SERIE DE TD N°02

Solution de L'exercice 01:

Résolution du système par la méthode de Cramer :

$$\begin{cases} 2x + y + 2z = 1 \\ x - y + 2z = 2 \\ -3x - z = -1 \end{cases}$$

Matrice associée au système $A \cdot X = B$

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & -1 & 2 \\ -3 & 0 & -1 \end{pmatrix} ; \begin{pmatrix} 2 & 1 & 2 \\ 1 & -1 & 2 \\ -3 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$det(A) = \begin{vmatrix} 2 & 1 & 2 \\ 1 & -1 & 2 \\ -3 & 0 & -1 \end{vmatrix} = -3\begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} - \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= -3(2+2) - (-2-1) - 12 + 3 = 9$$

Par la méthode de Cramer :

$$x = \frac{\begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & 2 \\ -1 & 0 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 2 \\ 1 & -1 & 2 \\ -3 & 0 & -1 \end{vmatrix}} = \frac{1}{9} \; ; \; y = \frac{\begin{vmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ -3 & -1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 2 \\ 1 & -1 & 2 \\ -3 & 0 & -1 \end{vmatrix}} = -\frac{5}{9}$$

$$y = \frac{\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ -3 & 0 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 2 \\ 1 & -1 & 2 \\ -3 & 0 & -1 \end{vmatrix}} = \frac{2}{3}$$

Solution de L'exercice 02:

Résolution du système par la méthode de la matrice inverse :

$$\begin{cases} 2x + y + z = 1 \\ x - y + z = 0 \\ x + y + 3z = 2 \end{cases}$$

Matrice associée au système $A \cdot X = B$

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix} ; \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$det(A) = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = 2\begin{vmatrix} -1 & 1 \\ 1 & 3 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= 2(-3-1) - (3-1) + (1+1) = -8$$

$det(A) = -8 \neq 0$; A est inversible

Alors : $A^{-1} = \dfrac{1}{det(A)}\left(C^A\right)^t$

$$C^A = \begin{pmatrix} +\begin{vmatrix} -1 & 1 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} \\ +\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} \end{pmatrix} ;$$

$$A^{-1} = \begin{pmatrix} 1/2 & 1/4 & -1/4 \\ 1/4 & -5/8 & 1/8 \\ -1/4 & 1/8 & 3/8 \end{pmatrix}$$

On a : $A \cdot X = B \Rightarrow X = A^{-1} \cdot B$

$$X = \begin{pmatrix} 1/2 & 1/4 & -1/4 \\ 1/4 & -5/8 & 1/8 \\ -1/4 & 1/8 & 3/8 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix}$$

Solution de L'exercice 03:

Résolution du système par la méthode de GAUSS :

$$\begin{cases} x - 2y + t = 2 \\ x - y - z + 4t = 2 \\ x - 3y + z - 2t = 2 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 2 \\ 1 & -1 & -1 & 4 & 2 \\ 1 & -3 & 1 & -2 & 2 \end{array}\right) \begin{array}{l} L_1 \\ L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_3 - L_1 \end{array} \sim$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 2 \\ 0 & 1 & -1 & 3 & 0 \\ 0 & -1 & 1 & -3 & 0 \end{array}\right) \begin{array}{l} L_1 \\ L_2 \\ L_3 \leftarrow L_3 + L_2 \end{array} \sim$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 2 \\ 0 & 1 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{l} L_1 \\ L_2 \\ L_3 \leftarrow L_3 + L_2 \end{array}$$

Finalement, on obtient le système d'équations équivalent :

$$\begin{cases} x-2y+t=2 \\ y-z+3t=0 \end{cases}$$

Avec une infinité de solutions comme suit :

$$\begin{cases} x=2y-t+2 \\ z=y+3t \end{cases}$$

Solution de L'exercice 04:

Soit le système d'équations suivant :

$$\begin{cases} 2x+2y-\sqrt{2}z=1 \\ 2x+4y+2\sqrt{2}z=2 \\ -\sqrt{2}x+2\sqrt{2}y+z=\sqrt{2} \end{cases}$$

1) Forme matricielle :

$$\begin{pmatrix} 2 & 2 & -\sqrt{2} \\ 2 & 4 & 2\sqrt{2} \\ -\sqrt{2} & 2\sqrt{2} & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ \sqrt{2} \end{pmatrix}$$

2) Calcul de A^2 :

$$A^2 = A\times A = \begin{pmatrix} 2 & 2 & -\sqrt{2} \\ 2 & 4 & 2\sqrt{2} \\ -\sqrt{2} & 2\sqrt{2} & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 2 & -\sqrt{2} \\ 2 & 4 & 2\sqrt{2} \\ -\sqrt{2} & 2\sqrt{2} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 8 & \sqrt{2} \\ 8 & 28 & 8\sqrt{2} \\ \sqrt{2} & 8\sqrt{2} & 11 \end{pmatrix}$$

3) Le rang de la matrice A :

$$det(A) = 2\begin{vmatrix} 4 & 2\sqrt{2} \\ 2\sqrt{2} & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 1 \end{vmatrix} - \sqrt{2}\begin{vmatrix} 2 & 4 \\ -\sqrt{2} & 2\sqrt{2} \end{vmatrix}$$

$$= 2(4-8)-2(2+4)-\sqrt{2}(4\sqrt{2}+4\sqrt{2}) = -8-12-16$$

$det(A) = -36 \neq 0$; A est inversible et le rang de la matrice A est égal à 3 (le système admet une unique solution).

4) La matrice inverse de A :

$$A^{-1} = \frac{1}{det(A)}(C^A)^t = \frac{1}{-36}(C^A)^t$$

$$C^A = \begin{pmatrix} +\begin{vmatrix} 4 & 2\sqrt{2} \\ 2\sqrt{2} & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 1 \end{vmatrix} & +\begin{vmatrix} 2 & 4 \\ -\sqrt{2} & 2\sqrt{2} \end{vmatrix} \\ -\begin{vmatrix} 2 & -\sqrt{2} \\ 2\sqrt{2} & 1 \end{vmatrix} & +\begin{vmatrix} 2 & -\sqrt{2} \\ \sqrt{2} & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ -\sqrt{2} & 2\sqrt{2} \end{vmatrix} \\ +\begin{vmatrix} 2 & -\sqrt{2} \\ 4 & 2\sqrt{2} \end{vmatrix} & -\begin{vmatrix} 2 & -\sqrt{2} \\ 2 & 2\sqrt{2} \end{vmatrix} & +\begin{vmatrix} 2 & 2 \\ 2 & 4 \end{vmatrix} \end{pmatrix};$$

$$C^A = \begin{pmatrix} -4 & -6 & 8\sqrt{2} \\ -6 & 0 & -6\sqrt{2} \\ 8\sqrt{2} & -6\sqrt{2} & 4 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 1/9 & 1/6 & -2\sqrt{2}/9 \\ 1/6 & 0 & \sqrt{2}/6 \\ -2\sqrt{2}/9 & \sqrt{2}/6 & -1/9 \end{pmatrix}$$

5) Déduction des valeurs de x, y et z :

On a : $A\cdot X = B \Rightarrow X = A^{-1}\cdot B$

$$X = \begin{pmatrix} 1/9 & 1/6 & -2\sqrt{2}/9 \\ 1/6 & 0 & \sqrt{2}/6 \\ -2\sqrt{2}/9 & \sqrt{2}/6 & -1/9 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ \sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 1/2 \\ 0 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 \\ 1/2 \\ 0 \end{pmatrix}$$

------------------Intégrales------------------

Solution de L'exercice 05:

1) $\int (3x^2+4)^3 x\cdot dx = \frac{1}{6}\int (3x^2+4)^3(6x).dx$

$= \frac{1}{6}\left[\frac{1}{4}(3x^2+4)^4 + k\right]$

$= \frac{1}{24}(3x^2+4)^4 + C$

2) $\int \frac{x^3-2x+3}{x+1}dx$; nous avons :

$\frac{x^3-2x+3}{x+1} = x^2-x-1+\frac{4}{x+1}$, alors

$\int \frac{x^3-2x+3}{x+1}dx = \frac{1}{3}x^3-\frac{1}{2}x^2-x+4\ln|x+1|+C$

3) $\int \frac{2x+1}{\sqrt{x^2+x+1}}dx = \int (x^2+x+1)^{-1/2}(2x+1)dx$

$= 2\sqrt{x^2+x+1}+C$

4) $\int e^x\sqrt{e^x-1}\,dx = \int (e^x-1)^{1/2}e^x dx$

$= \frac{2}{3}(e^x-1)^{3/2}+C$

5) $\int \frac{x}{1+x^4}dx$, posons $t=x^2$

Alors : $dt = 2x\cdot dx \Rightarrow x\cdot dx = \frac{1}{2}dt$

$$\int \frac{x}{1+x^4}dx = \frac{1}{2}\int \frac{1}{1+t^2}dt = \frac{1}{2}\text{Arctg}(t)+C$$

6) $\int \frac{x-2}{(2x-3)^2}dx$, nous avons :

$$\frac{x-2}{(2x-3)^2} = \frac{1}{2}\cdot\frac{2x-4}{(2x-3)^2}$$

$$= \frac{1}{2}\cdot\left[\frac{2x-3}{(2x-3)^2} - \frac{1}{(2x-3)^2}\right]$$

$$\int \frac{x-2}{(2x-3)^2}dx = \frac{1}{2}\cdot\int \frac{2x-3}{(2x-3)^2}dx - \frac{1}{2}\cdot\int \frac{1}{(2x-3)^2}dx$$

$$= \frac{1}{8}\cdot\int \frac{4\cdot(2x-3)}{(2x-3)^2}dx - \frac{1}{4}\cdot\int (2x-3)^{-2}\cdot 2\cdot dx$$

$$= \frac{1}{4}\cdot ln|2x-3| + \frac{1}{4}(2x-3)^{-1} + C$$

7) $\int \sqrt{1+x^2}dx = ?$; on pose

$x = sh(t) \Rightarrow dx = ch(t)dt$

$$\int \sqrt{1+x^2}dx = \int \sqrt{1+sh^2(t)}ch(t)dx$$

Or $\quad ch^2(t) - sh^2(t) = 1$

$$\int \sqrt{1+x^2}dx = \int ch(t)\cdot ch(t)dx$$

$$= \int ch^2(t)dx$$

Or $\begin{cases} ch^2(t) = \dfrac{1+ch(2t)}{2} \\ sh(2t) = 2ch(t)sh(t) \end{cases}$

$$\int \sqrt{1+x^2}dx = \frac{1}{2}\int dt + \frac{1}{2}\int ch(2t)\,dt$$

$$= \frac{t}{2} + \frac{sh(2t)}{4} + C$$

Alors :

$$\int \sqrt{1+x^2}dx = \frac{1}{2}\text{Argsh}(x) + \frac{1}{2}ch(t)sh(t) + C$$

$$= \frac{1}{2}\text{Argsh}(x) + \frac{1}{2}x\sqrt{1+x^2} + C$$

8) $\int \frac{dx}{sh(x)}$; $sh(x) = \frac{e^x - e^{-x}}{2}$

$\int \frac{dx}{sh(x)} = \int \frac{2\cdot dx}{e^x - e^{-x}}$; posons : $t = e^x$

$$\Rightarrow dt = e^x dx$$

donc $\; x = ln(t) \Rightarrow dx = \frac{dt}{t}$

$$\int \frac{dx}{sh(x)} = \int \frac{2\cdot dt}{t\cdot\left(t - \frac{1}{t}\right)} = 2\int \frac{dt}{t^2 - 1}$$

$$\int \frac{dx}{sh(x)} = -2\text{Argth}(t) + C$$

9) $\int \frac{5x+2}{x^2-5x+4}dx$; $A = 5$, $B = 2$

$a = 1$, $b = -5$, $c = 4$

D'une manière générale, on a :

$$\int \frac{Ax+B}{ax^2+bx+c}dx = \frac{A}{2a}\int \frac{2ax+b}{ax^2+bx+c}dx + \left(B - \frac{Ab}{2a}\right)\int \frac{1}{ax^2+bx+c}dx$$

$$\int \frac{5x+2}{x^2-5x+4}dx = \frac{5}{2}\int \frac{2x-5}{x^2-5x+4}dx + \left(2 - \frac{5\cdot(-5)}{2}\right)\int \frac{1}{x^2-5x+4}dx$$

$$= \frac{5}{2}ln\left|x^2-5x+4\right| + \frac{29}{2}\int \frac{1}{\left(x-\frac{5}{2}\right)^2 - \frac{9}{4}}dx$$

Posons : $t = x - \frac{5}{2} \Rightarrow dt = dx$

$$\int \frac{5x+2}{x^2-5x+4}dx = \frac{5}{2}ln\left|x^2-5x+4\right| + \frac{29}{2}\int \frac{1}{t^2 - \left(\frac{3}{2}\right)^2}dt$$

$$= \frac{5}{2}ln\left|x^2-5x+4\right| + \frac{29}{2}\cdot\left(\frac{1}{2\cdot\left(\frac{3}{2}\right)}\right)ln\left|\frac{t-3/2}{t+3/2}\right| + C$$

$$= \frac{5}{2}ln\left|x^2-5x+4\right| + \frac{29}{6}\cdot ln\left|\frac{x-4}{x-1}\right| + C$$

Solution de L'exercice 06:

I)

1) $\int_1^2 \frac{dt}{t\cdot(1+t)} = \int_1^2 \left[\frac{1}{t} - \frac{1}{1+t}\right]dt$

$$= \left[ln|t|\right]_1^2 - \left[ln|1+t|\right]_1^2 = ln\,\frac{4}{3}$$

Puis $\int_1^2 \frac{ln(1+t)}{t^2}dt$;

Intégrons par parties :

$U = ln(1+t) \Rightarrow dU = \frac{dt}{1+t}$

$dV = \frac{dt}{t^2} \quad \Rightarrow V = -\frac{1}{t}$

$$\int_1^2 \frac{ln(1+t)}{t^2}dt = \left[-\frac{1}{t}ln(1+t)\right]_1^2 + \int_1^2 \frac{dt}{t\cdot(1+t)}$$

$$\left[-\frac{1}{t}ln(1+t)\right]_1^2 + ln\frac{4}{3} = ln\frac{8}{\sqrt{27}}$$

2) $\int_0^1 \frac{t^2}{1+t^2}\cdot \text{Arctg}(t)dt$

Nous avons : $\frac{t^2}{1+t^2} = 1 - \frac{1}{1+t^2}$

$$\int_0^1 \frac{t^2}{1+t^2}\cdot \text{Arctg}(t)dt = \int_0^1 \text{Arctg}(t)dt - \int_0^1 \frac{1}{1+t^2}\cdot \text{Arctg}(t)dt$$

$I_1 = \int_1^2 \text{Arctg}(t)dt$; Intégration par parties :

$$U = \text{Arctg}(t) \Rightarrow dU = \frac{dt}{1+t^2}$$

$$dV = dt \qquad \Rightarrow V = t$$

$$I_1 = \left[t\cdot \text{Arctg}(t)\right]_0^1 - \int_0^1 \frac{t}{1+t^2}dt$$

$$I_1 = \left[t\cdot \text{Arctg}(t)\right]_0^1 - \frac{1}{2}\left[ln(1+t^2)\right]_0^1 = \frac{\pi}{4} - ln\sqrt{2}$$

$$I_2 = \int_0^1 \frac{1}{1+t^2}\cdot \text{Arctg}(t)dt = \frac{1}{2}\left[\text{Arctg}^2(t)\right]_0^1 = \frac{1}{2}\left(\frac{\pi}{4}\right)^2 = \frac{\pi^2}{32}$$

Alors $\int_0^1 \frac{t^2}{1+t^2}\cdot \text{Arctg}(t)dt = \frac{\pi}{4} - ln\sqrt{2} - \frac{\pi^2}{32}$

3) $\int_0^{\pi/4} cos^3(t)\cdot sin^2(t)\,dt = \int_0^{\pi/4} cos^2(t)\cdot sin^2(t)cos(t)dt$

Posons $u = sin(t) \Rightarrow du = cos(t)dt$

$sin(t) = 0 \Rightarrow u = 0$

$sin(t) = \pi/4 \Rightarrow u = \sqrt{2}/2$

$$\int_0^{\pi/4} cos^3(t)\cdot sin^2(t)\,dt = \int_0^{\sqrt{2}/2}(1-u^2)\cdot u^2 du$$

$$= \left[\frac{u^3}{3}\right]_0^{\sqrt{2}/2} - \left[\frac{u^5}{5}\right]_0^{\sqrt{2}/2} = \frac{7\sqrt{2}}{120}$$

II)

$$I = \int_0^1 \frac{ln(x+1)}{1+x^2}dx = ?$$

Posons $x = \frac{1-t}{1+t} \Rightarrow t = \frac{1-x}{1+x}$

$$dx = \frac{-2}{(1+t)^2}dt \qquad \begin{cases} x = 0 \Rightarrow t = 1 \\ x = 1 \Rightarrow t = 0 \end{cases}$$

$$I = -\int_0^1 \frac{ln\left(\frac{1-t}{1+t}+1\right)}{1+\left(\frac{1-t}{1+t}\right)^2}\times\frac{-2}{(1+t)^2}dt$$

$$I = 2\int_0^1 \frac{ln\left(\frac{2}{1+t}\right)}{(1+t)^2+(1-t)^2}dt = \int_0^1 \frac{ln\left(\frac{2}{1+t}\right)}{1+t^2}dt$$

$$I = \int_0^1 \frac{ln(2)}{1+t^2}dt - \int_0^1 \frac{ln(1+t)}{1+t^2}dt$$

$$I = ln(2)\left[\text{Arctg}(t)\right]_0^1 - I$$

$$2I = ln(2)\left(\frac{\pi}{4}\right)$$

$$I = ln(2)\left(\frac{\pi}{8}\right)$$

$J = \int_0^1 \frac{\text{Arctg}(x)}{1+x^2}\cdot dx = ?$ Par intégrale par parties :

$$U = \text{Arctg}(x) \Rightarrow dU = \frac{dx}{1+t^2}$$

$$dV = \frac{dx}{1+x} \qquad \Rightarrow V = ln(1+x)$$

$$J = \left[ln(1+x)\cdot \text{Arctg}(x)\right]_0^1 - \int_0^1 \frac{ln(1+x)}{1+x^2}dx$$

$$J = ln(2)\left(\frac{\pi}{4}\right) - ln(2)\left(\frac{\pi}{8}\right)$$

$$J = ln(2)\left(\frac{\pi}{8}\right)$$

<u>**Solution de L'exercice 07**</u>:

$H = \int_0^{\pi/8} e^{-2t}\,cos(2t)\,dt = ?$ Par intégrale par parties :

$$U = e^{-2t} \qquad \Rightarrow dU = -2\,e^{-2t}dt$$

$$dV = cos(2t)\,dt \quad \Rightarrow V = \frac{1}{2}sin(2t)$$

$$H = \left[\frac{1}{2}e^{-2t}\,sin(2t)\right]_0^{\pi/8} + \int_0^{\pi/8} e^{-2t}\,sin(2t)\,dt$$

$$H = \frac{1}{2}e^{-\pi/4}\,sin(\pi/4) + \int_0^{\pi/8} e^{-2t}\,sin(2t)\,dt$$

$$H = \frac{\sqrt{2}}{4}e^{-\pi/4} + \int_0^{\pi/8} e^{-2t}\,sin(2t)\,dt$$

$$\int_0^{\pi/8} e^{-2t}\sin(2t)\,dt = ?$$

Par intégrale par parties :

$U = e^{-2t} \qquad \Rightarrow dU = -2\,e^{-2t}dt$

$dV = \sin(2t)\,dt \quad \Rightarrow V = -\frac{1}{2}\cos(2t)$

$$\int_0^{\pi/8} e^{-2t}\sin(2t)\,dt = -\frac{1}{2}\left[e^{-2t}\cos(2t)\right]_0^{\pi/8} - \int_0^{\pi/8} e^{-2t}\cos(2t)\,dt$$

$$\int_0^{\pi/8} e^{-2t}\sin(2t)\,dt = -\frac{1}{2}\left[e^{-2t}\cos(2t)\right]_0^{\pi/8} - H$$

$$H = \frac{\sqrt{2}}{4}e^{-\pi/4} - \frac{1}{2}\left[e^{-2t}\cos(2t)\right]_0^{\pi/8} - H$$

$$2H = \frac{\sqrt{2}}{4}e^{-\pi/4} - \frac{\sqrt{2}}{4}e^{-\pi/4} + \frac{1}{2}$$

$$H = \frac{1}{4}$$

$$I + J = \int_0^{\pi/8} e^{-2t}\left[\cos^2(t) + \sin^2(t)\right]dt$$

$$= \int_0^{\pi/8} e^{-2t}dt = -\frac{1}{2}\left[e^{-2t}\right]_0^{\pi/8} = \frac{1}{2}\left(1 - e^{-\pi/4}\right)$$

$$I - J = \int_0^{\pi/8} e^{-2t}\left[\cos^2(t) - \sin^2(t)\right]dt$$

$$= \int_0^{\pi/8} e^{-2t}\cos(2t)\,dt = H = \frac{1}{4}$$

$$\begin{cases} I + J = \frac{1}{2}\left(1 - e^{-\pi/4}\right) \\ I - J = \frac{1}{4} \end{cases}$$

$$\Rightarrow \begin{cases} I = \frac{3}{8} - \frac{1}{4}e^{-\pi/4} \\ J = \frac{1}{8} - \frac{1}{4}e^{-\pi/4} \end{cases}$$

<u>Solution de L'exercice 08</u>:

I)

$$I_1 = \int_0^{\pi/2} \frac{\cos(x)}{1 + 2\sin(x)}\,dx$$

$$I_2 = \int_0^{\pi/2} \frac{\sin(2x)}{1 + 2\sin(x)}\,dx$$

Calcul de I_3 :

$$I_3 = I_1 + I_2 = \int_0^{\pi/2} \frac{\cos(x) + \sin(2x)}{1 + 2\sin(x)}\,dx$$

$$= \int_0^{\pi/2} \frac{\cos(x) + \sin(2x)}{1 + 2\sin(x)}\,dx$$

$$= \int_0^{\pi/2} \frac{\cos(x) + 2\sin(x)\cos(x)}{1 + 2\sin(x)}\,dx$$

$$= \int_0^{\pi/2} \cos(x)\,dx = \left[\sin(x)\right]_0^{\pi/2} = 1$$

Calcul de I_1 :

$$I_1 = \int_0^{\pi/2} \frac{\cos(x)}{1 + 2\sin(x)}\,dx = \frac{1}{2}\int_0^{\pi/2} \frac{2\cdot\cos(x)}{1 + 2\sin(x)}\,dx$$

$$= \frac{1}{2}\left[\ln\left|1 + 2\sin(x)\right|\right]_0^{\pi/2} = \ln\sqrt{3}$$

$$I_2 = I_3 - I_1 = 1 - \ln\sqrt{3}$$

II)

$$I = \int \frac{\cos(x)}{\cos(x) + \sin(x)}\,dx \ ; \ J = \int \frac{\sin(x)}{\cos(x) + \sin(x)}\,dx$$

$$I + J = \int \frac{\cos(x) + \sin(x)}{\cos(x) + \sin(x)}\,dx = \int dx = x + c_1$$

$$I - J = \int \frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)}\,dx = \ln\left|\cos(x) + \sin(x)\right| + c_2$$

$$\begin{cases} I + J = x + c_1 \\ I - J = \ln\left|\cos(x) + \sin(x)\right| + c_2 \end{cases}$$

$$\Rightarrow \begin{cases} I = \frac{1}{2}x + \frac{1}{2}\ln\left|\cos(x) + \sin(x)\right| + C \quad ;avec\ C = \frac{c_1 + c_2}{2} \\ J = \frac{1}{2}x - \frac{1}{2}\ln\left|\cos(x) + \sin(x)\right| + C' \quad ;avec\ C' = \frac{c_1 - c_2}{2} \end{cases}$$
