very close to the fundamental theorem. However, the deep concepts of this mathematics did not develop until centuries later due to the pioneering work of both the physicist Newton and the mathematician Leibniz.

In the second half of the seventeenth century, independently, Isaac Newton in England and Gottfried Wilhelm Leibniz in Germany synthesized the scattered ideas of calculus and integral calculus and developed them into a powerful, integrated mathematical system. Although they worked concurrently, each had its own approach to introducing calculus.

<u>Newton and Leibniz work</u>(Reyes, 2004)<u>:</u>

Newton's interest was focused on calculating speeds and distances. For example, to calculate the distance traveled by an object moving at a constant speed, say 2 m/s, over a period of 20 seconds, we multiply the speed by the time. The result is the area of the rectangle below the velocity graph, as shown in the following figure. Calculating distances is similar and easy if the speed of the moving object is constant, but the problem lies in calculating distances traveled if the speed of the moving object is varying.

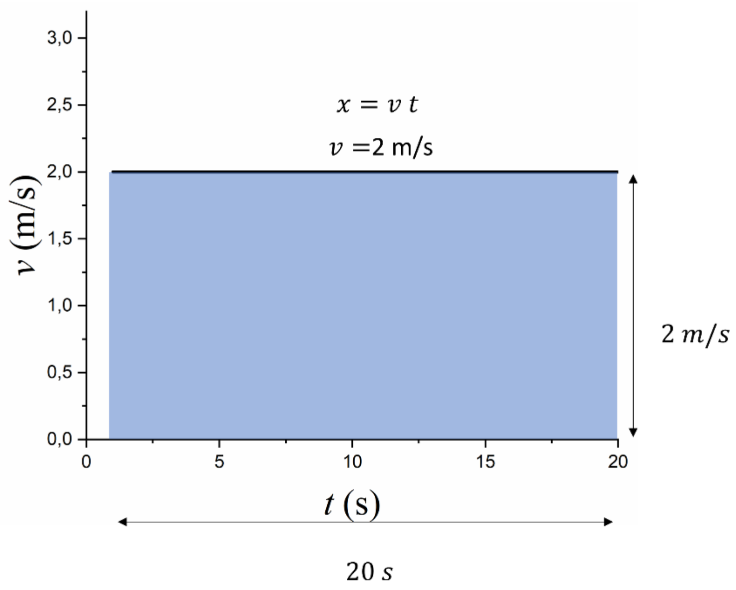


Figure 14 The variation of the velocity as a function of time

Let's take the following example:

A moving object moves at a speed that varies with time according to the following equation:

$$v(t) = t^2$$

To calculate the distance traveled by the object over a time period from t_a to time t_b, we should calculate the area under the curve of the function.

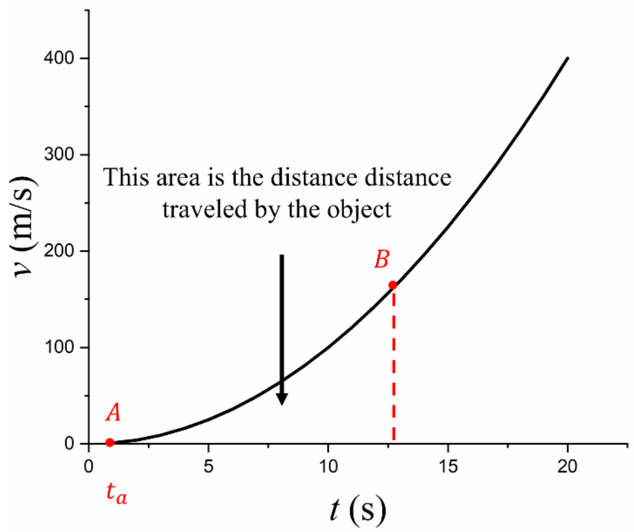


Figure 15 The variation of the velocity as a function of time

This geometric problem is similar to the one Archimedes encountered while working on calculating the area of a disc.

Newton used a similar approach to calculate the area under the curve. He approximated the area under the curve using rectangles of equal width, Δx , while the length of the rectangle is the velocity at instant t ($v(t)$).

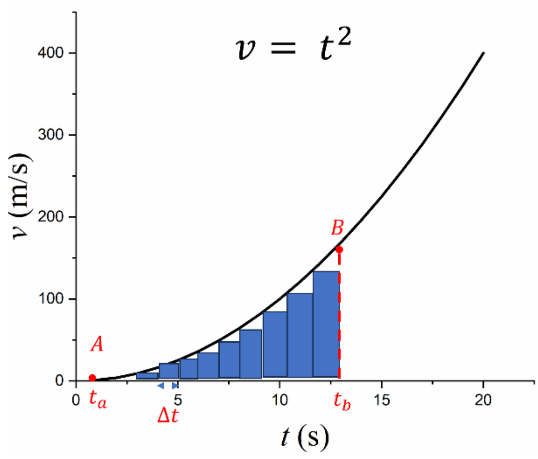


Figure 16 Approximation of the surface under the curve

The distance traveled will be approximately the sum of the areas of the rectangles.

$$\Delta x = x_b - x_a = \sum_{i}^{n} v(t_i)\, \Delta t$$

Although this is a good approximation, it is clear that the area under the curve is not exactly equal to the sum of the rectangles.

To increase the accuracy of the approximation, the length of the side must be reduced, just as Archimedes did when trying to approximate the area of a disk.

When the length of the side is reduced enough, the area of the sum of the rectangles equals the area under the curve. The value Δt took on another definition, dt, which is the ***infinitesimal difference***, while Leibniz's symbol for the sum was the integral $\int$. This symbol was in fact, just the way Leibniz wrote the first letter of the word **sum**.

Therefore, the expression of the distance Δx becomes:

$$\Delta x = x_b - x_a = \int_{t_a}^{t_b} v(t)\, dt$$

Newton didn't care much for the mathematical definition of infinitesimals, and the mathematics he derived was able to analyze the motion of bodies and planets well. On the other hand, Leibniz's definition of infinitesimal numbers presented a significant problem. He considered a quantity like dT to be so small that it was smaller than any positive real value but bigger than zero. This definition blatantly contradicted the definition of the real numbers, since between every real number and another real number there are an infinity of real numbers. This posed a difficult mathematical problem, and despite the excellent performance of calculus in both mathematics and physics, a deep understanding of it required the development of the concept of limits by the genius Cauchy.

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Cauchy developed the concept of limits, a revolutionary approach to a precise and rigorous understanding of integration.

The integration of the function $v(t)$ becomes:

$$\lim_{\Delta t \to 0} \sum_{i}^{n} v(t_i)\, \Delta t \;=\; \int_{t_a}^{t_b} v(t)\, dt$$

This formula, which relies on the limit of Riemann sums, is the essence of the precise definition of definite integrals, and this method owes its rigor to the work of Cauchy in the early nineteenth century.

<u>Some primitive functions:</u>

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C \quad \text{for } n \neq -1$$

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int e^x\, dx = e^x + C$$

$$\int a^x\, dx = \frac{a^x}{\ln a} + C \quad \text{for } a > 0, a \neq 1$$

$$\int \sin x \ dx = -\cos x + C$$

$$\int \cos x \ dx = \sin x + C$$

$$\int \sec^2 x \ dx = \tan x + C$$

$$\int \csc^2 x \ dx = -\cot x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} \ dx = \sin^{-1} x + C$$

$$\int \frac{1}{1+x^2} \ dx = \tan^{-1} x + C$$

$$\int \frac{1}{x\sqrt{x^2-1}} \ dx = \sec^{-1}|x| + C$$

$$\int \tan{(x)} \ \ dx \ = \ -\ln\ |\cos{\{x\}}| \ + \ C$$

$$\int \ln x \ dx = x\ln x - x + C$$

2.5.6. The electrical field created from a continuous distribution of charge:

We suppose that we have an object A (see Figure 17) charged by a continuous electrical charge Q.

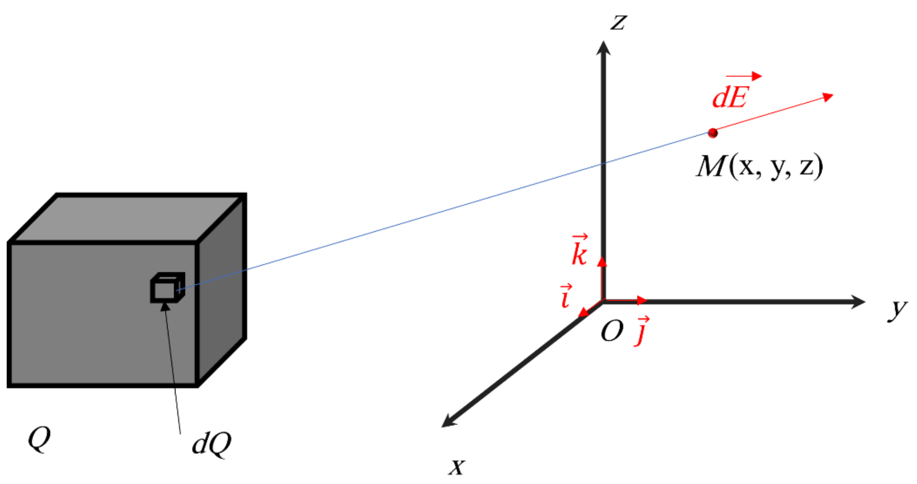


Figure 17 The electric field created from a continuous charge distribution

To calculate the electrical field created from this charge at the point M (x,y,z), we can first take an infinitesimal quantity from the total charge noted as dQ; this infinitesimal charge dQ creates an infinitesimal electrical field $d\vec{E}$ given as:

$$d\vec{E} = K \frac{dQ}{r^2} \overrightarrow{u_r}$$

Where, $\overrightarrow{u_r}$ is the unit vector of the vector r, which is oriented from dQ to the point M. The total electrical field created from all other elements of object A is the following integral:

$$\vec{E} = \int\int\int K \frac{dQ}{r^2} \overrightarrow{u_r}$$

To calculate this sum (integral), we need to present the quantity dQ as a function of the density of charge ρ. Also, we need to present the unit vector u as a function of the Cartesian, polar, cylindrical, or spherical coordinates.

2.5.7. The electrical field created by a circle:

In this section, we deal with the electrical field created from a circle of radius R charged with a linear density λ on a point M as shown in Figure 18.

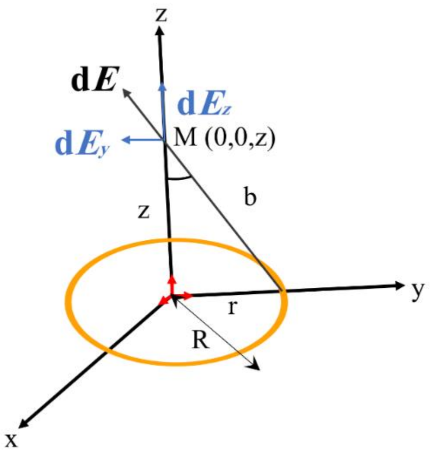


Figure 18 The electric field created by a circle

<u>Solution</u>

We take an infinitesimal charge dq from the total charge of the circle Q. The relation between the infinitesimal charge dq of the charge and the linear density λ is:

$$dq = \lambda \, dl.$$

Where dl is an infinitesimal arc from the total arc of the circle l.

$$dq = \lambda \, r \, d\alpha.$$

This infinitesimal charge will induce the creation of an infinitesimal electrical field $d\vec{E}$ with two compounds along the z and y axes.

$$d\vec{E} = d\overrightarrow{E_y} + d\overrightarrow{E_z}$$

We can neglect calculating the electric fields in the direction y due to the symmetry of the electrical field with the *y* direction. For the compounds of the electric field in the direction *z*, it takes the following form:

$$d\vec{E} = k\frac{dq}{b^2}\cos(\theta)\ \vec{k}$$

$$d\vec{E} = k\frac{r\lambda\, d\alpha}{b^2}\cos(\theta)\ \vec{k}$$

We can also write the cos(θ) as a function of *b* and *r*, therefore, the relation of the infinitesimal electrical field becomes:

$$d\vec{E} = k\frac{r\lambda z\, d\alpha}{b^3}\vec{k}$$

The total electrical field created from the circle is:

$$d\vec{E} = \int_0^{2\pi} k\frac{zr\lambda\, d\alpha}{(z^2+r^2)^{3/2}}\ \vec{k}$$

Therefore,

$$d\vec{E} = k\frac{zr\lambda\, 2\pi}{(z^2+r^2)^{3/2}}\ \vec{k}$$

2.5.8. The electric field created by a disc:

A disc of a center o and a radius *R* charged uniformly by a surface density charge σ>0 as shown in Figure 19.

Calculate the electrical field created from this disc at the point *M* (*0*,*0*,*z*) a function of *z*.

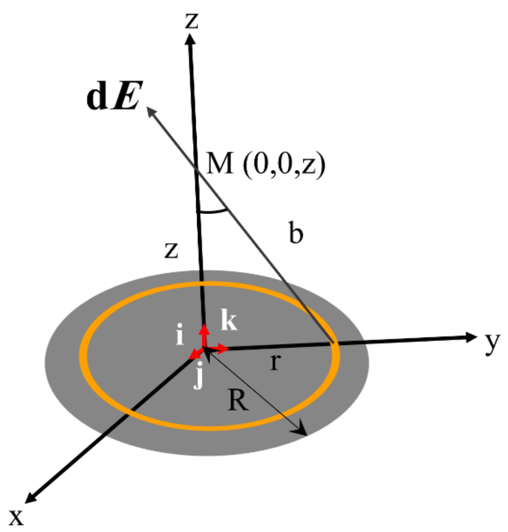


Figure 19 The electric field created by a disc

<u>The solution:</u>

We start first by taking an infinitesimal electrical charge dq, which is distributed on a small ring in the disc as shown in Figure 20.

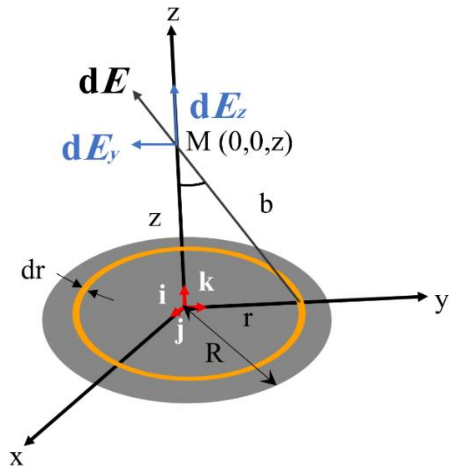


Figure 20 The infinitesimal electric field created by a disc

The relation between the infinitesimal quantity dq of the charge and the surface density σ is:
dq = σ ds.
where ds is an infinitesimal surface, which is the surface of the ring.
ds = 2 π r dr.
Therefore: dq = σ 2 π r dr.
This infinitesimal charge will induce the creation of an infinitesimal electrical field $d\vec{E}$ with two compounds along the z and y axes.

$$d\vec{E} \ = d\overrightarrow{\mathrm{E}_y} + d\overrightarrow{\mathrm{E}_z}$$

We can neglect calculating the electric fields in the y direction due to the symmetry of these fields. For the compounds of the electric field in the direction z, it takes the following form:

$$d\vec{E} = K\frac{dq}{b^2}\cos(\theta)\ \vec{k}$$

θ represents the angle between the Z axis and the direction of the infinitesimal electric field, as shown in figure 20.
By using the relation of the infinitesimal electric charge, we have:

$$d\vec{E} = K\frac{2\ \pi\ r\sigma\ dr}{b^2}\cos(\theta)\ \vec{k}$$

We can also write the cos(theta) as a function of b and r, therefore, the relation of the infinitesimal electrical field becomes:

$$d\vec{E} = K\frac{2\ \pi\ r\sigma\ dr}{b^2}\frac{\mathrm{z}}{b}\ \vec{k}$$

Also,

$$d\vec{E} = K\frac{2\,\pi\, z\, r\sigma\, dr}{b^3}\,\vec{k}$$

The total electric field created by the disc (***E***) is the integral of the partial electric field $d\vec{E}$ created by the infinitesimal ring of the disc.

$$d\vec{E} = \int_0^R K\frac{2\,\pi\, z\, r\sigma\, dr}{b^3}\,\vec{k}$$

Therefore, we obtain the total electrical field created from the disc at the point M(0,0,z):

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left[\frac{z}{|z|} - \frac{z}{\sqrt{z^2 + R^2}}\right]\vec{k}$$

2.6. Electrostatic potential:

We have previously dealt with electrostatic forces and electrostatic fields resulting from the interaction of electric charges and the electric charges themselves.

Both physical quantities, "electrostatic forces and electrostatic fields", are vector physical quantities.

In some of the problems we encountered, calculating these quantities was a complex matter because it required many mathematical calculations arising from the differential nature of the physical problem at hand and also arising from the vector nature of these two quantities.

In fact, obtaining these two physical quantities through scalar calculations instead of vector calculations may facilitate the solution of many of the physical problems that we will encounter in this course.

We consider that in space A, there is an electric field resulting from a constant electric charge Q_0. Any other electric charge in this space, let's call it Q, will be subject to an electric force **F** given by the following relation:

$$\vec{F} = Q\,\vec{E}$$

If the two charges Q and Q_0 are of the same sign, this will lead to the movement of the electric charge Q in the direction of the electric force $\vec{F}$**.** To stop the movement of this electric charge, we need to apply another force opposite to the force $\vec{F}$ and has the same magnitude.

The new force, let's call it $\boldsymbol{F}_d$, has the following expression:

$$\overrightarrow{F_d} = -Q\,\vec{E}$$

For an infinitesimal displacement d*l*, the infinitesimal work resulting from the force $\overrightarrow{F_d}$ is given by the following relation:

$$w = -Q \int_A^B \overrightarrow{E}\, d\vec{l}$$

The following integral $\int_A^B \overrightarrow{E}\ d\vec{l}$ is called the the circulation of the electric field from point A to point B.

This circulation is conservative; that is, its value is constant and not related to the path followed In case the charge Q is equal to one Coulomb, the work done is called the electromotive force:

$$w = \int_A^B \overrightarrow{E}\, d\vec{l}$$

In the previous example, we had the electric circulation as a constant value independent of the path. That means the work done to move an electrical field from a point A to a point B is also independent of the path. The expression, $-\overrightarrow{E}\ d\vec{l}$ called ***the variation of electrical potential dV***, and it is a scalar quantity. Therefore, the electrical field is the derivative of the electrical potential:

We can rewrite the expression of the work as follows:

$$w = -Q \int_A^B \overrightarrow{E}\, d\vec{l} = \int_A^B dV = -Q\,(V_B - V_A)$$

We can now define the difference in electric potential (EP) is the work required for a single charge to move it from point A to point B.

2.6.1. The electrical potential created from a charge q:

The electrical field created from a charge q is given as:

$$\vec{E} = K\,\frac{q}{r^2}\,\vec{u}$$

Because the electrical field and the displacement dl have the same direction the relation of the difference potential becomes:

dV = - E dl

And dl is parallelled to dr,

Therefore;

$$dV = \frac{dr}{4\,\pi\,\varepsilon_0}\,\frac{q}{r^2}$$

The expression of electrical potential becomes:

$$V = \frac{1}{4\,\pi\,\varepsilon_0}\,\frac{q}{r}$$

2.6.2. The calculation of the electrical field from the electrical potential:

We have seen before that the relation between the difference of potential and the electrical field is:

$$dV = -\vec{E}\, d\vec{l}$$

If we suppose that the charge q is moving in the 3D space, therefore,

$$d\vec{l} = dx\,\vec{\imath} + dy\,\vec{\jmath} + dz\,\vec{k}$$

We can now obtain the relation between the electrical field and the electrical potential as follows:

$$\vec{E} = -\vec{\nabla} V = -(\frac{\partial V}{\partial x}\vec{\imath} + \frac{\partial V}{\partial y}\vec{\jmath} + \frac{\partial V}{\partial z}\vec{k})$$

<u>Example:</u>

Find the relation of the electrical field if V the electrical potential given as:

V (x, y, z) = 3xy + $2z^2$

<u>Solution:</u>

Using the above relation:

$$\vec{E} = 3y\,\vec{\imath} + 3x\,\vec{\jmath} + 4z\,\vec{k}$$

2.6.3. The electrical potential created from a set of electrical charges:

If we suppose that we have n electrical charges q1, q2, q3, …, qn

The electrical potential created from this set is the sum of the electrical potential created from each charge:

$$V_T = \sum_{q=1}^{n} V_i$$

2.6.4. The electrical potential created from a continuous distribution of electrical charges:

If we have a continuous distribution Q of electrical charges, the electrical potential created from this distribution is the integral of the electrical potential created from an infinitesimal charge dq.

$$V = \int \frac{1}{4\,\pi\,\varepsilon_0}\frac{dq}{r}$$

We return here to remind you of the importance of using electric potential in calculations because it is a scalar quantity compared to the electric field, which is a vector quantity. After finding the electric potential (EP) relationship, we can apply the gradient operator to find the electric field.

<u>The work :</u>

When a charge is among a configuration of charges, it experiences an electric force exerted by the other charges. When the charge moves, the electric force induces a work. The work is a is a transfer of energy by means of a force. For a constant force that is exerted by an agent on a particle that has a rectilinear displacement dl, the work is given by:

$$W = F\, dl\, cos\,(\theta)$$

The work is positive if θ is between 0 and 90° and negative if theta is between 90 and 180°

2.7. Electric dipole:

An electric dipole is an electrostatic system consisting of two equal and opposite electric charges, +q and -q, separated by a small distance d. Although the electric dipole has zero charge, it creates an electric field between the two charges and possesses potential energy when placed in an external electric field due to the separation of its charges.

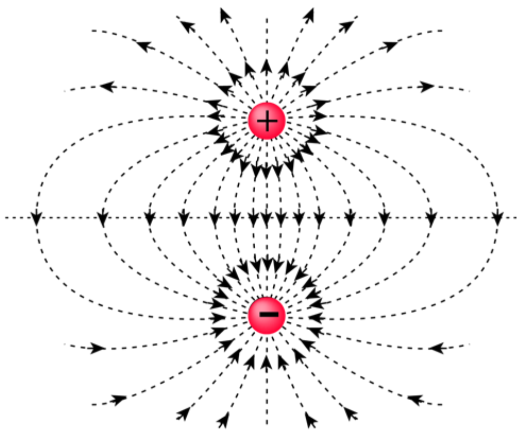


Figure 21 Electric dipole

The key characteristic of an electric dipole is its **electric dipole moment**, denoted by the vector $\vec{p}$. The magnitude of the dipole moment is given by the product of the charge magnitude and the separation distance:

$$\vec{p} = q\, \vec{d}$$

2.8. The flux of the electrical field:

Electric flux is a measure of the total electric field $\vec{E}$ passing through a given surface $\vec{S}$. The mathematical expression of the electric field is defined as the integral of the vector product of the electric field and an infinitesimal surface element vector of the surface S ($\overrightarrow{dS}$).

$$\Phi_E = \oint \vec{E}\ \overrightarrow{dS} = E\ dS \cos(\theta)$$

Here, E is the electrical field, dS is an infinitesimal surface, θ is the angle between the vector $\vec{E}$, and the vector $\overrightarrow{dS}$, remember that the vector $\overrightarrow{dS}$ is perpendicular to dS (the infinitesimal surface element).

If the angle θ is zero (that means the electrical field is in the same direction of the dS) the electrical field become 1 (that means the electrical field is in the opposite direction to dS), the electrical flux becomes -1.

If the angle is pi/2 the that means the electrical field is perpendicular to the dS, the electrical flux becomes nil.

Example:

Calculate the electric flux through a sphere that has a radius of r = 2 m and carries a charge of q = 4 µC at the center of the sphere.

Solution:

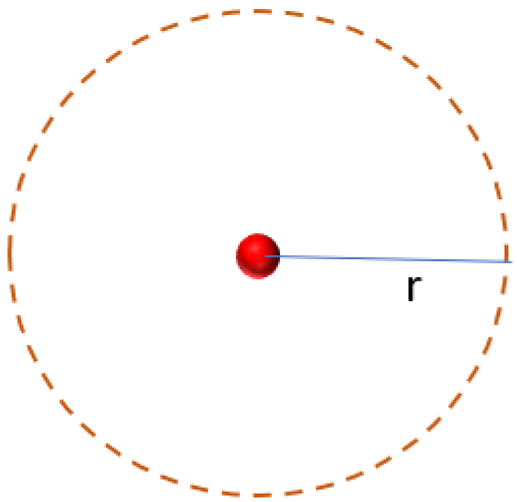


Figure 22 Electric field created from a point charge

The electric field vector is parallel to the surface vector, then:

$$\Phi_E = \oiint \vec{E}\ d\vec{S} = E \oiint dS$$

So :

$$S = \oiint dS = 4\pi r^2$$

$$\Phi_E = \frac{k\ q}{R^2}\ 4\pi r^2 = 4\ \pi\ k\ q = \frac{q}{\varepsilon_0}$$

because $k = \frac{1}{4\pi\varepsilon_0}$

2.9. Gauss theory:

Gauss's law, or Gauss's theorem, is a fundamental principle of classical electromagnetism, providing a close relationship between electric fields and the distribution of electric charges. The beginnings of Gauss's law are associated with the German mathematician and physicist Carl Friedrich Gauss (1777–1855). Gauss formulated this law in 1835, during a period that witnessed many scientific breakthroughs in the fields of electricity and magnetism. Although the law itself was not widely known at that time, it represented one of the pillars of classical physics when the great scientist Charles Maskwell incorporated it into his famous equations in 1861. Maxwell's equations combined the concepts of electricity and magnetism, and Gauss's law was one of its pillars. This new framework remained robust until the beginning of the twentieth century when scientific developments in the quantum energy field led to the emergence of a new scientific revolution: quantum mechanics, which radically reshaped our understanding of physics.

2.9.1. Concept of Electric Flux:

During its work on calculating electric fields, Carl Friedrich Gauss needed to develop a new concept, called now ***the electric flux.*** Electric flux is a fundamental concept in electromagnetism that represents the measurement of the electric field passing through a given surface. This concept was already known in other fields of physics, such as fluid dynamics, where it was used to calculate the amount of flux through surfaces. However, Gauss was the first to apply it to electricity in a precise quantitative manner.

For a small area element dA, the electric flux dΦ_E is defined as:

$$d\emptyset_E = \vec{E}\, d\vec{A}$$

$\vec{E}$: The electric field vector.

$d\vec{A}$: The vector area element, with magnitude equal to the area and direction normal to the surface.

For a closed surface, the total electrical flux passing through the area A is given as:

$$\emptyset_E = \int \vec{E}\, d\vec{A}$$

2.9.2. Units

Electric flux is a scalar quantity with units of volt-meters (V·m) or newton-meters squared per coulomb ($N \cdot m^2/C$).

Physical Interpretation of the mathematical expression of the electric flux:

If the electrical field $\vec{E}$ is perpendicular to the surface A, the electrical flux becomes the product of the magnitude of the electrical field and the surface area of A. In this case, the electrical flux is maximal. If the electrical field $\vec{E}$ is parallel to the surface A, the electrical flux becomes nil. For a closed surface, flux is positive when field lines exit the surface and negative when they enter.

2.9.3. Applications of Gauss's Law:

Gauss's law is very effective for calculating electric fields arising from highly symmetric geometric structures, such as spherical, cylindrical, or plane objects, or even systems with linear charge distributions, such as a piece of iron of negligible thickness.

How do we use the Gauss theory?

Step 1: Identify the Symmetry of the Charge Distribution

To apply Gauss's Law effectively, the charge distribution should have ***high symmetry***.

Step 2: Choose a Gaussian Surface

The Gaussian surface should be closed (enclose charge completely), and align with the symmetry of the charge distribution.

Step 3: Evaluate the Flux Integral

If the electrical field is constant over the Gauss surface, then the Gauss law becomes,

$$\emptyset_E = \int \vec{E}\, d\vec{A}$$

Step 4: Compute the Enclosed Charge Q_{enc}

If the charge is a point charge, $Q_{enc} = q$.

If the charge is distributed, integrate over volume or surface ... etc

Step 5: Solve for E

After substituting Q_{enc}, solve for E.

<u>Examples of application of Gauss' theory:</u>

1- A sphere of radius R carries a uniform volume charge density ρ. Find the electric field:

(a) Inside the sphere ($r < R$)

(b) Outside the sphere ($r > R$)

Using Gauss theory

$$\oint E \cdot dA = \varepsilon_0 Q$$

(a) Inside the sphere (r<R):

Consider a Gaussian sphere of radius r.

The enclosed charge is:

$$Q = \rho V_r$$

So,

$$Q = \rho \cdot \frac{4}{3}\pi r^3$$

From Gauss' law:

$$E\, 4\,\pi r^2 = \rho \cdot \frac{4}{3\varepsilon_0}\pi r^3$$

Therefore:

$$E = \frac{\rho\, r}{3\varepsilon_0}$$

(b) Outside the sphere (r>R):

The total charge enclosed is:

$$Q = \rho \cdot \frac{4}{3}\pi R^3$$

By Gauss's Law:

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Electric Field of a Uniformly Charged Cylinder

A long cylinder of radius R has a uniform volume charge density ρ. Find the electric field:

(a) Inside the cylinder (r < R)

(b) Outside the cylinder (r > R)

Using Gauss's Law, we choose a cylindrical Gaussian surface of radius r and length L:

(a) Inside the cylinder (r < R)

The charge enclosed is:

$$Q_{\text{enc}} = \rho \cdot \pi r^2 L$$

Applying Gauss's Law:

$$E(2\pi rL) = \frac{\rho\pi r^2 L}{\varepsilon_0}$$

Solving for E:

$$E = \frac{\rho r}{2\varepsilon_0}$$

(b) Outside the cylinder (r>R)

Total charge enclosed is

$Q = \rho\pi R^2 L,$

So,

$$E(2\pi rL) = \frac{\rho\pi R^2 L}{\varepsilon_0}$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\rho\pi R^2}{r}$$

$$E = \frac{\rho R^2}{2\varepsilon_0 r}$$

Which decreases as 1/r

Electric Field of a Charged Spherical Shell

A thin spherical shell of radius R carries a surface charge density σ. Find the electric field:

(a) Inside the shell (r < R)

(b) Outside the shell (r > R)

Solution:

Using Gauss's Law:

(a) Inside the shell (r < R)

Since there is no enclosed charge,

$$E = 0$$

(b) Outside the shell (r > R)

Total charge is

$Q = \sigma 4\pi R^2$

Using a Gaussian sphere of radius r,

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Electric Field of a Uniformly Charged Cylindrical Shell:

A long thin cylindrical shell of radius R carries a surface charge density σ.

Find the electric field:

(a) Inside the shell (r < R)

(b) Outside the shell (r > R)

Solution

Using Gauss's Law, take a cylindrical Gaussian surface of radius r and length L:

(a) Inside the shell (r<R)
Since there is no enclosed charge,

$$E = 0$$

(b) Outside the shell

The total charge enclosed is

$Q = \sigma(2\pi RL)$

Applying Gauss's Law:

$$E(2\pi rL) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{\sigma R}{\varepsilon_0 r}$$

Which behaves as 1/r.

2.10. Conductors at equilibrium:

In electrostatic equilibrium, electrical conductors exhibit a set of distinct properties that govern the behavior of electric fields, charge distribution, and electrostatic potentials. These principles, reflected in Gauss's law and charge conservation, form the basis of technologies ranging from Faraday cages to lightning protection systems. This section of the course provides theoretical frameworks, experimental verifications, and practical applications to illustrate the equilibrium state of conductors.

Definition: Let us remember first that a conductor is an object, generally metallic, inside which electric charges, such as free electrons or other kinds of charges, can move easily under the effect of an electric field, even of low intensity. This electrical conductor is said to be in electrostatic equilibrium when no net movement of charges (usually electrons) occurs within it. The free electrons redistribute themselves until they cancel any electric field within the material.

Properties of a conductor in equilibrium:

1- Zero internal electric field.
2- Based on the law, $\vec{E} = -\vec{\nabla} V$, the potential is constant inside the conductor and, by continuity, on its surface. In other words, a conductor in equilibrium is an equipotential surface.
3- Since the number of protons is equal to the number of electrons, the total charge inside the conductor is zero.
4- External electrical field perpendicular to the surface.

2.10.1. Electrostatic pressure:

For a conductor with surface charge density σ, the electric field on the surface of the conductor is given by:

$$E_{\text{surface}} = \frac{\sigma}{\varepsilon_0}$$

As we mentioned before, inside the conductor, the electric field is zero.

Let us now consider a small charge element dq = σ dS, the force dF acting on it is the product of dq and the effective electric field it experiences. it is clear that the infinitesimal charge cannot exert a net force on itself; therefore, the effective field is half the total surface field:

$$E_{\text{effective}} = \frac{\sigma}{2\varepsilon_0}$$

Thus:

$$dF = dq\, E_{\text{effective}} = \sigma\, ds \frac{\sigma}{2\varepsilon_0} = \frac{\sigma^2}{2\varepsilon_0} ds$$

The pressure becomes:

$$P = \frac{dF}{ds} = \frac{\sigma^2}{2\varepsilon_0} = \frac{q^2}{2\varepsilon_0 s^2} = \sigma\, E_{\text{effective}}$$

Mechanical Stress: Electrostatic pressure mimics tensile stress in materials, attempting to expand the conductor's surface.

Discontinuity in Field: The abrupt change in electric field at the surface (from outside) creates a net outward force.

<u>Examples</u>

Charged Spherical Conductor

For a sphere with charge Q and radius R:

$$P = \frac{\sigma^2}{2\varepsilon_0} = \frac{Q^2}{32\,\pi^2\,\varepsilon_0\,\mathrm{R}^4}$$

This pressure causes mechanical stress, potentially deforming the sphere if structural strength is exceeded.

2.10.2. Capacitance of a conductor:

Capacitance C measures the charge stored per unit of potential:

$$C = \frac{Q}{V}$$

Where Q is the charge and V is the volume. This capacitance depends only on the geometry of the conductor.

Capacitance C is a positive quantity; the unit of this quantity is called the Farad (F). The farad is thus defined as the capacitance of an insulated conductor whose potential is 1 volt when it receives a charge of 1 coulomb.

The farad is a very large unit; sub-multiples are commonly used:

Microfarad: $1\mu F = 10^{-6}$ F, nano-farad: $1nF = 10^{-9}F$, picofarad: $1\ pF = 10^{-12}F$.

2.10.3. Electrostatic Influence Effect:

The electrostatic effect is the redistribution of electrical charges on an electrical conductor under the influence of an external electric field. These interactions are fundamental to understanding many electrical systems. If we place a neutral object A in an external electrical field created by object B, the charges inside object A will be redistributed.

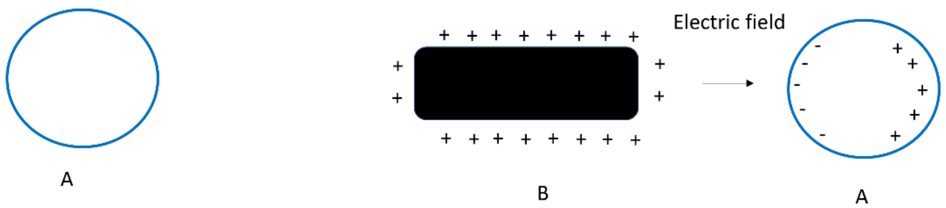


Figure 23 Electrostatic influence effect

How does this happen?

When a piece of macroscopic insulating material is placed in an external electric field (which may be generated by an electric current or a conductor), the atoms and molecules that make up this material interact with this electric field. There are two basic mechanisms for this response at the atomic level:

<u>Polar Materials:</u> These materials are composed of inherently polar molecules (such as water), due to the non-uniform distribution of electric charges. In the absence of external electric fields,

the orientations of these dipoles are random, and therefore the total dipole moment of the material is zero. When an external electric field is applied, the torque acting on each dipole causes it to tend to align in the direction of the external electric field. ***This alignment does not occur totally*** due to the random thermal motion of the molecules, but there is a tendency, or average, alignment in the direction of the field.

Nonpolar molecular materials: These materials consist of molecules that do not have permanent dipoles and have a more homogeneous charge distribution within their molecules. When an external electric field is applied, a relative displacement occurs between the center of positive charge (the nucleus) and the center of negative charge (the electron cloud) within each atom or molecule, leading to the induction of a temporary electric dipole that points in the same direction as the external field.

The effect of an external electric field on a large number of atoms or molecules that make the material leads to a phenomenon at the macroscopic level known as ***electric polarization***, symbolized as:

$$P = \frac{\sum p_i}{\sum V}$$

Where, p_i is the dipole moment of each molecule or atom in the substance. The summation is done over a large number of molecules or atoms present within the atomic volume in a small volume (ΔV). This volume is small macroscopically but very large at the microscopic level, containing a huge number of atoms or molecules.

2.10.4. Capacitor:

A capacitor is an electronic component that stores electrical energy in an electric field by accumulating positive and negative charges on two conductive objects that are generally plates elements separated by an insulating dielectric material. Its ability to store charge, quantified as capacitance, enables critical functions in electronic circuits, such as energy buffering, noise filtering, and timing control.

How does the capacitor work

Charge Storage Mechanism:

When a voltage is applied between the plates of the capacitor, electrons accumulate on one plate (negative charge), while the other plate loses electrons (positive charge). The dielectric prevents direct current flow between the two plates, which allows the charge separation. For example, the capacitor can be connected to a battery, which causes a transient current until the voltage across the capacitor matches the battery voltage.

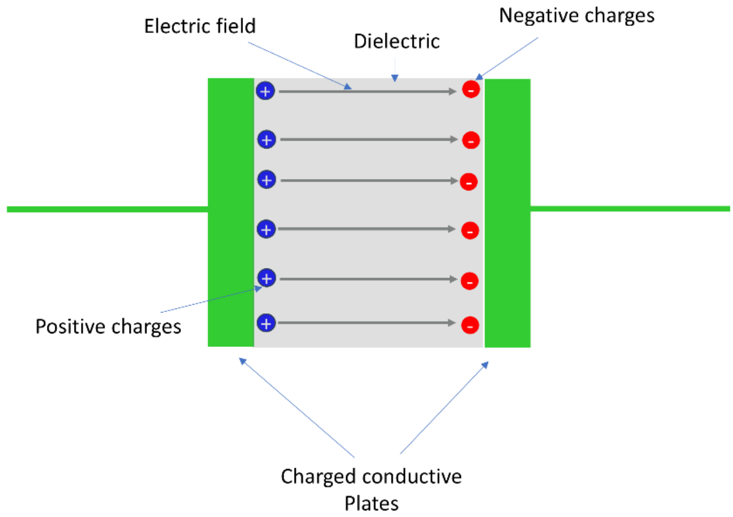


Figure 24 Capacitor

Electric Field Formation:

The charge separation between the two plates creates an electric field, storing energy as:

$$E = \frac{1}{2} C\, V^2$$

Where V is the voltage and C is the capacity of the capacitor.

The capacity of the capacitor:

The capacitance of a capacitor is defined as:

$$C = \frac{Q}{\Delta V}$$

Here, Q is the charge carried by each of the plates ($+Q$ for one and $-Q$ for the other). Capacitance is a constant specific to each capacitor. Its value depends on the shape, dimensions, and relative position of the two conductors' nature that constitute it, and the nature of the dielectric element.

The capacity of a plane capacitor:

Consider a plane capacitor, consisting of two plane conductors, carrying charges +Q and -Q respectively, with surfaces S, separated by a distance e. The distance e is very small compared to the dimensions of the two plane conductors. Due to the symmetry of the distribution, the electric field between the plates of this capacitor is uniform, and it is given by:

$$E = \frac{\sigma}{\varepsilon_0}$$

The distribution is uniform; we have:

$$\sigma = \frac{Q}{S}$$

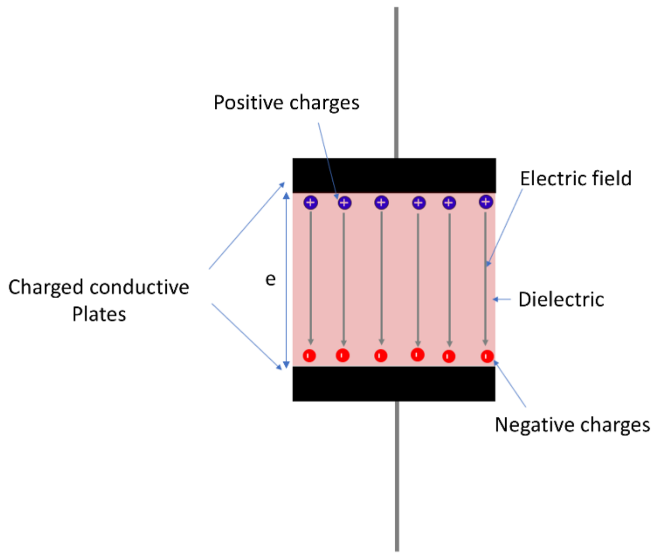


Figure 25 Plane capacitor

The relation of the capacity of the capacitor becomes:

$$C = \frac{Q}{\Delta V} = \frac{\sigma\, S}{E\; e} = \frac{\sigma\, S}{\frac{\sigma}{\varepsilon_0}\; e} = \frac{S\, \varepsilon_0}{e}$$

<u>Capacity of a spherical capacitor:</u>

A spherical capacitor (Figure 26) consists of two concentric conducting spheres. The first, for example, of radius R1, carries a positive charge of +Q and its potential is V1; the second, of radius R2 (R1 < R2), carries a charge of -Q and its potential is V2.

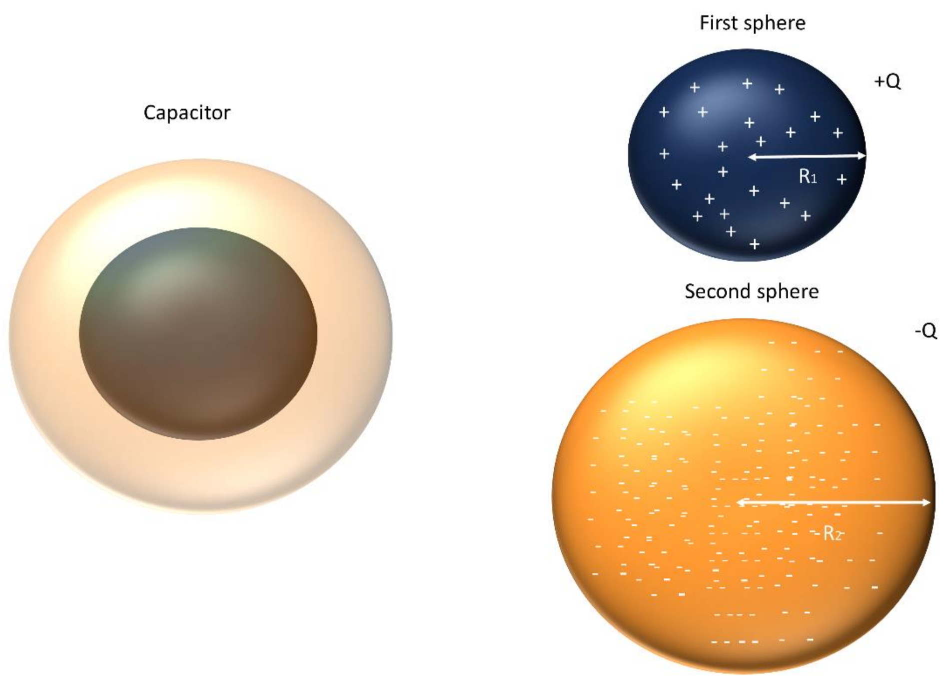


Figure 26 Spherical capacitor

Applying Gauss's theorem, we obtain the electric field between the plates of such a capacitor:

$$\vec{E} = k \frac{Q}{r^2} \overrightarrow{u_r}$$

We know that,

$$E = -\frac{dV}{dr}$$

Therefore,

$$V = \int_{V1}^{V2} -dV = k\,Q \int_{R1}^{R2} \frac{dr}{r^2}$$

Therefore, the capacity of the capacitor becomes:

$$C = 4\,\pi\,\varepsilon_0 \frac{R1\,R2}{R2 - R1}$$

2.10.5. Grouping of the capacitors:

Series Connection:

Consider the group of N capacitors in series shown in the following figure.

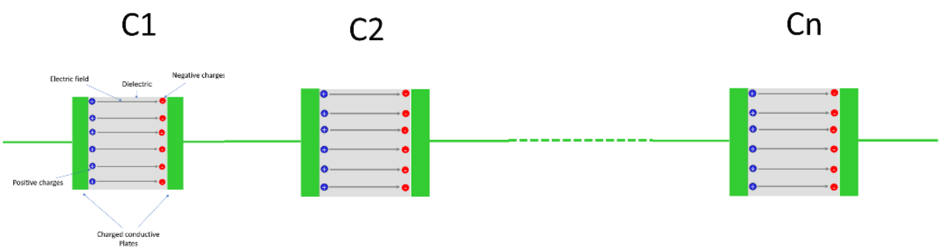


Figure 27 Grouping of capacitors "series connection"

When a potential difference AV = V0 - Vn is applied between the endpoints of the set of capacitors, the left-hand plate of the first capacitor will acquire a charge Q.

The total potential difference across the terminals of the set of capacitors can then be simply written:

$$\Delta V = (V_0 - V_1) + (V_1 - V_2) + (V_2 - V_3) + \dots + (V_{n-1} - V_n)$$

$$\Delta V = \frac{Q}{C_1} + \dots + \frac{Q}{C_n} = Q \sum_{i=1}^{n} \frac{1}{C_i}$$

Therefore:

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$$

Parallel Association:

Let there be N capacitors, placed in parallel, with the same potential difference V.

Let Qi and Ci are the electric charge and capacitance of the ith capacitor, and we have:

$$Q_i = C_i V$$

The total electric charge carried by all the capacitors is then given by:

$$Q = \sum_{i=1}^{n} Q_i = V \sum_{i=1}^{n} C_i$$

So:

$$C_{eq} = \sum_{i=1}^{n} C_i$$

Chapter 3

Electrokinetic

3. Electrokinetic

3.1 Introduction:

Our lives today depend primarily on the use of electricity to operate many electronic and electrical devices. Devices such as heaters, televisions, and washing machines cannot function without an electric current, or, more simply, without the movement of electric charges. In scientific and academic fields, the movement of this charge, plays a fundamental role in our understanding of the behavior of the materials around us and our biological nature. In biology, for example, the human nervous system consists of electrical circuits that transmit commands from the brain to the rest of the organs through the movement of charged particles like electrons or ions.

In classical physics, the science that studies the movement of these electric charges is called electrokinetics. Electrokinetic is a set of phenomena resulting from the movement of charged particles in conductor elements, particularly the electrical circuits. In this chapter we will study the fundamental principles and laws that govern the behavior of the electrical current, voltages, and charges in the electrical circuit.

3.2 Electrical conductor:

At their core, conductors are materials that allow the free movement of electric charges. Free movement is relative because electric charges generally remain subject to the influences of their environment, such as the electrical potential of atoms, and the vibrations of crystals (temperature effect). This ability is due to "l*oosely bound*" valence bond electrons in metals or mobile charge carriers in solutions, such as ionic conductors. When an electric potential difference is applied, these electrons begin to move, creating an electric current.

Examples of Conductors:

1. Metals like copper, silver, gold, and aluminum are the best conductors due to relatively free-moving electrons.
2. Plasma, which is a state of matter where ions and electrons move freely.
3. Ionic solutions like salt solutions conduct electricity through ion movement.
4. Graphene and carbon nanotubes are new 2D materials with excellent conductors at the nanoscale.

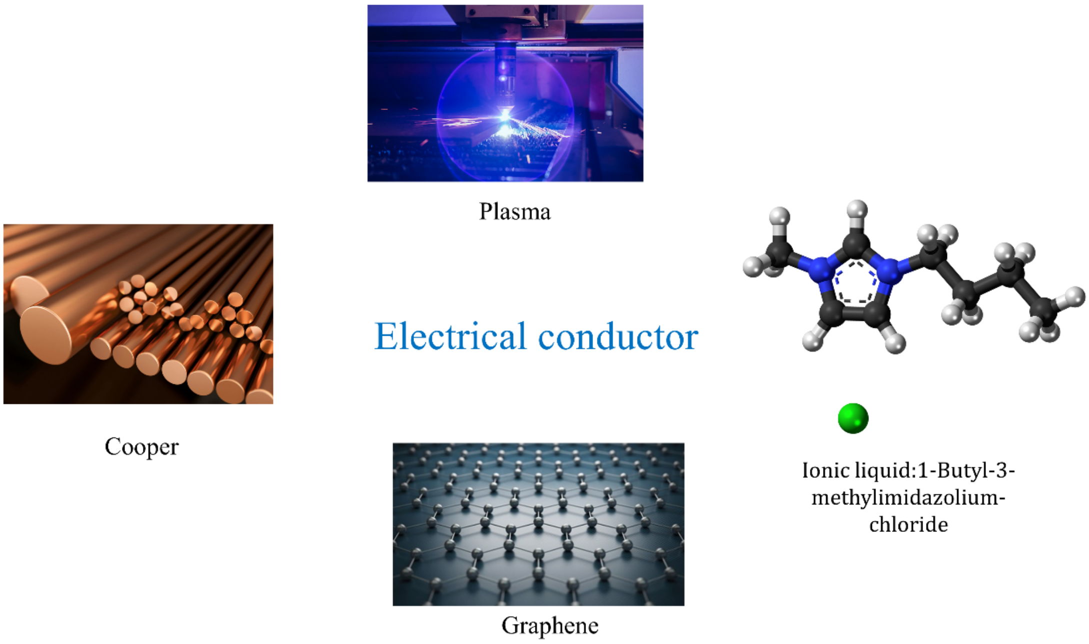


Figure 28 Electric conductor

3.3. The origin of the electrical current:

Consider two conductors, A and B, initially in electrostatic equilibrium. Conductor A carries a charge Q_A, and conductor B carries a charge Q_B. Let's say that the charge Q_A is greater than the charge Q_B. The presence of these two charges creates two electric potentials in A and B, and also creates an electric field between these two charges, which we call E.

When conductor A is connected to conductor B via an electrically conductive wire, for example, copper, the electrostatic equilibrium is disturbed, and the electric charges move under the influence of an electric force resulting from the superposition of electric fields from each of the conductors. The movement of electrons continues from the two conductors until a new state of electrical equilibrium is reached in a conductor, this time composed of both conductors. The movement of electrons over time is called electric current, and it is a very important physical quantity in understanding the behavior of electrokinetic systems. This resulting current is temporary, meaning it will disappear once a state of electrical equilibrium is reached see Figure 29.

In practical applications of electricity, we require permanent electric currents. Therefore, it is necessary to maintain an imbalance between conductors A and B. This can be achieved using batteries or electric generators.

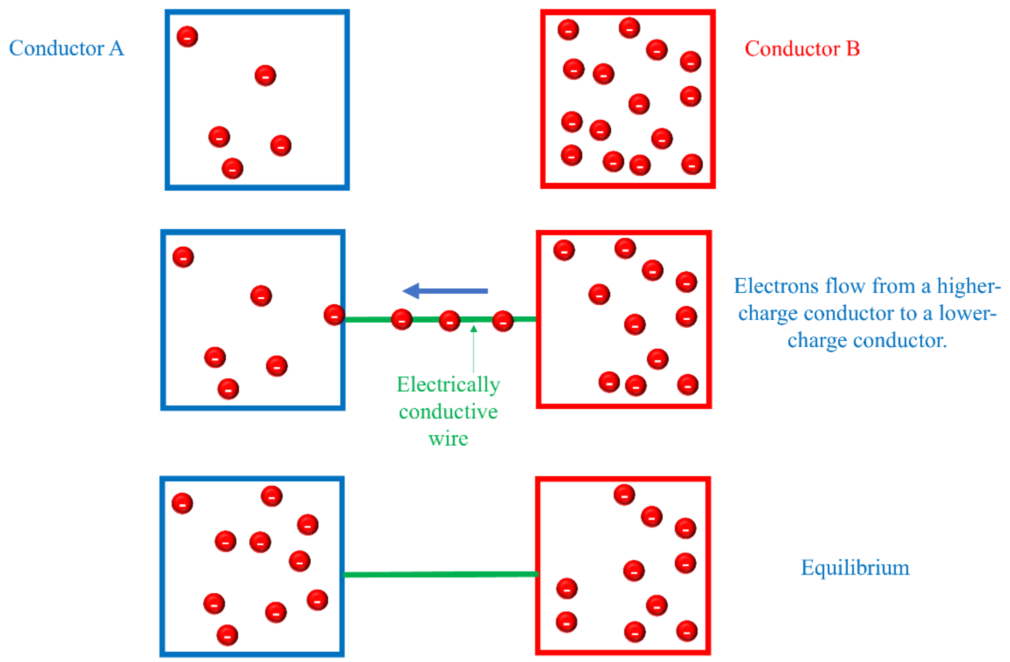


Figure 29 The origin of the electric current

3.4. Electrical current:

The electrical current is defined as the flow of electric charges through a conductor over time. The electrical current has a direction, and it is measured in amperes (A), "a fundamental unit in SI", where 1 ampere is defined as 1 coulomb of charge passing through a point in a circuit every second.

From a historical point of view, it is extremely difficult to pinpoint the exact time when the concept of electric current developed. The works of scientists such as William Gilbert in his book "De Magnete" or Benjamin Franklin on lightning, while studying static electricity, laid the groundwork for a deeper understanding of the movement of electric charges, thus establishing the concept of electric current. In this lesson, we will argue that the concept of electric current was most clearly defined when Alessandro Volta invented the voltaic pile. This device allowed for a continuous flow of electric charge instead of intermittent discharges of static electricity, which represented a qualitative leap in the applications of electricity and in our understanding of the nature of electricity. Based on Volta's work and "battery", many scientists in the following years studied the electrokinetic, including the brilliant scientist Ampere, who conducted a wide range of experiments on both electricity and magnetism and the relationship between them. In these experiments, Ampere introduced the concept of electric current in all its dimensions. He considered it a continuous flow of electricity and demonstrated through his experiments that it has a definite direction. He also demonstrated through his law of force and of electric circuits that electric current is a measurable quantity. In fact, we can say that Ampere, through his work, introduced the concept of electric current in the same concept used today.

3.4.1. Types of electrical current:

1. **Direct current:** Direct current (DC), is an electric current in which electric charges (mostly electrons) flow in one direction. This current remains constant over time. This electrical current can be generated by batteries or DC generator.
2. **Alternating Current:** Alternating current (AC), is an electric current where the flow of electric charge reverses direction periodically.

3.4.2. Electrical Current Intensity:

Current intensity, is a measure of the flow of electric charge through an electrical conductor per unit time. It represents the total amount of electric charge passing through a point in the conductor every second. It is symbolized by the symbol I and measured in amperes (A), which is equal to coulomb per second.

The mathematical definition of this quantity is given by the following relationship:

$$I = \frac{Q}{t}$$

Q is the electric charge (in coulombs, C),

t is time (in seconds, s).

Rq: The ampere (A) is a fundamental unit in the SI.

3.4.3. The direction of the electrical current:

Electric current flows in the direction opposite to the negative charges, that is, in the direction of the electric field, from the positive pole to the negative pole. This is the conventional direction of current, which was chosen by Ampere at the beginning of the nineteenth century

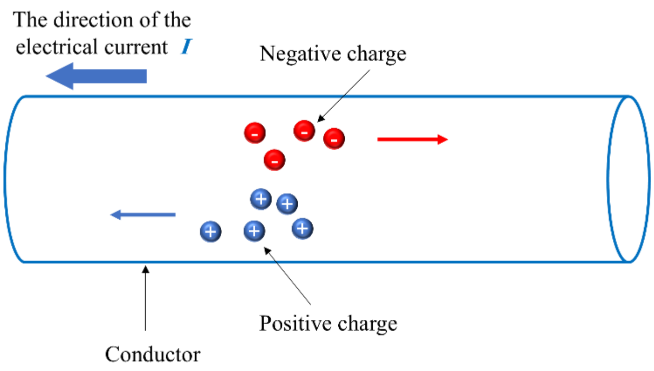


Figure 30 The direction of the electric current

3.5. Ohm law:

Ohm's law is one of the most famous laws of electromagnetism. It was published by the German physicist Georg Simon Ohm in 1827 after conducting numerous experiments using voltaic batteries to study the behavior of electric potential in circuits. The law states that electric potential increases with the linear increase of current and resistance. The mathematical relation of this law is:

$$V = RI$$

V is the voltage (potential), R is the resistance, and I is the current.

3.6. Density of Current:

We consider a conductor of section dS as shown in Figure 31.

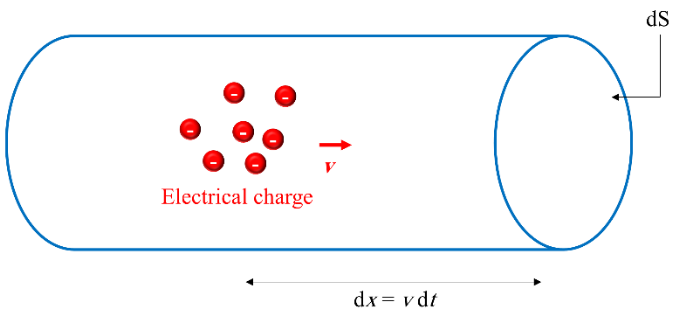


Figure 31 The movement of electric charge

The quantity of charge dq, which crosses the section dS perpendicular to the axis of the flow tube for a time dt, is:

$$dq = \rho\, dV = \rho\, v\, dt\, dS$$

In our case, both the velocity and the direction vector are parallel. In the case where these two quantities are no longer parallel, this expression becomes:

$$dq = \rho\, dV = \rho\, \vec{v}\,\overrightarrow{dS}\, dt$$

We now defined the vector $\vec{J}$:

$$\vec{J} = \rho \, \vec{v}$$

This vector, called the density of current vector, is the amount of electric current flowing per unit area of a conductor's cross-section (surface). The unit of this quantity is A/m^2.

From the expression of the infinitesimal quantity of charge, we can rewrite the expression of the density of current as:

$$\frac{dq}{dt} = \rho \, \vec{v} \, \overrightarrow{dS} = \vec{J} \, \overrightarrow{dS}$$

Therefore,

$$I = \int \int \vec{J} \, \overrightarrow{dS}$$

The density of current is a very useful quantity in many fundamental needs, and technological applications, such as the design of electronic conductors and wires, where the density of current helps in determining the appropriate thickness of wires to avoid overheating.

Example:

Calculating the current density passed through a copper wire with a diameter of 2 mm carrying a steady current of 5 A.

Solution:

$$J = \frac{I}{S}$$

Therefore,

$$J = 1.59 \; 10^6 \; A/m^2$$

The relation between the electrical density and the electrical field:

We obtain in Chapter I that the electrical potential can be obtained as:

$$V = V_A - V_B = \int_A^B \vec{E} \, d\vec{l}$$

If the conductor is a wire with a section S, the electrical field is uniform from A to B.

$$V = E \, l = R \, I = R \, J \, S = E \, l$$

Therefore,

$J = \frac{E \, l}{S \, R}$

3.7. The movement of electric charge and conductivity:

To explain the mechanism of electrical conduction in electrical conductors, we suggest the following model:

Let's suppose that we have a conductor metal. We also assume that the conduction electrons in this metal move randomly, affected by collisions with positive ions, which cause them to lose speed and change direction. Between two collisions, the electron moves in a straight line. If we calculate the average velocity of a large number of electrons moving randomly and changing their speed and direction after each collision, it will be zero. Therefore, there is no electrical current in this metal.

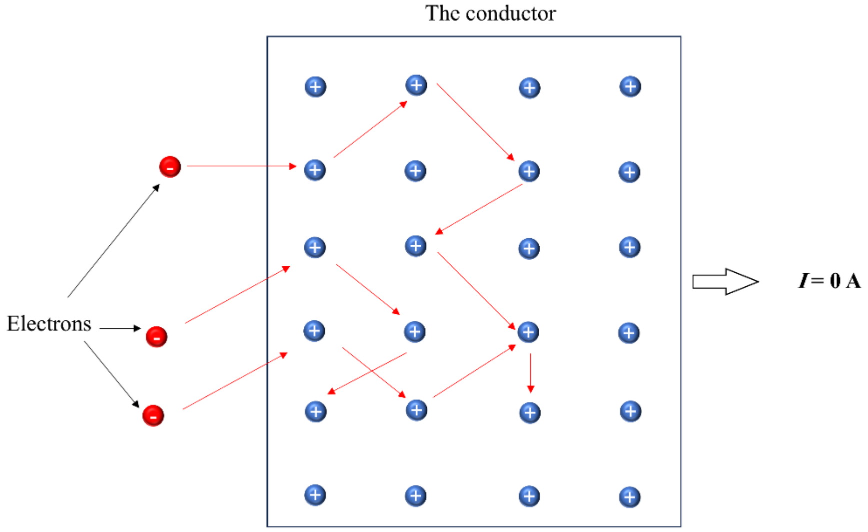


Figure 32 The movement of charges in a conductor

If we applied a homogeneous external electrical field on the conductor, between two collisions, the electrons experience acceleration in the opposite direction of the applied electric field as shown in Figure 32. The acceleration α of the electron given as:

$$\sum \vec{F} = m\,\vec{\alpha} = q\,\vec{E}$$

$$\vec{\alpha} = q\,\frac{\vec{E}}{m} = -e\,\frac{\vec{E}}{m}$$

The velocity of the electron is given as:

$$\vec{v} = \vec{v}_{th} + \vec{\alpha}\,t = \vec{v}_{th} - e\,\frac{\vec{E}}{m}\,t$$

Were $\vec{v}_{th}$ is the random velocity due to the thermal agitation.

We need to calculate the average velocity of the electrons. The average velocity of the thermal agitation is nil. The second side of the electron velocity equation is a quantity common to all electrons. Let τ be the average time between two collisions of an electron with a positive ion. Therefore, the average electron velocity is:

$$\overrightarrow{v_a} = -e\,\frac{\vec{E}}{m}\tau$$

The average velocity is proportional to the electric field, because the factors e, τ, and m are constants.

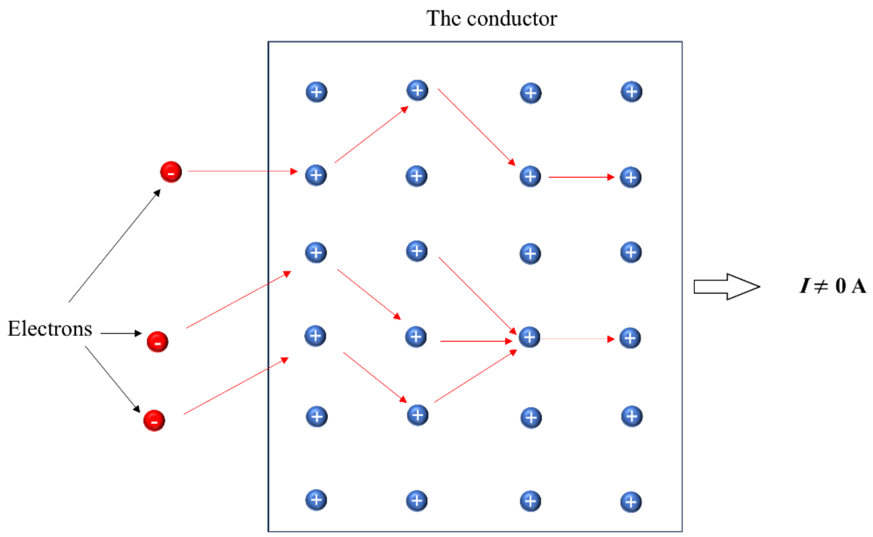


Figure 33 The movement of charges in a conductor under the effect of electric field

We can now represent the current density as:

$$\vec{J} = n\, e^2\ \frac{\vec{E}}{m} \tau$$

From the relation between the current density and the electrical field, $\vec{J} = \sigma\, \vec{E}$. The conductivity can present as:

$$\sigma = \frac{n\, e^2}{m} \tau$$

We can also present the resistivity, which is the inverse of the conductivity:

$$\rho = \frac{m}{n\, e^2\, \tau}$$

The unit of resistivity is Ω.

The values of the conductivity of some metals are given in the following table.

Table 1 The conductivity of some chemical elements

Element	Symbol	Conductivity
Silver	Ag	$6.30\ 10^7$
Copper	Cu	$5.96\ 10^7$
Gold	Au	$4.52\ 10^7$
Aluminum	Al	$3.77\ 10^7$
Calcium	Ca	$2.98\ 10^7$
Beryllium	Be	$2.50\ 10^7$
Magnesium	Mg	$2.24\ 10^7$
Rhodium	Rh	$2.20\ 10^7$
Sodium	Na	$2.10\ 10^7$
Iridium	Ir	$2.06\ 10^7$
Tungsten	W	$1.82\ 10^7$
Zinc	Zn	$1.69\ 10^7$
Nickel	Ni	$1.43\ 10^7$
Lithium	Li	$1.08\ 10^7$
Iron	Fe	$1.00\ 10^7$
Platinum	Pt	$9.40\ 10^6$
Palladium	Pd	$9.30\ 10^6$
Cobalt	Co	$6.00\ 10^6$
Titanium	Ti	$2.40\ 10^6$
Lead	Pb	$4.80\ 10^6$

3.8. Electrical power

Electrical power is the rate of transferring or consuming electrical energy in a circuit. It is calculated as:

Power (P) = Voltage (V) × Current (I) (the unit of electrical power is the Watt, W)

3.9. Joule law:

The Joule effect, also called Joule heating or resistive heating, is a phenomenon in which the passage of an electric current through a conductor increases the temperature of the conductor. As previously explained, an electric current results from the regular movement of electrical charges, usually electrons, within a conductor. As they move within a conductor, the electrons interact with the conductor's crystal lattice, as well as with impurities, leading to "collisions" at the microscopic level. Part of the kinetic energy of the electrons is transferred to the conductor's crystal lattice through these "collisions", increasing their vibration and, consequently, increasing the conductor's temperature.

The mathematical relationship for this phenomenon is given as follows:

$W = R\,I^2\,t = P\,t$

W is the heat energy generated over time, R is the resistance, I the applied current, and t is the time.

It is clear that increasing resistance or electrical current leads to an increase in the temperature of an electrical conductor. Resistance is primarily created by the impediment of electron movement by the conductor's atoms. However, when large currents are applied to a conductor, this can lead to changes in the nature of the conductor and damage it due to increased temperature. This means that increasing the current does not necessarily mean an increase in temperature, but rather the destruction of the conductor. This is a common problem in modern technological applications, especially within the limits of currents applied to nano and micro conductors, which do not cause a temperature increase sufficient to change their properties.

3.10. Electrical circuit:

An electrical circuit is a set of conductors and electrical compounds that are connected in a closed loop pathway. The electrical current flows in this loop, enabling the transfer of the electrical energy to the electrical compounds for useful work.

3.10.1. Electrical Circuit Elements

Elements of an electrical circuit are electrical components that work in concert to control the flow of electrical current to perform specific tasks. These components differ based on their assigned tasks, such as providing power, directing current, consuming power, converting power, etc.

Examples:

1. Power Source: power sources, such as batteries and generators, provide the electrical energy needed to operate a circuit.
2. Conductors: conductors are materials that conduct electrical current between components in an electrical circuit, such as wires. These wires are typically made of copper and offer low resistance to electrical current, reducing energy loss.
3. Resistors: restrict the flow of current and convert electrical energy into heat.
4. Light bulbs: convert electrical energy into light and heat.
5. Cotors: convert electrical energy into mechanical motion.

6. Transistors: Act as electronic switches or amplifiers in complex circuits.
7. Heaters: Generate heat from electrical energy.

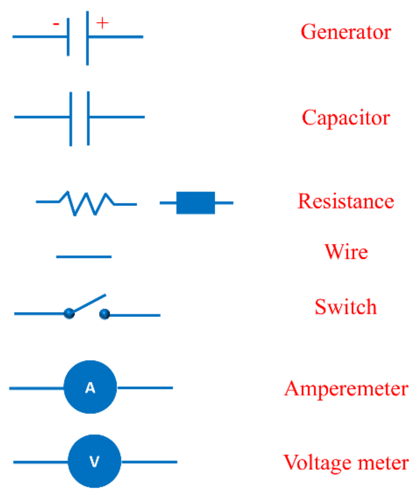


Figure 34 Electrical Circuit Elements

3.10.1.a) Generators:

An electrical generator is a device that converts energy such as mechanical energy or chemical energy into electrical energy through the process of electromagnetic inductions or chemical reactions. In the case of electromagnetic inductions, the process involves a changing magnetic field inducing an electric current in a conductor, enabling the generator to produce electricity. In the case of chemical reactions, redox reactions transfer electrons, generating electricity. Note here that there are other type of source energy such as, solar energy, geothermal energy, and nuclear energy.

3.10.1.b) Generators types:

<u>DC generator:</u>

The DC generators create a direct current (DC), where the electric current flows in a single direction, and its intensity is constant. DC arises from the constant flow of electrons and is usually generated by batteries. The chemical reactions inside the battery create a potential difference. This potential difference causes electrons to flow between the negative and positive electrodes. As they flow, the electrons create electromagnetic fields that excite electrons in a heater, for example. The electrons in the heater move randomly, and their movement causes collisions with atoms, which leads to greater vibration of the atoms and consequently higher temperature (the Joule effect).

<u>AC generator:</u>

The AC generators create an alternative current (AC), where the electric current change its direction as a function of time. Alternating current is generated from several sources, such as dams. Dams store large quantities of water, and due to the height of the dams, the water acquires significant potential energy. Releasing water from the dam causes it to flow at high speed, colliding with the dam's turbine wheels. This water transfers its kinetic energy to the turbines, causing them to rotate. The turbines are connected to the shaft of an electric generator, which consists of the rotor: a large electromagnet rotating at a constant speed (e.g., 3,000 rpm for 50 Hz). The stator: stationary copper coils surrounding the rotor. When the rotor rotates, the changing magnetic field induces an alternating current in the stator coils, according to Faraday's law of magnetic induction. This induces the creation of an alternative electrical current.

3.10.2. The electrometric force EMF:

Electromotive force (EMF) is the energy provided by an electrical source, such as a battery or generator, per unit charge.

Sum of Electromotive Forces:

When we connect several generators in a circuit, the resulting electrical power depends on the nature of the connection, whether in series or parallel.

<u>Generators in Series:</u>

When generators are connected in series, where the positive terminal of one generator connects to the negative terminal of the next generator (See Figure 35), the total EMF adds algebraically (we take into account their polarities).

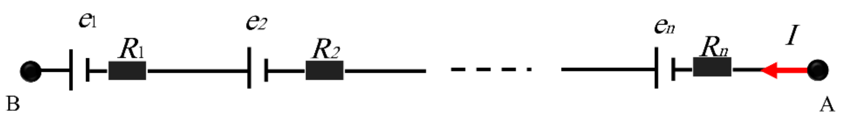


Figure 35 The grouping of generators in series

$$E_T = V_B - V_A = e_1 + e_2 + \cdots + e_n = R_1 I + R_2 I + .. + R_n I$$

If polarities oppose (e.g., one generator is reversed), subtract the opposing EMF:

$$E_T = e_1 - e_2 + \cdots + e_n$$

If e_2 is reversed.

Example: Two generators with E_1 = 12, internal resistance R_1=1 Ω and E_2 = 6 V, internal resistance R_2=0.5 Ω in series:

E_{total} = 12 + 6 = 18 V.

<u>Generators in Parallel:</u>

In this scenario, all positive terminals of the generators are connected together, and all negative terminals of the generators are connected together.

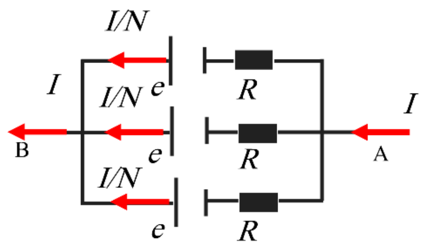


Figure 36 The grouping of generator in parallel

If the generators have the same EMF and are connected with the same polarity, the total EMF equals the EMF of one generator.

$$E_T = V_B - V_A = e - \frac{R}{N}\,I$$

Receiver:

A receiver is a device that consumes electrical energy and converts it into another form of energy, such as light, heat, or mechanical work.

Key differences between the generator and the receiver:

Table 2 The difference between the generator and receiver

Aspect	**Generator**	**Receiver**
Function	Produces electrical energy	Consumes electrical energy
EMF	Provides EMF	May have a counter-EMF (e.g., motors)
Energy Flow	Supplies energy to the circuit	Dissipates energy from the circuit
Examples	Battery, generator, solar cell	Resistor, bulb, motor
Voltage	$V_B - V_A = e - I\,\mathrm{R}$	$V_B - V_A = \mathrm{I\,R}$

3.10.3. Electrical resistance:

Resistance is a physical phenomenon that shows opposition to the flow of electric current. It is a fundamental concept in electricity and electronics and is measured in ohms (Ω).

Resistor:

The resistor is an electrical component designed to provide specific resistance in an electrical circuit. The resistor is used for different objectives such as controlling current, voltage division, and signal processing.

Resistance grouping:

Before explaining how to calculate the sum of resistances connected in parallel or series, we will briefly introduce two laws. We will return to explaining these two laws in more detail in the following sections of this lesson.

First law: In any electrical circuit, the sum of the intensities of the electrical currents flowing into a node is equal to the sum of the intensities of the electrical currents flowing out of the same node.

Second law: When resistors are connected in parallel to each other. The voltage across each resistor is the same, and the current splits up and takes different paths through each resistor.

Parallel grouping:

As shown in Figure 37, the total electrical current I is the sum of the electrical currents I_1, I_2, …, I_n, and the voltage between each parallel resistance is the same $V_1 = V_2 = V_3 = .. = V_n$.

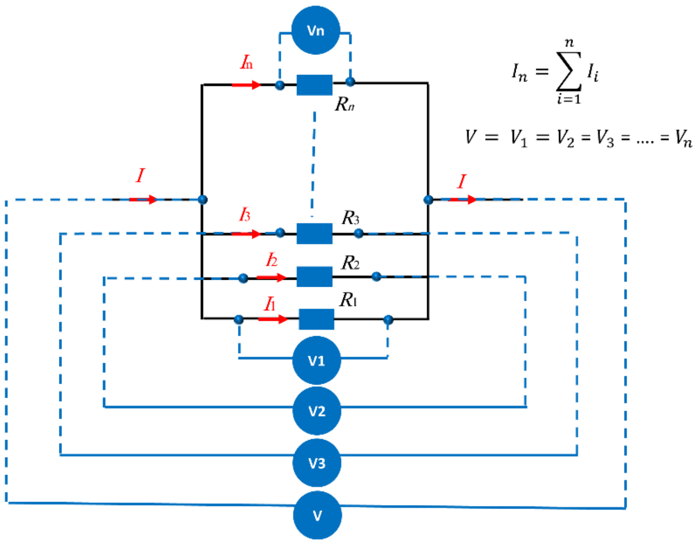


Figure 37 The parallel grouping of resistors

Using Ohm's law, $V = R_T\, I$, $V_1 = I_1\, R_1$, $V_2 = I_2\, R_2$, …, $V_n = I_n\, R_n$.

$V/R_T = I = I_1 + I_2 + \ldots + I_n = V_1/R_1 + V_2/R_2 + \ldots + V_n/R_n = V\,(1/R_1 + 1/R_2 + .. + 1/R_T)$

Therefore,

$$\frac{1}{R_T} = \sum_{i=1}^{n} \frac{1}{R_i}$$

Series grouping:

As shown in Figure 38. the electrical current passing through each resistance is the same, $I = I_1 = I_2 ..= I_n$. The potential inside the circuit is the sum of the voltage between each resistance. $V_T = V_1 + V_2 + .. + V_n$.

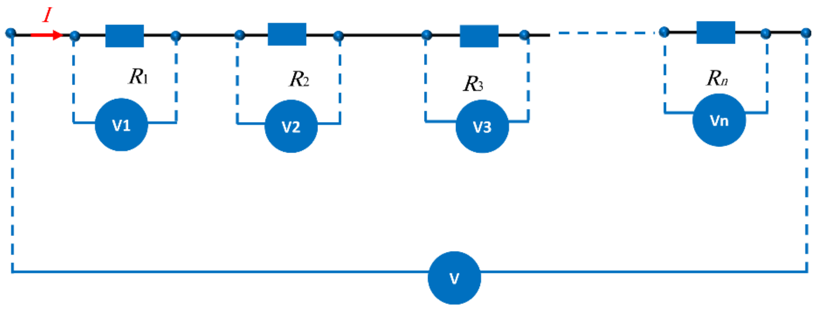


Figure 38 The serie groupement of resistors

$V_T = I_T\, R_T = V_1 + V_2 + .. + V_n = R_1\, I + R_2\, I + .. + R_n\, I = I\,(R_1 + R_2 + .. + R_n)$

Therefore, $R_T = R_1 + R_2 + .. + R_n$

Example:

Four resistances are connected to a 12.0 V battery, as shown in the following figure. R1 = 500, R2 = 800, R3 =1100, R4 = 400

a. What is the power supplied by the battery?

b. Determine the current and potential difference across each resistance.

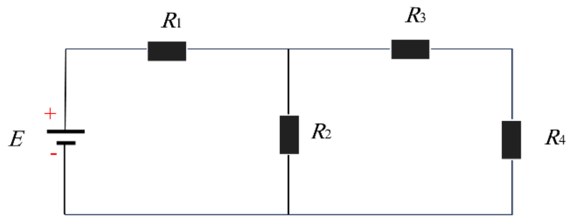


Figure 39 Description of the circuit

3.10.4. Analysis of an electrical circuit:

As we mentioned before, an electrical circuit is a set of conductors and electrical compounds that are connected in a closed loop pathway. The electrical current flows in this loop, enabling the transfer of electrical energy to the electrical compounds for useful work.

<u>Node:</u>

A node is a point in an electrical circuit where at least three dipoles meet.

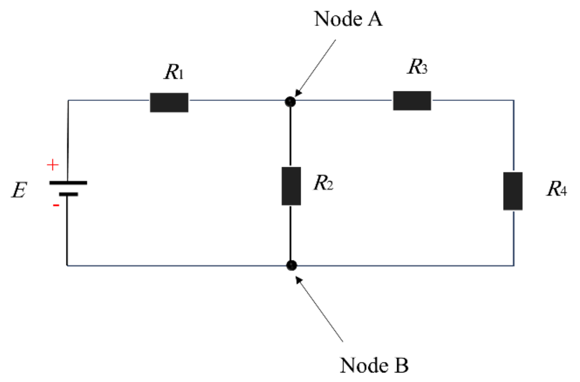
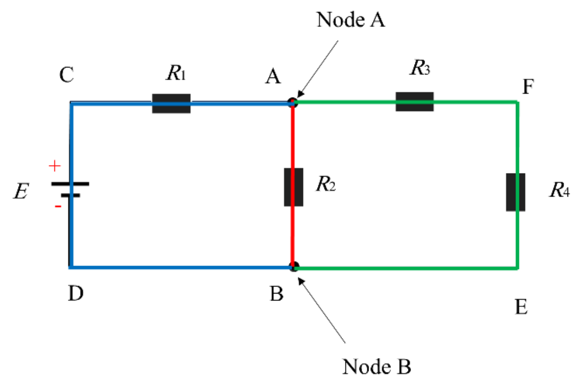


Figure 40 The nodes (junctions)

<u>Branch:</u>

A branch is a portion of the circuit between two nodes.



Branch 1: AB
Branch 2: ACDB
Branch 3: AFEB

Figure 41 The branches

<u>Mesh:</u>

a mesh is made up of a set of branches, forming a closed circuit.

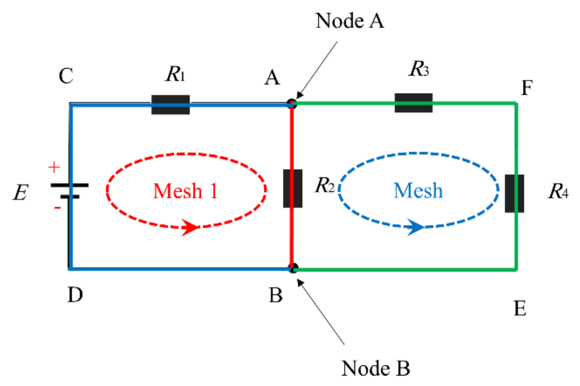


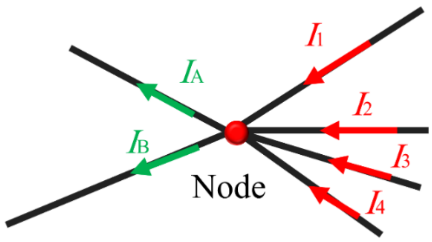
Figure 42 Mesh

3.11. Kirchhoff Laws:

3.11.1. Law of Current Conservation:

The algebraic sum of currents entering a node is equal to the sum of currents leaving the same node, see Figure 43.

$$\sum I_{in} = \sum I_{out}$$



$$I_1 + I_2 + I_3 + I_4 = I_A + I_B$$

Figure 43 Law of nodes

This law is based on the conservation of electric charge—charge cannot accumulate at a node in a steady-state circuit

3.11.2. Law of Voltage Conservation:

The algebraic sum of all voltages (potential differences) around any closed loop in a circuit is zero. This includes EMFs from generators and voltage drops across receivers or resistances.

$$\sum_{i=1}^{n} V_i = 0$$

Chapter 4

Magnetism

4. Magnetism

4.1 Introduction:

Magnetism is one of the most important physical phenomena in our daily lives. Many instruments used throughout history have relied on magnetic phenomena. For example, the traditional compass, designed to explore the world, relied on a needle that moved in the direction of magnetic north. In our current world, magnetism is relied upon in many modern applications. For example, current computers rely on magnetism to store data, and modern trends in nanomedicine rely on magnetic nanoparticles to destroy cancer cells or selectively transport drugs.

4.2 The Historical Aspect of Magnetism

Observations of magnetism date back thousands of years, when humans discovered the magical properties of magnetite. Magnetite is composed primarily of iron oxide Fe_3O_4, which enables it to attract iron. This mineral was primarily used in the development of the compass in China. This compass, called "Si Nan", we can consider as a primitive instrument. Meanwhile, thousands of kilometers away in ancient Greece, the Greek philosopher Thales of Miletus wrote in the 6th century BC about the miraculous ability of magnetite to attract iron. His explanations for this phenomenon were spiritual, as he believed that a kind of spirit resided within the stone.

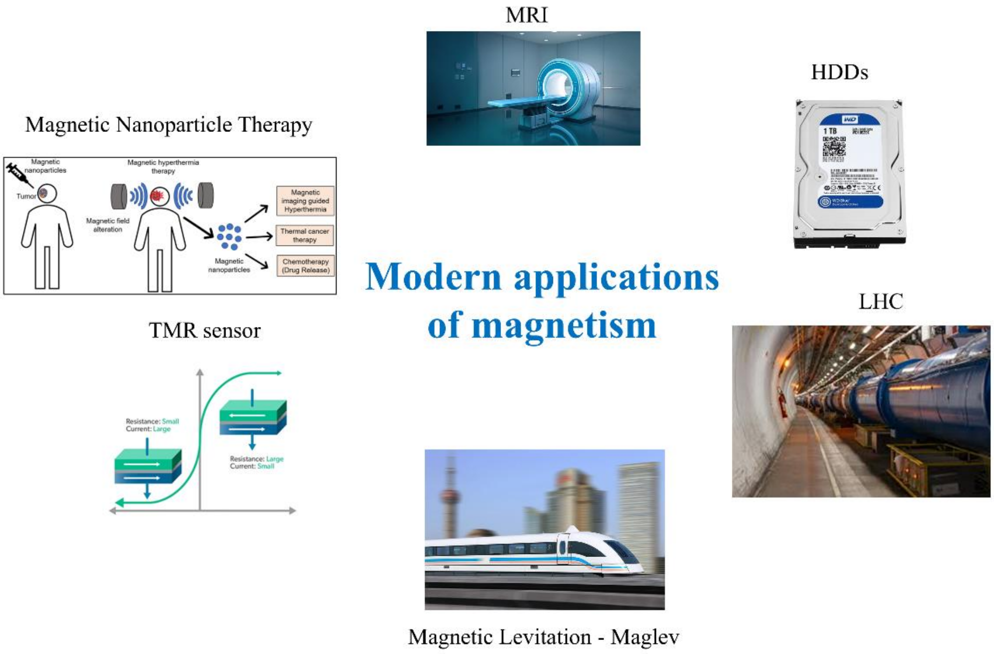


Figure 44 Applications of magnetism

4.3 Magnetic field:

Similar to the electric field, a magnetic field (also called magnetic flux density or magnetic induction) is defined as a ***mediator of the magnetic force***. A magnetic field is a vector field, which means it has both a magnitude and a direction, and is presented as $\vec{B}$. In classical physics, the magnetic field is generated by the movement of electrical charge (current in a wire). However, the magnetic field can also be generated by magnetic materials like ferromagnetic materials (the ferromagnetic behavior cannot be described in classical physics), and the intrinsic magnetic moment of particles, called spin.

A magnet or an electrical field creates a magnetic field around itself. If an object is placed in this field, it experiences a magnetic force. The magnetic field can be presented by its magnetic field lines, similarly to the case of the electrical field. The magnetic field lines create complete loops around current sources or magnets, never beginning or ending at a point (See Figure 45).

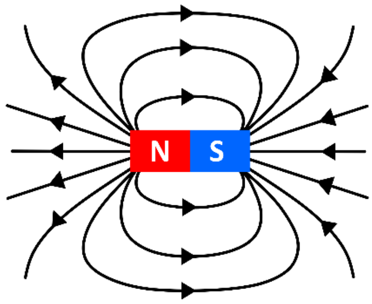


Figure 45 Magnetic field lines

Magnetic field lines possess the following characteristics:

1. The magnetic field is always tangent to the field lines.
2. The magnetic field table is directly proportional to the field line density.
3. The magnetic field lines originate from the north pole of the magnet and move to the south pole of the magnet.
4. The magnetic field lines never intersect.

4.4 Supposition principle:

The superposition principle of magnetic fields asserts that the cumulative magnetic field at a given site M, generated by several magnetic fields $\vec{B}_1, \vec{B}_2, \ldots \vec{B}_n$ (each with distinct direction and magnitude), is the vector sum of the individual magnetic fields from each source at that location.

$$\vec{B}_T = \sum_{i=1}^{n} \vec{B}_i$$

This holds true assuming the medium is linear and the sources are independent.

4.5. Earth's magnetic field

Earth's magnetic field is a geomagnetic field created by molten iron currents in its outer core of the earth. This magnetic field extends from the planet's interior to 100,000 km from the planet's center. Also, the Earth's magnetic field has north and south magnetic poles, shielding Earth from solar wind. These dipoles are not fixed; they move slowly. Figure 46. presents the movement of the north dipole of Earth's magnetic field from 1500 to 2024(Alexander, 2025).

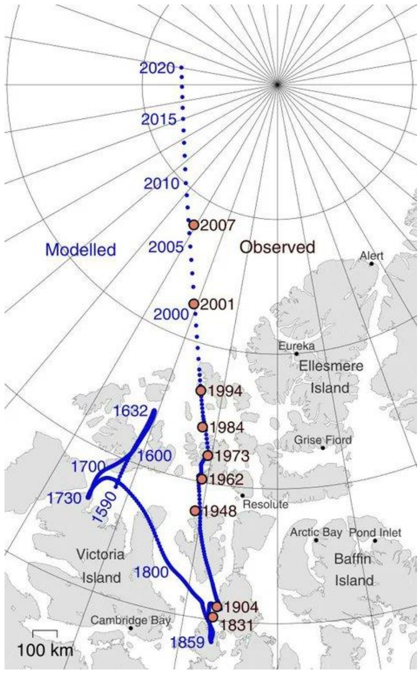


Figure 46 Earth's magnetic field

4.6. The magnetic field produced by a long wire:

In 1820, Danish physicist Hans Christian Ørsted made a very important discovery while studying electric currents passing through conductive wires. ***An electric current passing through a conductive wire causes a change in the direction of a magnetic compass needle placed nearby***. This may be one of the first observations linking electricity and magnetism.

Let's take the following example:

A group of magnetic needles is distributed around an electrically conducting wire, see Figure 47. When no electric current is applied to the wire, the needles do not change their direction. When an electric current is applied, the magnetic needles deflect due to the formation of a magnetic field around the wire.

The magnetic field produced by the wire is tangent to a circle centered on the wire, and the direction of the magnetic field is obtained using the right-hand rule.

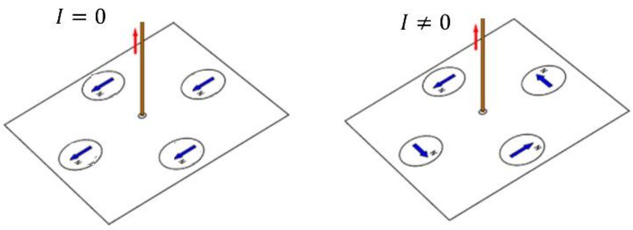


Figure 47 The effect of electric field on needles

4.6.1. The unit of the magnetic field:

In the IS system, the magnetic field is measured in tesla (T).

4.6.2. The right-hand rule:

For a long wire carrying a direct current i, the magnetic field is tangent to a circle centered on the wire. The direction is that indicated by the fingers of the right hand when the thumb is placed in the direction of the current.

4.6. The magnetic force:

Magnetic force is one of the fundamental concepts in physics and a fundamental component of the electromagnetic force, which is one of the four fundamental forces in nature. This force arises from the interaction between the magnetic fields and moving electric charges. When an electrical charge moves, it generates a magnetic field. This magnetic field interacts with other moving electrical charges, which result in the creation of a magnetic force. This interaction is responsible for a diverse range of phenomena, from the ordinary attraction of magnets to iron to the complex functioning of spintronic devices such as solid-state drives (SSDs).

Similarly to the magnetic field, the electrical force is inherently a vector quantity, possessing both magnitude and direction. Due to its vector nature, magnetic force also depends on its direction relative to the electric charge and the direction of the magnetic field.

Effect of a magnetic field on the movement of an electric charge: Consider a particle with a charge of q moving at a speed v. We place an electrical current close to the particle. Due to both the electrical field $\vec{E}$ and the magnetic field $\vec{B}$, the particle experiences a magnetic force that depends on the charge q and the magnetic field, and the particle's speed v. The Dutch physicist Hendrik Lorentz gives the expression for the force exerted on this particle as follows:

$$\vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B}\,)$$

In the presence of only the magnetic field (let say that we have a magnet close to the particle), the Lorentz force becomes:

$$\vec{F} = q\,(\vec{v} \times \vec{B})$$

This magnetic force possesses the following characteristics:

1. The magnetic force is zero if the velocity is zero or if the velocity has the same direction as the magnetic field.
2. The modulus of the force is:

   $$F = |q|\, v\, B \sin(\theta)$$

   Where θ is the angle between the vector of the particle velocity and the magnetic field
3. The force is perpendicular to the plane formed by the vectors $\vec{v}$ and $\vec{B}$.
4. The direction of the magnetic force is obtained by the right-hand rule. The direction of the force is the same as the thumb in the case of a positive charge, and it is opposite to the direction of the thumb in the case of a negative charge.

4.6.1. The Laplace force:

The Laplace force is a magnetic force that arises from the Lorentz force. Let's suppose we have a conductive wire of length L and cross-sectional area A, carrying a current I. The electrical current is due to the movement of charge carriers, for example, electrons with an average drift velocity v. Let's take the number of charge carriers per unit volume to be n, and each carrier has a charge q. We applied a magnetic field on this wire.

The total number of electrical charges in the conductor is N = n A L. Also, the total magnetic force on all these electrical charges can be obtained using the Lorentz law as:

$$\vec{F} = N\left(q\,\vec{F} \times \vec{B}\right) = n\,A\,L\left(q\,\vec{v} \times \vec{B}\right)$$

The current density expression is:

$$\vec{J} = n\,q\,\vec{v}$$

So,

$$\vec{F} = AL\,(\vec{J} \times \vec{B})$$

Therefore:

$$\vec{F} = I\,(\vec{L} \times \vec{B})$$

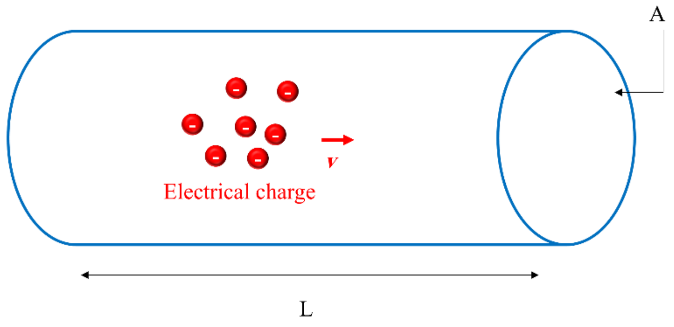


Figure 48 The movement of the electric charges

This equation represents the Laplace force, which is ***the total magnetic force on a current-carrying conductor***.

4.7. Hall effect:

The Hall effect(Chien, 2013) is a phenomenon widely used in modern physics to study the properties of materials or for use as a magnetic sensor. This effect is observed when a current-carrying conductor is placed in an external magnetic field, resulting in a voltage difference perpendicular to the current.

Consider the following figure, which shows a conductive metal piece in which an electric current is applied in the direction of the x-axis, and a magnetic field in z-axis. Electrons move along the x-axis with a velocity $\vec{v}$. The magnetic field perpendicular to the metal piece causes the electrons to experience a magnetic force, the Lorentz force.

$$\vec{F} = q\,(\vec{v} \times \vec{B}\,)$$

Under the influence of this force, the electrons are deflected in the direction perpendicular to both the electric current and the magnetic field, resulting in a difference in the field between the two ends of the metal sheet along the y-axis.

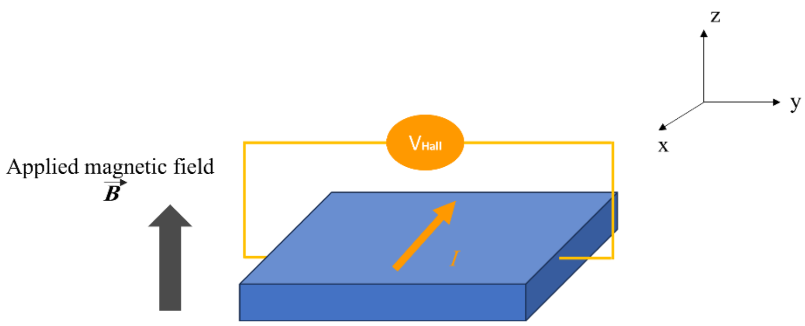


Figure 49 Hall effect

4.8. Ampère's circuital law

Ampère's circuital law is a fundamental law in magnetism and one of Maxwell's equations, which relates the integrated magnetic field around a closed loop to the electric current passing through the loop. We can consider this law as analogous to Gauss's law in electrostatics, "see the electrostatic chapter".

Ampère's Law: the line integral of the magnetic field B around a closed loop (called an Amperian loop) is proportional to the total electric current, the expression of the law is given as:

$$\oint \vec{B}\, d\vec{l} = \mu_0\, I$$

Here,

$\vec{B}$ is the magnetic field vector.

μ_0 is the magnetic permeability of the space.

4.9. The Biot-Savart law:

The Biot-Savart law is a fundamental principle of electromagnetism, derived experimentally in 1820. This law relates the magnetic field produced by a small strip of current-carrying conductor at a specific point in a vacuum to the electric current. This law allows for the calculation of magnetic fields in systems where symmetry does not allow for easy application of Ampere's law. Although the law was derived two centuries ago, it is still used today to analyze many magnetic systems, including modern spintronic sensors used to study the properties of nanoparticles. The law states that an electric current of intensity I passes through an infinitesimal element d$\vec{l}$ of an electrical conductor generates a magnetic field d$\vec{B}$ where:

$$\overrightarrow{dB} = \frac{\mu_0\, I}{4\,\pi\, r^2}\, d\vec{l} \times \overrightarrow{u_r}$$

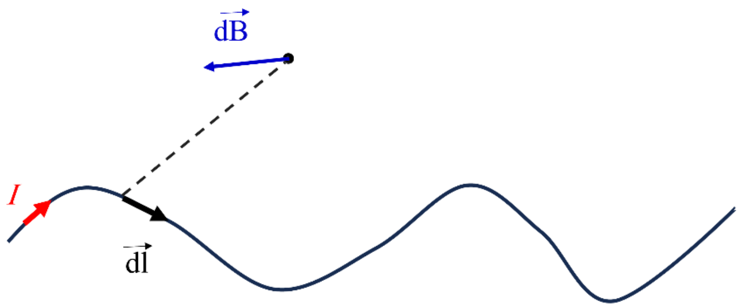


Figure 50 The magnetic field created from an infinitesimal current

The total magnetic field created from all the l is:

$$\vec{B} = \frac{\mu_0 \, I}{4\,\pi} \int \frac{d\vec{l} \times \overrightarrow{u_r}}{r^2}$$

4.10. Magnetic dipole:

A magnetic dipole is a simple magnetic configuration that produces a magnetic field. It can be visualized as either a pair of equal and opposite magnetic poles separated by a small distance or a closed loop of electric current. Unlike electric dipoles, magnetic monopoles do not exist in nature, making magnetic dipoles the primary building blocks of magnetic phenomena.

Problems with the solutions:

Problem 01:

Calculate the partial derivatives of the following functions and then give their gradients:

$$f(x,y) = 2x + 3y + 5$$
$$g(x,y) = 3\,x^2y + 4xy^3 + 2x - 5y + 7$$
$$h(x,y) = \exp(x)\sin(y) + x^2y + 3y$$
$$L(x,y) = \ln(x^2 + y^2) + x\exp(xy) + \frac{y}{x}$$

Solution :

1/

$$\boldsymbol{f(x,y) = 2x + 3y + 5}$$

Partial Derivatives and Gradients

$$\frac{\partial f}{\partial x} = 2, \quad \frac{\partial f}{\partial y} = 3$$
$$\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = (2,3)$$

2/

$$\boldsymbol{g(x,y) = 3\,x^2y + 4xy^3 + 2x - 5y + 7}$$

$$\frac{\partial g}{\partial x} = 6xy + 4y^3 + 2, \quad \frac{\partial g}{\partial y} = 3x^2 + 12xy^2 - 5$$
$$\nabla g(x,y) = (6xy + 4y^3 + 2,\ 3x^2 + 12xy^2 - 5)$$

3/

$$\boldsymbol{h(x,y) = \exp(x)\sin(y) + x^2y + 3y}$$
$$\frac{\partial h}{\partial x} = \exp(x)\sin(y) + 2xy, \quad \frac{\partial h}{\partial y} = \exp(x)\cos(y) + x^2 + 3$$
$$\nabla h(x,y) = (\exp(x)\sin(y) + 2xy,\ \exp(x)\cos(y) + x^2 + 3)$$
$$\frac{\partial}{\partial x}\ln(x^2 + y^2) = \frac{2x}{x^2 + y^2}, \quad \frac{\partial}{\partial y}\ln(x^2 + y^2) = \frac{2y}{x^2 + y^2}$$
$$\frac{\partial}{\partial x}(x\exp(xy)) = \exp(xy) + xy\exp(xy), \quad \frac{\partial}{\partial y}(x\exp(xy)) = x^2\exp(xy)$$
$$\frac{\partial}{\partial x}\left(\frac{y}{x}\right) = -\frac{y}{x^2}, \quad \frac{\partial}{\partial y}\left(\frac{y}{x}\right) = \frac{1}{x}$$

4/

$$\boldsymbol{L(x,y) = \ln(x^2 + y^2) + x\exp(xy) + \frac{y}{x}}$$

$$\frac{\partial L}{\partial x} = \frac{2x}{x^2 + y^2} + \exp(xy) + xy\exp(xy) - \frac{y}{x^2}$$
$$\frac{\partial L}{\partial y} = \frac{2y}{x^2 + y^2} + x^2\exp(xy) + \frac{1}{x}$$
$$\nabla L(x,y) = \left(\frac{2x}{x^2 + y^2} + \exp(xy) + xy\exp(xy) - \frac{y}{x^2},\quad \frac{2y}{x^2 + y^2} + x^2\exp(xy) + \frac{1}{x}\right)$$

Problem 02:

1- Give the expression for the position vector of a point *M* in the three coordinate systems: Cartesian, cylindrical (polar), and spherical.
2- Deduce the expressions for displacement, surface elements, and volume elements in each coordinate system.
3- Calculate the surface area of the portion of a circular cylinder with a radius $R = 2$ m and height h=5m, limited by $45° \leq \theta \leq 120°$
4- Find the surface area of the band cut on a sphere of radius R, defined by $\alpha \leq \theta \leq \beta$
 What does the result become if $\alpha=0$ and $\beta=\pi$?
5- Calculate the volume of a sphere.

Solution:

Vector and Geometric Calculations in Different Coordinate Systems:

Cartesian Coordinates:

$$\overrightarrow{OM} = x\,\vec{\imath} + y\,\vec{\jmath} + z\,\vec{k}$$

Cylindrical Coordinates:

$$\vec{r} = \rho\,\overrightarrow{U_\rho} + z\,\vec{k}$$

Spherical Coordinates:

$$\vec{r} = r\,\overrightarrow{U_r}$$

Differential Displacement, Surface and Volume Elements:

$$\mathrm{d}\vec{r} = \mathrm{d}x\,\vec{\imath} + \mathrm{d}y\,\vec{\jmath} + \mathrm{d}z\,\vec{k}$$

$$\text{Surface elements: } \mathrm{d}S_x = \mathrm{d}y\,\mathrm{d}z, \quad \mathrm{d}S_y = \mathrm{d}x\,\mathrm{d}z, \quad \mathrm{d}S_z = \mathrm{d}x\,\mathrm{d}y$$

Volume element:

$$\mathrm{d}V = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

Cylindrical:

$$\mathrm{d}\vec{r} = \mathrm{d}\rho\,\overrightarrow{U_\rho} + \rho\,\mathrm{d}\theta\,\overrightarrow{U_\theta} + \mathrm{d}z\,\vec{k}$$

Side surface:

$$\mathrm{d}\overrightarrow{S_\rho} = \rho\,\mathrm{d}\theta\,\mathrm{d}z\,\overrightarrow{U_\rho}$$

Top and bottom surfaces:

$$\mathrm{d}\overrightarrow{S_z} = \rho\,\mathrm{d}\rho\,\mathrm{d}\theta\,\vec{k}$$

Radial surface:

$$\mathrm{d}\overrightarrow{S_\theta} = \mathrm{d}\rho\,\mathrm{d}z\,\overrightarrow{U_\theta}$$

Volume element:

$$\mathrm{d}V = \rho\,\mathrm{d}\rho\,\mathrm{d}\theta\,\mathrm{d}z$$

Spherical:

$$\mathrm{d}\vec{r} = \mathrm{d}r\,\overrightarrow{U_r} + r\,\mathrm{d}\theta\,\overrightarrow{U_\theta} + r\sin\theta\,\mathrm{d}\varphi\,\overrightarrow{U_\varphi}$$

$$\mathrm{d}\vec{S} = r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi\,\overrightarrow{U_r}$$

Volume element:

$$\mathrm{d}V = r^2\sin\theta\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\varphi$$

Surface Area of a Sector of a Cylinder:

Lateral surface area element:

$$\mathrm{d}S = R\,\mathrm{d}\theta\,\mathrm{d}z$$
$$A = \int_{z=0}^{5}\int_{\theta=\frac{\pi}{4}}^{\frac{2\pi}{3}} R\,\mathrm{d}\theta\,\mathrm{d}z$$
$$= Rh\left(\frac{2\pi}{3} - \frac{\pi}{4}\right)$$
$$A = 2\cdot 5\cdot\left(\frac{8\pi - 3\pi}{12}\right) = 10\cdot\frac{5\pi}{12}$$
$$= \frac{50\pi}{12} = \frac{25\pi}{6}\ \mathrm{m}^2$$

Surface Area of a Spherical Band

$$\mathrm{d}\vec{S} = r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi\,\overrightarrow{U_r}$$
$$A = \int_{\varphi=0}^{2\pi}\int_{\theta=\alpha}^{\beta} R^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi$$
$$= 2\pi R^2[-\cos\theta]_{\alpha}^{\beta}$$
$$A = 2\pi R^2(\cos\alpha - \cos\beta)$$
$$A = 2\pi R^2(\cos 0 - \cos\pi) = 2\pi R^2\big(1 - (-1)\big) = 4\pi R^2$$

Volume of a Sphere:

$$V = \int_{r=0}^{R}\int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi} r^2\sin\theta\,\mathrm{d}\varphi\,\mathrm{d}\theta\,\mathrm{d}r$$
$$V = \left[\frac{r^3}{3}\right]_0^R\cdot[-\cos\theta]_0^{\pi}\cdot[\varphi]_0^{2\pi}$$
$$= \frac{2R^3}{3}\cdot 2\pi = \frac{4\pi R^3}{3}$$

Problem 03:

Calculate by integration:

1. The perimeter of a circle with radius *R* (figure a).
2. The area of a disk with radius *R* (figure b).
3. The volume *V* of a cylinder with radius *R* and height *h* (figure c).
4. The surface area of a hemisphere with radius *R* (figure d).
5. The volume *V* of a sphere with radius *R* (figure e).
6. The volume of a cone with height *h* and a circular base of radius *R* (figure f).

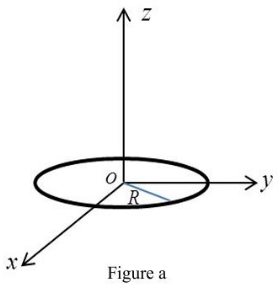


Figure a

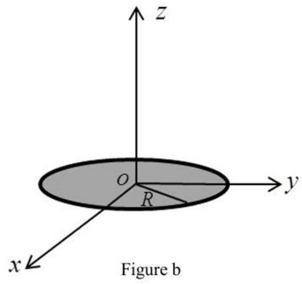


Figure b

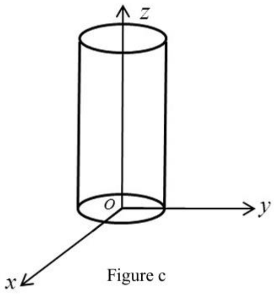


Figure c

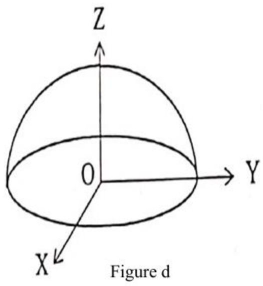


Figure d

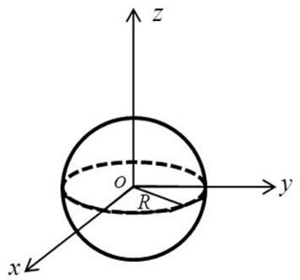


Figure e

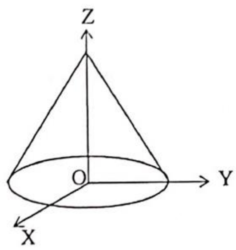


Figure f

Solution:
Perimeter of a Circle:

$$P = \int_{0}^{2\pi} R \, \mathrm{d}\theta = 2\pi R$$

Area of a Disk

$$A = \int_{\theta=0}^{2\pi} \int_{r=0}^{R} r \, \mathrm{d}r \, \mathrm{d}\theta$$

$$= \frac{R^2}{2} \cdot 2\pi = \pi R^2$$

Volume:

$$V = \int_{z=0}^{h} \int_{\theta=0}^{2\pi} \int_{r=0}^{R} r \, \mathrm{d}r \, \mathrm{d}\theta \, \mathrm{d}z$$

$$= \pi R^2 h$$

Surface Area of a Hemisphere:

$$A = \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\frac{\pi}{2}} R^2 \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi$$

$$= 2\pi R^2 (0 + 1) = 2\pi R^2$$

Volume of a Sphere:

$$V = \int_{r=0}^{R} \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} r^2 \sin\theta \, \mathrm{d}\varphi \, \mathrm{d}\theta \, \mathrm{d}r$$

$$= \frac{4\pi R^3}{3}$$

Volume of a Cone:

Equation of cone:

$$y = \frac{R}{h}x, \quad x \in [0, h]$$

$$V = \pi \int_0^h \left(\frac{R}{h}x\right)^2 \mathrm{d}x$$

$$= \frac{1}{3}\pi R^2 h$$

Problem 04:

The expression of a vector $\vec{V}$ is given as:

$$\vec{V} = (2xy + z^3)\vec{\imath} + (x^2 + 2y)\vec{\jmath} + (3xz^2 - 2)\vec{k}$$

Prove that $\overrightarrow{grad} \times \vec{V} = \vec{0}$

Solution:

$$\left(\partial_y V_z - \partial_z V_y\right) =$$

$$\partial_y(3xz^2 - 2) - \partial_z(x^2 + 2y) = 0 - 0 = 0$$

$$(\partial_z V_x - \partial_x V_z) =$$

$$\partial_z(2xy + z^3) - \partial_x(3xz^2 - 2)$$

$$= 3z^2 - 3z^2 = 0$$

$$\left(\partial_x V_y - \partial_y V_x\right) =$$

$$\partial_x(x^2 + 2y) - \partial_y(2xy + z^3)$$

$$= 2x - 2x = 0$$

Therefore,

$$\vec{\nabla} \times \vec{V} = 0$$

Problem 06:

Considering an orthonormal coordinate system (oxyz) with basis vectors $(o, \vec{\imath}, \vec{\jmath}, \vec{k}\,)$. At any point M(x,y,z) in space, a physical quantity f is defined as:

f(M) = r^2.

Where $\vec{r} = OM = x\,\vec{\imath} + y\,\vec{\jmath} + z\,\vec{k}$.

1- Determine the gradient of the scalar field f.

2- Calculate the differential df

3- Prove that at any point M in space, the differential df of the function f is related to the gradient of f elementary displacement vector $d\vec{r}$ by the relation df = $\overrightarrow{grad}f\ d\vec{r}$.

Solution:

The computation of the gradient in Cartesian coordinates:

$$\nabla f = \frac{\partial f}{\partial x}\vec{\imath} + \frac{\partial f}{\partial y}\vec{\jmath} + \frac{\partial f}{\partial z}\vec{k}$$

$$\frac{\partial f}{\partial x} = 2x, \quad \frac{\partial f}{\partial y} = 2y, \quad \frac{\partial f}{\partial z} = 2z$$

Thus, the gradient is:

$$\nabla f = 2x\vec{\imath} + 2y\vec{\jmath} + 2z\vec{k} = 2\vec{r}$$

The total differential of

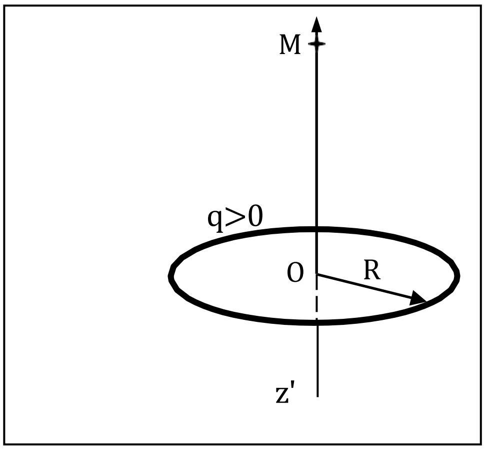
$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

$$df = 2x\ dx + 2y\ dy + 2z\ dz$$

<u>Problem 07:</u>

A circular loop of radius R and center O carries a uniformly distributed positive charge q.

1- Determine the expression for the electric field E(z) at a point M located on the z'0z axis perpendicular to the plane of the loop.
2- Provide the expression for the electric potential V(z) at point M using:
a) Direct calculation.
b) The expression for the electric field E(z). (We will assume the potential is zero at infinity).



Solution:

Electric Field and Potential of a Charged Circular Loop:

The charge is distributed uniformly along the circumference be the linear charge density.

$$\mathrm{d}\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\mathrm{d}q}{(R^2 + z^2)^{3/2}}\left(-R\cos\theta\ \vec{\imath} - R\sin\theta\ \vec{\jmath} + z\,\vec{k}\right)$$

Due to symmetry, the $(\vec{\imath})$ and $(\vec{\jmath})$ components cancel. The total field is only $along(\vec{k})$:

$$\vec{E}(z) = \int \mathrm{r}d\vec{E}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{z}{(R^2+z^2)^{3/2}} \int_0^{2\pi} \lambda R \ \mathrm{d}\theta \, \vec{k}$$

$$\vec{E}(z) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qz}{(R^2+z^2)^{3/2}} \, \vec{k}$$

Electric Potential V(z) by Direct Calculation:

$$V(z) = \int \mathrm{r}dV$$

$$= \int \frac{1}{4\pi\varepsilon_0} \cdot \frac{\mathrm{d}q}{\sqrt{R^2+z^2}}$$

q= $\frac{1}{4\pi\varepsilon_0\sqrt{R^2+z^2}} \int_0^{2\pi} \lambda R \ \mathrm{d}$

$$V(z) = \frac{1}{4\pi\varepsilon_0\sqrt{R^2+z^2}} \cdot \lambda R \cdot 2$$

$$= \frac{q}{4\pi\varepsilon_0\sqrt{R^2+z^2}}$$

Electric Potential from the Electric Field:

$$\vec{E}(z) = -\frac{\mathrm{d}V}{\mathrm{d}z} \, \vec{k}$$

$$\Rightarrow \quad \frac{\mathrm{d}V}{\mathrm{d}z} =$$

$$-E_z = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{qz}{(R^2+z^2)^{3/2}}$$

Integrate from z to (∞), assuming ($V(\infty) = 0$):

$$V(z) = -\int_z^{\infty} \frac{\mathrm{d}V}{\mathrm{d}z'} \, \mathrm{d}z'$$

$$= \frac{q}{4\pi\varepsilon_0} \int_z^{\infty} \frac{z'}{(R^2+z'^2)^{3/2}} \, \mathrm{d}z'$$

Make the substitution ($u = R^2 + z'^2$), so ($\mathrm{d}u = 2z'\mathrm{d}z'$), then:

$$V(z) = \frac{q}{4\pi\varepsilon_0} \left[-\frac{1}{\sqrt{R^2+z^2}} \right]$$

$$= \frac{q}{4\pi\varepsilon_0\sqrt{R^2+z^2}}$$

Problem 08:

a) Find the expression for the electric field created in M, by a uniform linear distribution λ<0 distributed on a length L.

b) Calculate this field when a = 40 cm, b=10cm, λ=−2μC/m, and L=50cm.

Solution:

1/

$$\mathrm{d}E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda\,\mathrm{d}x}{(a+b-x)^2}$$

The field points along the (x)-axis (repulsive if ($\lambda > 0$), attractive if ($\lambda < 0$)):

$$E = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{1}{(a+b-x)^2}\,\mathrm{d}x$$

Change variables: let ($u = a + b - x$), so ($\mathrm{d}u = -\mathrm{d}x$) and limits $become$ ($u = a + b$) to ($u = a + b - L$)

$E = \frac{\lambda}{4\pi\varepsilon_0}\int_{a+b}^{a+b-L}\frac{-1}{u^2}\,\mathrm{d}u$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{u}\right]_{a+b-L}^{a+b}$$

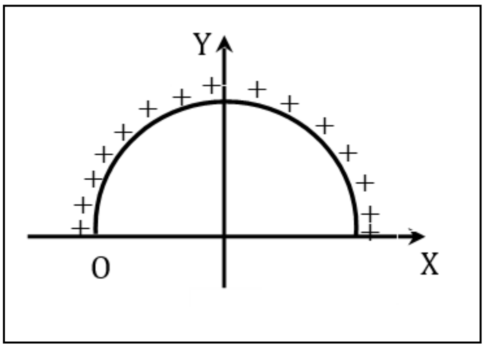
$$E = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{a+b-L} - \frac{1}{a+b}\right)$$

2/

$$E \approx -1.76 \times 10^6\ \mathrm{V/m}$$

Problem 09:

Consider a non-conductive wire in the shape of a semicircle with radius R, uniformly charged with a linear density λ>0 (see the figure on the side).



Given:

R=15cm and λ=10μC/m.

Calculate:

a. The total charge, Q, of the semicircle.

b. The potential produced at the center O.

c. The total electric field is at the center O, and represented.

Solution:

$$L = \pi R$$

The total charge is:

$$Q = \lambda L = \lambda \pi R = 10 \times 10^{-6} \cdot \pi \cdot 0.15 = \boxed{Q = 1.5\pi \times 10^{-6}\,\mathrm{C} \approx 4.712 \times 10^{-6}\,\mathrm{C}}$$

Electric Potential at the Center:

Since the potential due to a line charge element $(\mathrm{d}q)$ at a distance (R) is:

$$\mathrm{d}V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\mathrm{d}q}{R} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda\,\mathrm{dl}}{R}$$

Integrating over the whole arc:

$$V = \frac{\lambda}{4\pi\varepsilon_0 R} \int_0^{\pi R} \mathrm{dl}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R} \cdot \pi R$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \cdot$$

$$= \frac{\lambda}{4\varepsilon_0}$$

Substitute values:

$$V = \frac{10 \times 10^{-6}}{4 \cdot 8.85 \times 10^{-12}} = \boxed{V = 282{,}485\,\mathrm{V} \approx 2.82 \times 10^{5}\,\mathrm{V}}$$

Electric Field at the Center:

Consider a charge element $(\mathrm{d}q = \lambda R\,\mathrm{d}\theta)$ located at angle $(\theta \in [0, \pi])$.

The position vector of the element:

$$\vec{r} = R\cos\theta\ \hat{\imath} + R\sin\theta\ \hat{\jmath}$$

Magnitude of the element field:

$$\mathrm{d}\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\mathrm{d}q}{R^2} \cdot \hat{r}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda R\,\mathrm{d}\theta}{R^2} \cdot (-\cos\theta\ \hat{\imath} - \sin\theta\ \hat{\jmath})$$

Now integrate over the semicircle:

$$\vec{E} = -\frac{\lambda}{4\pi\varepsilon_0 R} \int_0^{\pi} (\cos\theta\ \hat{\imath} + \sin\theta\ \hat{\jmath})\,\mathrm{d}\theta$$

$$\vec{E} = -\frac{\lambda}{4\pi\varepsilon_0 R} \cdot (0\,\hat{\imath} + 2\,\hat{\jmath}) = -\frac{\lambda}{2\pi\varepsilon_0 R}\hat{\jmath}$$

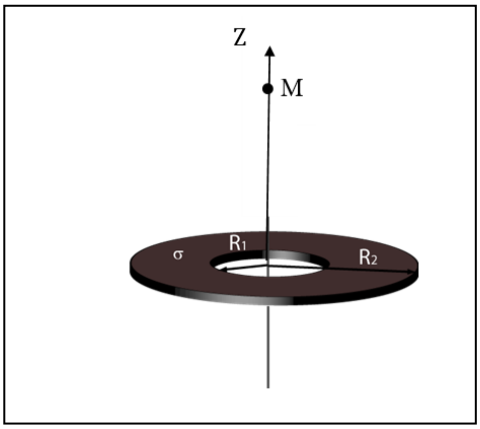
$$\vec{E} = -\frac{\lambda}{2\pi\varepsilon_0 R}\hat{\jmath}$$

$$E = \frac{10 \times 10^{-6}}{2\pi \cdot 8.85 \times 10^{-12} \cdot 0.15} \approx \boxed{1.19 \times 10^{5}\ \mathrm{V/m}}$$

Problem 10:

A circular disk of negligible thickness bounded by two circles of radius R1and R2 with a constant surface charge density σ (see figure on the right).

1- Calculate the electric field created by this charge distribution at point M(0,0,z).
2- What is the expression of the electrical field when $R1$ tends toward zero? Plot the curve E(z)=f(z). What can you conclude about E(0)?

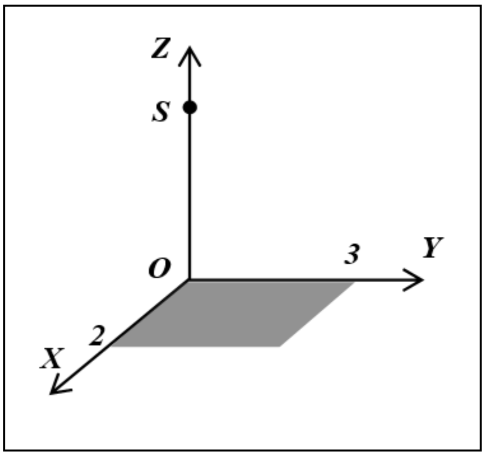


Problem 11:

A finite charge sheet has a surface charge density given by:

$$\sigma = 2x(x^2 + y^2 + 9)^{3/2}\ \mathrm{C/m^2}$$

This sheet extends in the plane z=0 within the region $0 \leq x \leq 2$ m, and $0 \leq y \leq 3$ m. Determine the electric field created at the point **S(0, 0, 3).**



Solution:

We are given a surface charge density:

$$\sigma(x, y) = \frac{2x}{(x^2 + y^2 + 9)^{3/2}} \quad (\mathrm{C/m^{2}})$$

The charge lies in the ($z = 0$) plane over the region ($0 \leq x \leq 2$), ($0 \leq y \leq 3$). We compute the electric field at point ($S(0,0,3)$).

Let a surface element ($\mathrm{d}S = \mathrm{d}x\, \mathrm{d}y$), and consider a point ($P(x, y, 0)$) on the surface. The vector from (P) to (S) is:

$$\vec{r} = \overrightarrow{r_S} - \overrightarrow{r_P} = -x\,\hat{\vec{\imath}} - y\,\hat{\vec{\jmath}} + 3\,\hat{\vec{k}}$$

$$r = \sqrt{x^2 + y^2 + 9}$$

The infinitesimal electric field due to ($\mathrm{d}q = \sigma(x, y)\, \mathrm{d}x\, \mathrm{d}y$) is:

$$\mathrm{d}\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\sigma(x, y)\, \mathrm{d}x\, \mathrm{d}y}{r^3}\vec{r}$$

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2x}{(x^2 + y^2 + 9)^{3/2}} \cdot \frac{\mathrm{d}x\, \mathrm{d}y}{(x^2 + y^2 + 9)^{3/2}} \left(-x\,\hat{\vec{\imath}} - y\,\hat{\vec{\jmath}} + 3\,\hat{\vec{k}}\right)$$

$$\mathrm{d}\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2x}{(x^2 + y^2 + 9)^3} \left(-x\,\hat{\vec{\imath}} - y\,\hat{\vec{\jmath}} + 3\,\hat{\vec{k}}\right) \mathrm{d}x\, \mathrm{d}y$$

Now integrate over the region:

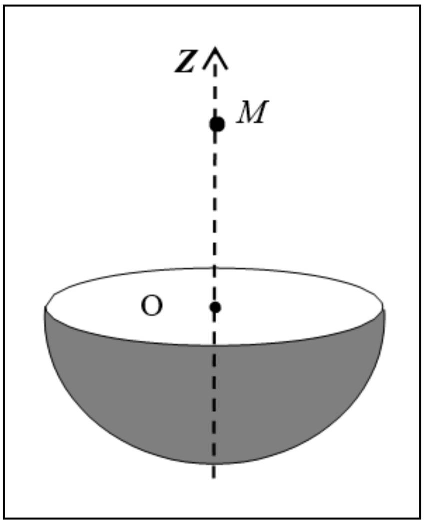
$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \iint_{0 \le x \le 2,\, 0 \le y \le 3} \frac{2x}{(x^2 + y^2 + 9)^3} \left(-x\,\hat{\vec{\imath}} - y\,\hat{\vec{\jmath}} + 3\,\hat{\vec{k}}\right) \mathrm{d}x\, \mathrm{d}y$$

$$E_x = -\frac{1}{4\pi\varepsilon_0} \iint \frac{2x^2}{(x^2 + y^2 + 9)^3} \mathrm{d}x\, \mathrm{d}y$$

$$E_y = -\frac{1}{4\pi\varepsilon_0} \iint \frac{2xy}{(x^2 + y^2 + 9)^3} \mathrm{d}x\, \mathrm{d}y$$

$$E_z = +\frac{1}{4\pi\varepsilon_0} \iint \frac{6x}{(x^2 + y^2 + 9)^3} \mathrm{d}x\, \mathrm{d}y$$

Problem 12:



Consider a hemisphere of radius R, centered at O, with a uniform surface charge density □>0

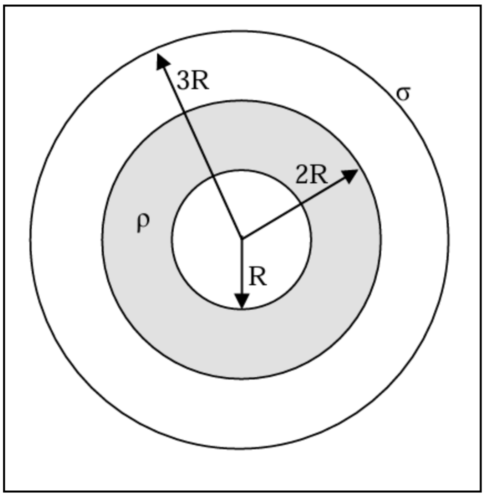
1- Determine the electrostatic potential at a point MMM along the symmetry axis (OzOzOz) of this hemisphere.
2- Derive the expression for the **electric field** at point MMM.
3- Compute the electrostatic potential and field at point O.

Problem 13:

At every point in space, determine the electrostatic field created by an infinite plane carrying a uniform surface charge density σ > 0.

Problem 14:

The figure opposite represents three concentric spheres S1, S2, and S3 with respective radii R, 2R, and 3R. The space between S1 and S2 is charged with a uniform volume charge density ρ. The surface of sphere S3 carries a uniform surface charge density σ.

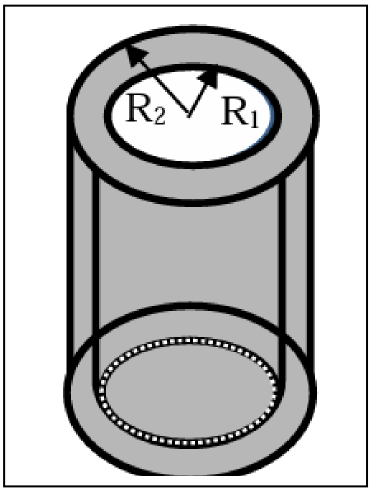


1. Determine the electrostatic field at every point in space.

2. Deduce the expression for the electrostatic potential, up to a constant.

Problem 15:

We consider a volume charge distribution with density ρ, uniformly distributed between two coaxial cylinders of infinite length and respective radii R1 and R2.



1. What is the Gaussian surface suitable for this system? Justify your answer.
2. At every point in space, determine the electrostatic field created by this charged system.
3. Deduce the expression for the electrostatic potential created in the different regions of space.

Problem 16:

A sphere with center O and radius R is charged in volume with a charge density given by:

$$\rho\,(r) = \rho_0\left(1 - \frac{r}{R}\right), where\ \rho_0 = constant$$

1. Determine the electric field at every point in space.
2. At which point in space does the field reach its maximum value?
3. Represent $E(r)$.

Problem 17:

Consider the electrical circuit shown in the figure below, consisting of three resistors: R_1=6 Ω, R_2=4 Ω, and R_3=12 Ω, powered by a real battery with an electromotive force (e.m.f.) E=20 V and an internal resistance r=1 Ω.

1. Draw the equivalent resistance of the association of resistors R_2 and R_3 ($R_{2,3}$). Find the equivalent resistance Req of the association of resistors R_1, and $R_{2,3}$.
2. Using Ohm's Law, calculate the current intensities through resistors R_1, $R_{2,3}$. Then deduce the currents flowing through resistors R_2 and R_3.
3. Calculate the potential difference $V_B - V_A$ across the terminals of the battery.
4. What energy is dissipated by the Joule effect in this circuit after 10 minutes of operation?

Problem 18:

The electrical circuit in the figure below consists of resistors: R_1 =10 Ω, R_2 =20, R_3=15 Ω, R_4 = 10, R_5= 25 Ω, and two generators with negligible internal resistances: E_1=12 V and E_2=6 V. Identify the number of: (a) nodes, (b) branches, (c) independent loops. How many currents need to be determined?

1. Establish the node and loop equations for the circuit.
2. Determine the current intensities in each branch.
3. Which of the two generators operates as a receiver? Justify.
4. Assuming the potential at point A is VA=+12 V, determine the potential VC at point C.
5. Calculate the power supplied or consumed by each dipole, then perform an energy balance. What do you conclude?

Problem 19:

In the circuit shown in the figure opposite, E = 250 V and R = 1 kΩ .

1. Calculate the intensities of the electric currents circulating in each resistor.
2. Deduce the value and direction of the current in the horizontal wire between points A and F.
3. Determine the voltage $U_{BD}=V_B-V_D$ between points B and D.
4. Calculate the power supplied by each electromotive force.

Problem 20:

Five resistors, R_1=2 Ω, R_2=3 Ω, R_3=1 Ω, R_4=5 Ω , and R_5=4 Ω , are connected as shown in the circuit diagram opposite.

1. Using the laws of nodes and meshes, express the currents flowing through each resistor as a function of the current I supplied by the source E.
2. Determine the equivalent resistance R_{eq} of the circuit.

Problem 21:

A battery with an electromotive force (e.m.f.) E=6 V and negligible internal resistance powers the circuit shown in the figure below. When switch K is open, as shown in the figure, the electric current intensity measured through the battery is 1 mA. When the switch is closed in position 1, the current intensity through the battery is 1.2 mA. When the switch is closed in position 2, the current intensity through the battery is 2 mA. Calculate the values of the resistances R1, R2, and R3.

Problem 22:

Consider the electric circuit shown below, powered by a generator with an e.m.f. E=200 V, E=200V and five resistors R1=10 Ω, R2=40 Ω, R3=20, R4=10, R5=30 Ω. Calculate:

1. The equivalent resistance R_{AB} between terminals A and B.

2. The current intensity I delivered by the generator.
3. The electric voltages across the terminals of each resistor.
4. The intensities of the currents passing through each resistor.

Problem 23:

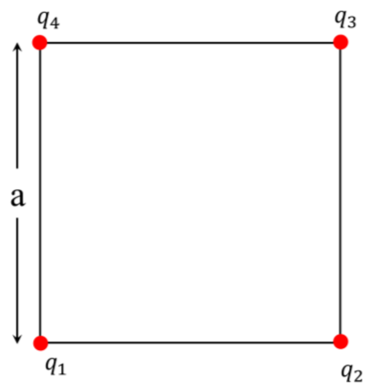
Four point charges are placed at the corners of a square of side length a as follows:

.$q_1 = +q \ at \ A(0,0)$

.$q_2 = +q \ at \ B(a,0)$

$q_3 = -q \ at \ C(a,a)$.

.$q_4 = -q \ at \ D(0,a)$



Let point M be located at the center of the square.

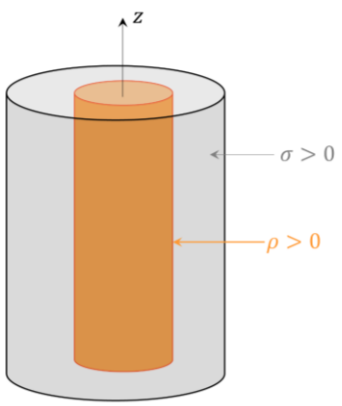
1- Find the net electric field $\vec{E}_M$ at point M due to the four charges.

2- Find the electrostatic potential V_M at point M.

3- Suppose a charge q_0 is placed at point M, calculate the force acting on it.

Problem 24:

We consider two cylinders, one of which is charged volumetrically with a positive electric charge and the other is charged surface-wise with a positive electric charge. The charge densities of the two cylinders are ρ and σ.



1- What is the Gaussian surface SG suitable for this system?

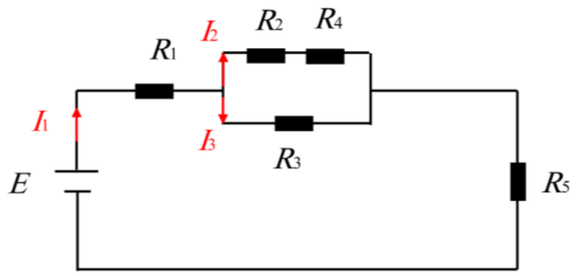
2- Determine, using Gauss's theorem, the electrostatic field created by this system at any point in space.

3- Deduce the expression of the electrostatic potential created in the different regions of space.

Problem 25:

Consider the electrical circuit composed of an ideal generator (no internal resistance) with electromotive force E = 10 V, and five resistors with the following values:

$R_1 = 3\Omega$, $R_2 = 4\ \Omega$, $R_3 = 3\Omega$, $R_4 = 1\Omega$, and $R_5 = 1\ \Omega$.



1- Find the number of nodes (junctions), branches, and independent loops.
2- Calculate the equivalent resistor Req of all the resistors of the circuit.
3- Calculate the intensity of currents I_1, I_2, and I_3.
4- Refined the current I_1 by using the node law.
5- Calculate the electrical power P_f supplied by the generator, then the total power P_d dissipated in the resistors.

Key words

Arabic (العربية)	French (Français)	English
الكهرباء	Électricité	Electricity
الدارة الكهربائية	Circuit électrique	Electrical circuit
الإلكترون	Électron	Electron
البروتون	Proton	Proton
النيوترون	Neutron	Neutron
المقاومة الكهربائية	Résistance électrique	Electrical resistance
الموصل الكهربائي	Conducteur électrique	Electrical conductor
العازل الكهربائي	Isolant électrique	Electrical insulator
الحقل الكهربائي	Champ électrique	Electric field
الحقل المغناطيسي	Champ magnétique	Magnetic field
فرق الجهد	Différence de potentiel	Potential difference
التدفق المغناطيسي	Flux magnétique	Magnetic flux
القوة الكهربائية	Force électrique	Electric force
القوة المغناطيسية	Force magnétique	Magnetic force
القدرة الكهربائية	Puissance électrique	Electrical power
التيار الكهربائي	Courant électrique	Electric current
التيار المستمر	Courant continu (DC)	Direct current (DC)
التيار المتردد	Courant alternatif (AC)	Alternating current (AC)
الشحنة الكهربائية	Charge électrique	Electric charge
فرق الجهد الكهربائي	Tension électrique	Voltage
قانون أوم	Loi d’Ohm	Ohm’s law
الموصلية الكهربائية	Conductivité électrique	Electrical conductivity
الاستقطاب الكهربائي	Polarisation électrique	Electric polarization
المقاومة المتغيرة	Résistance variable	Variable resistance
القدرة الفعالة	Puissance active	Active power
القدرة الظاهرة	Puissance apparente	Apparent power
القدرة غير الفعالة	Puissance réactive	Reactive power
المغناطيسية	Magnétisme	Magnetism
المغناطيس	Aimant	Magnet
القطب المغناطيسي	Pôle magnétique	Magnetic pole
التدفق المغناطيسي	Flux magnétique	Magnetic flux
الحث المغناطيسي	Induction magnétique	Magnetic induction
المواد المغناطيسية	Matériaux magnétiques	Magnetic materials

التأثير الكهرومغناطيسي	Effet électromagnétique	Electromagnetic effect
النفاذية المغناطيسية	Perméabilité magnétique	Magnetic permeability
خطوط المجال المغناطيسي	Lignes de champ magnétique	Magnetic field lines
القوة الدافعة المغناطيسية	Force magnétomotrice	Magnetomotive force (MMF)
الحث الكهرومغناطيسي	Induction électromagnétique	Electromagnetic induction
قانون فاراداي	Loi de Faraday	Faraday's law
قانون لنز	Loi de Lenz	Lenz’s law
المحول الكهربائي	Transformateur électrique	Electrical transformer
المولد الكهربائي	Générateur électrique	Electrical generator
المحرك الكهربائي	Moteur électrique	Electric motor
الدارة المغناطيسية	Circuit magnétique	Magnetic circuit
المحاثة	Inductance	Inductance
الملفات الكهربائية	Bobines électriques	Electrical coils
التردد الكهربائي	Fréquence électrique	Electrical frequency
المكثف الكهربائي	Condensateur électrique	Electrical capacitor
السعة الكهربائية	Capacité électrique	Capacitance
المحاثة الذاتية	Auto-induction	Self-induction
المحاثة المتبادلة	Induction mutuelle	Mutual induction
ثابت العزل الكهربائي	Permittivité électrique	Electrical permittivity
قانون كيرشوف للتيار	Loi des courants de Kirchhoff	Kirchhoff's Current Law (KCL)
قانون كيرشوف للجهد	Loi des tensions de Kirchhoff	Kirchhoff's Voltage Law (KVL)
قانون جاوس للكهرباء	Loi de Gauss pour l’électricité	Gauss’s law for electricity
قانون جاوس للمغناطيسية	Loi de Gauss pour le magnétisme	Gauss’s law for magnetism
قانون أمبير	Loi d'Ampère	Ampère's law
قانون كولومب	Loi de Coulomb	Coulomb's law
قانون فاراداي للحث	Loi de Faraday sur l’induction	Faraday’s law of induction

References:

Alexander, M.A.M., 2025. Structure of Earth's Magnetic Field and Solar Wind Interaction. International Journal of Advanced Research and Interdisciplinary Scientific Endeavours 2, 587–596. https://doi.org/10.61359/11.2206-2521

Bleaney, B.I., Bleaney, B., 2013. Electricity and Magnetism, Volume 2: Third Edition. OUP Oxford.

Burke, L., 1952. On the tunnel effect. Quarterly Journal of Experimental Psychology 4, 121–138. https://doi.org/10.1080/17470215208416611

Chien, C., 2013. The Hall Effect and Its Applications. Springer Science & Business Media.

Faraday, M., 1991. The Correspondence of Michael Faraday. IET.

Lévy, T., 2011. La mesure du cercle d'Archimède au moyen age: Le témoignage des textes hébreux. Brill.

Millikan., R.A., 1913. On the Elementary Electrical Charge and the Avogadro Constant. Phys. Rev. 2, 109–143. https://doi.org/10.1103/PhysRev.2.109

Morgan, E.S., 2002. Benjamin Franklin. Yale University Press.

Reyes, M.G., 2004. The rhetoric in mathematics: Newton, Leibniz, the calculus, and the rhetorical force of the infinitesimal. Quarterly Journal of Speech 90, 163–188. https://doi.org/10.1080/0033563042000227427